Basic Nuclear Physics and Decays

- Basic Ingredients

- Models (Fermi, Droplet, Shell)

- Decays and Stability

Source: Chapter 1 &2 , andCameron (1984)

Players and Properties of Nuclei

Nucleons:

	Mass	Spin	Charge
Proton	938.272 MeV/c^2	1/2	+1e
Neutron	939.565 MeV/c^2	1/2	0

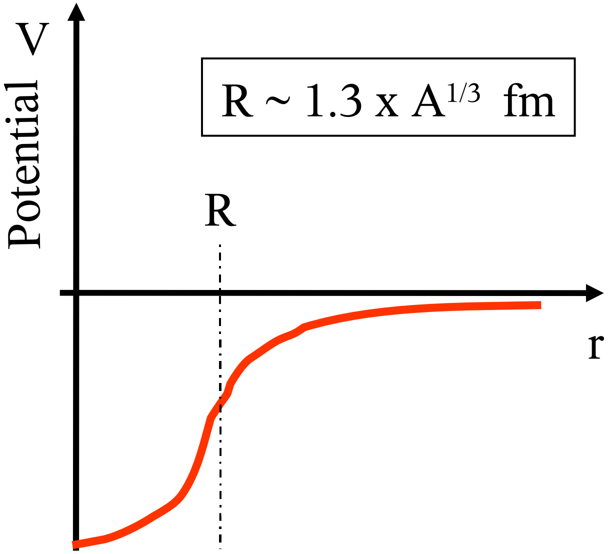
size: ~1 fm

Forces

Strong force (range ~ 1 fm) and electromagnetic force

Nuclear Potentials

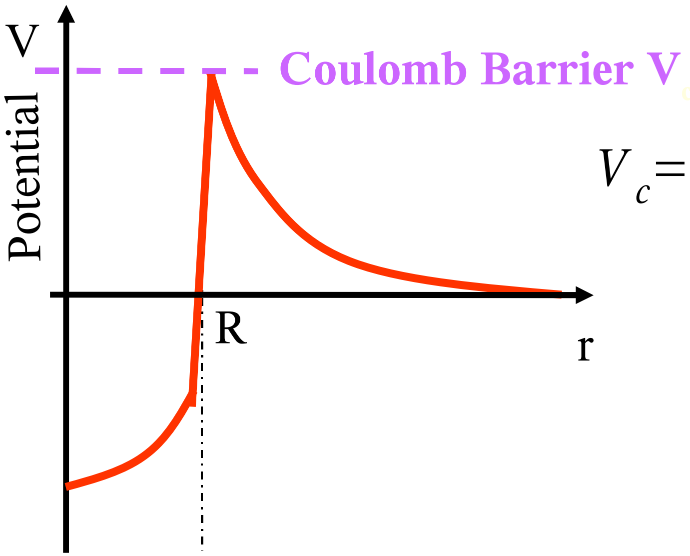
neutron



R ~ 1.3 x $A^{1/3}$ fm

proton

(or any other charged particle)



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

Rem: Nuclear radii R are determined by electron scattering experiments

Measured Core Radii from Electron Scattering

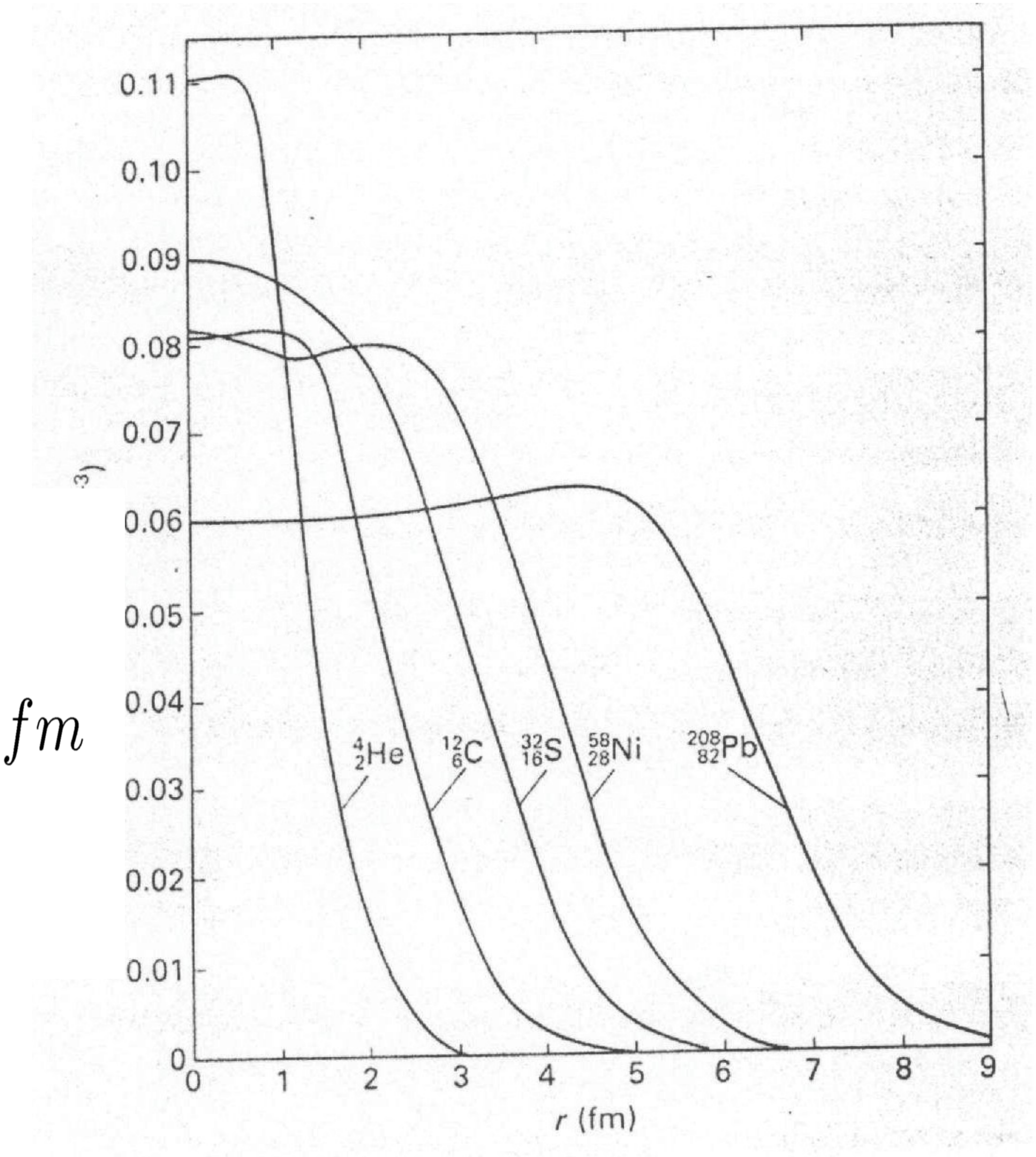
(see also Cross Sections)

Charge density can be approximated by a Saxon-Woods shape within about 10 %

$$\rho(r) = \frac{\rho_o}{1+e^{r-R}/a_V}$$

with $R = 1.18 \times A^{1/3} - 0.048 fm$

and $a_V = 0.055 \pm 0.07 fm$



Binding Energy B of Nuclei with Z protons, N neutrons and Mass M(Z,N)

$$\boxed{B(Z,N) = Zm_p + Nm_n - M(Z,N).}$$

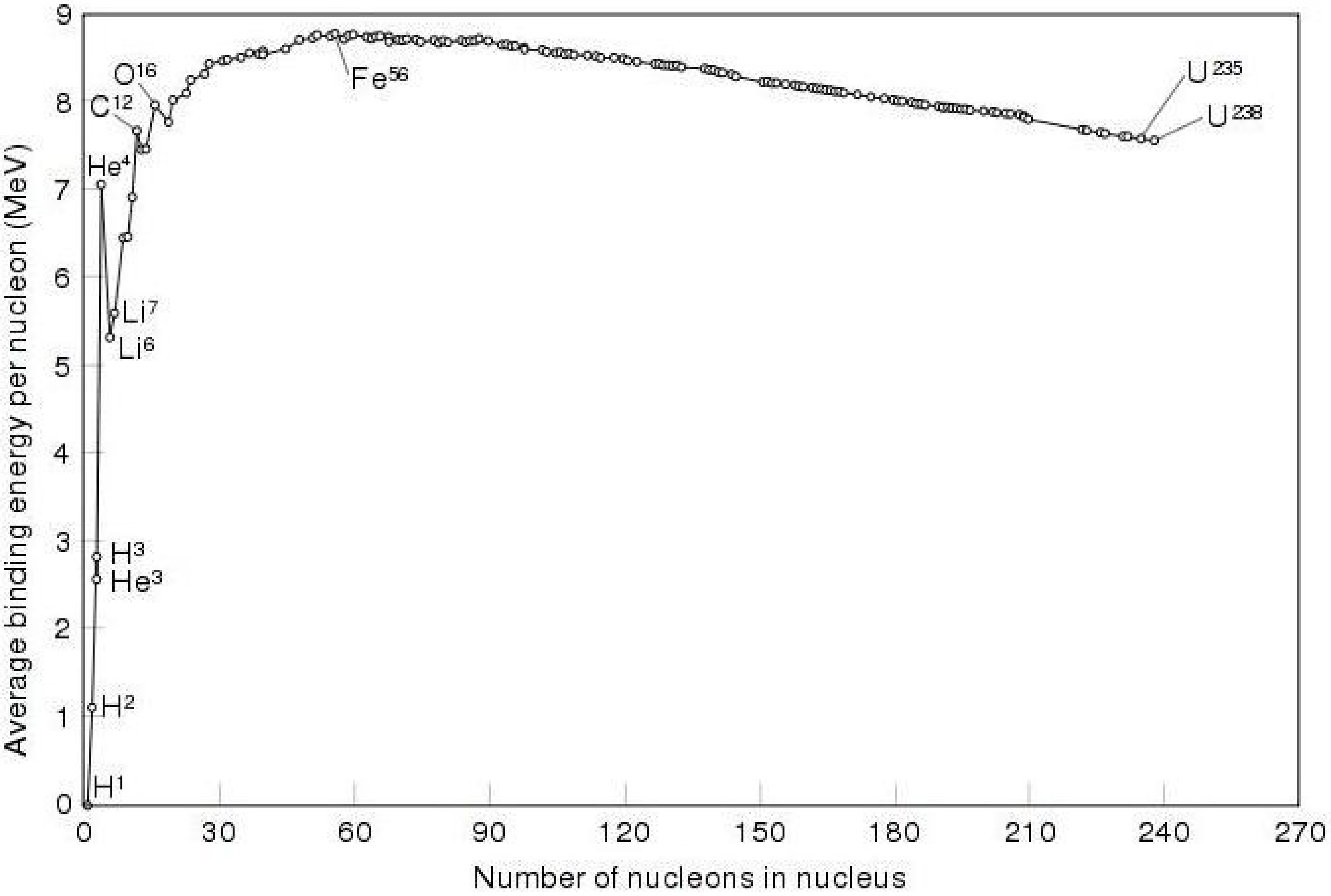
with E=m c^2, masses are measured in MeV.

Remark: Mass excess and atomic mass unit

$M_{ex}(Z,N) = M(Z,N) - A\, m_u$

u

Measured Nuclear Binding Energies of Stable Nuclei



Remark: Binding energy is released by nuclear burning

The Independent Particle Model

Assumption: Equilibrium distribution of cold Fermi-gas in a box like potential (last week, and see blackboard for details)

Phase Space

$$\Phi(E) = \frac{4\pi}{3}\frac{g}{h^3}(2m)^{3/2}E^{3/2}$$

$$\omega(E) = 2\pi\frac{g}{h^3}(2m)^{3/2}E^{1/2}.$$

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\frac{N}{V}\right)^{2/3}.$$

Mean binding energy per nucleon:

$$\bar{E} = \frac{1}{N}\int_0^{E_F} E \leq (E)dE = \frac{1}{\Phi(E_F)}\int_0^{E_F} E \leq (E)dE$$

$$= \left(\frac{8\pi}{3}\frac{(2m)^{3/2}}{h^3}E_F^{3/2}V\right)^{-1} \cdot \frac{2}{5}\frac{4\pi}{h^3}(2m)^{3/2}E_F^{5/2} \cdot V$$

$$\bar{E} = \frac{3}{5}E_F.$$

=> Mean energy/nucleon 8-12MeV

Potential V = 40MeV

Droplet Model (Nuclear Mass Formula)

(Weizsacker, 1935, Bethe and Backer 1936)

$$B(Z,A) = a_V A$$

Volume Effect (B = A (Potential – Ebar)

$$-a_s A^{2/3}$$

Surface Effect (less neighbors for nuclear at surface)

$$-a_C \frac{Z^2}{A^{1/3}}$$

Coulomb repulsion of protons

$$-a_A \frac{(Z - A/2)^2}{A}$$

Asymmetry Effect from Pauli exclusion (see blackboard)

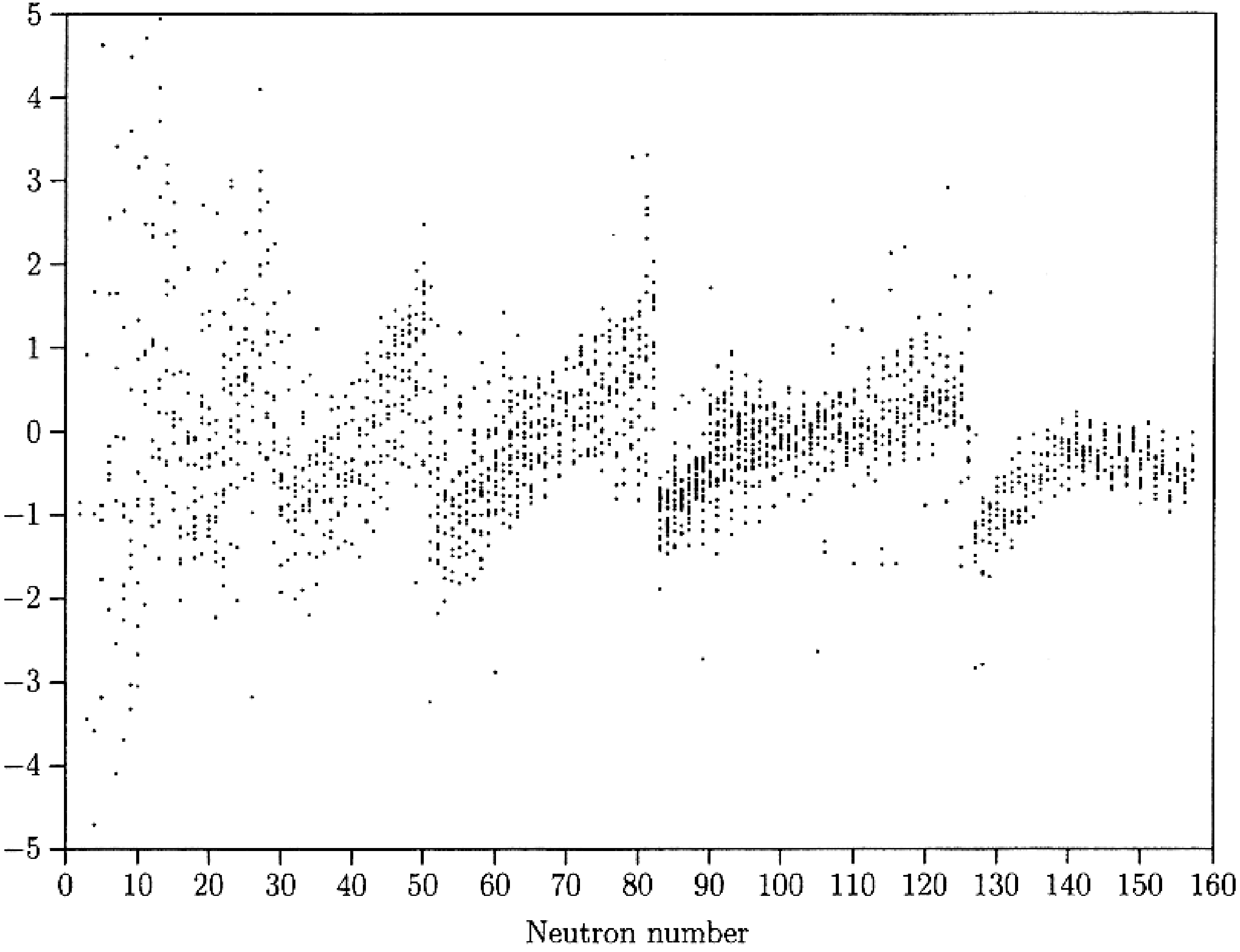
$$\begin{array}{ll} +\frac{12}{\sqrt{A}} & \text{if even-even} \\ 0 & \text{if odd A} \\ -\frac{12}{\sqrt{A}} & \text{if odd-odd.} \end{array}$$

Cooper Pairs (pairing gap of Nucleons)

$$B(Z,N) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_{sym} \frac{(N-Z)^2}{A} + B_{pair}(Z,N)$$

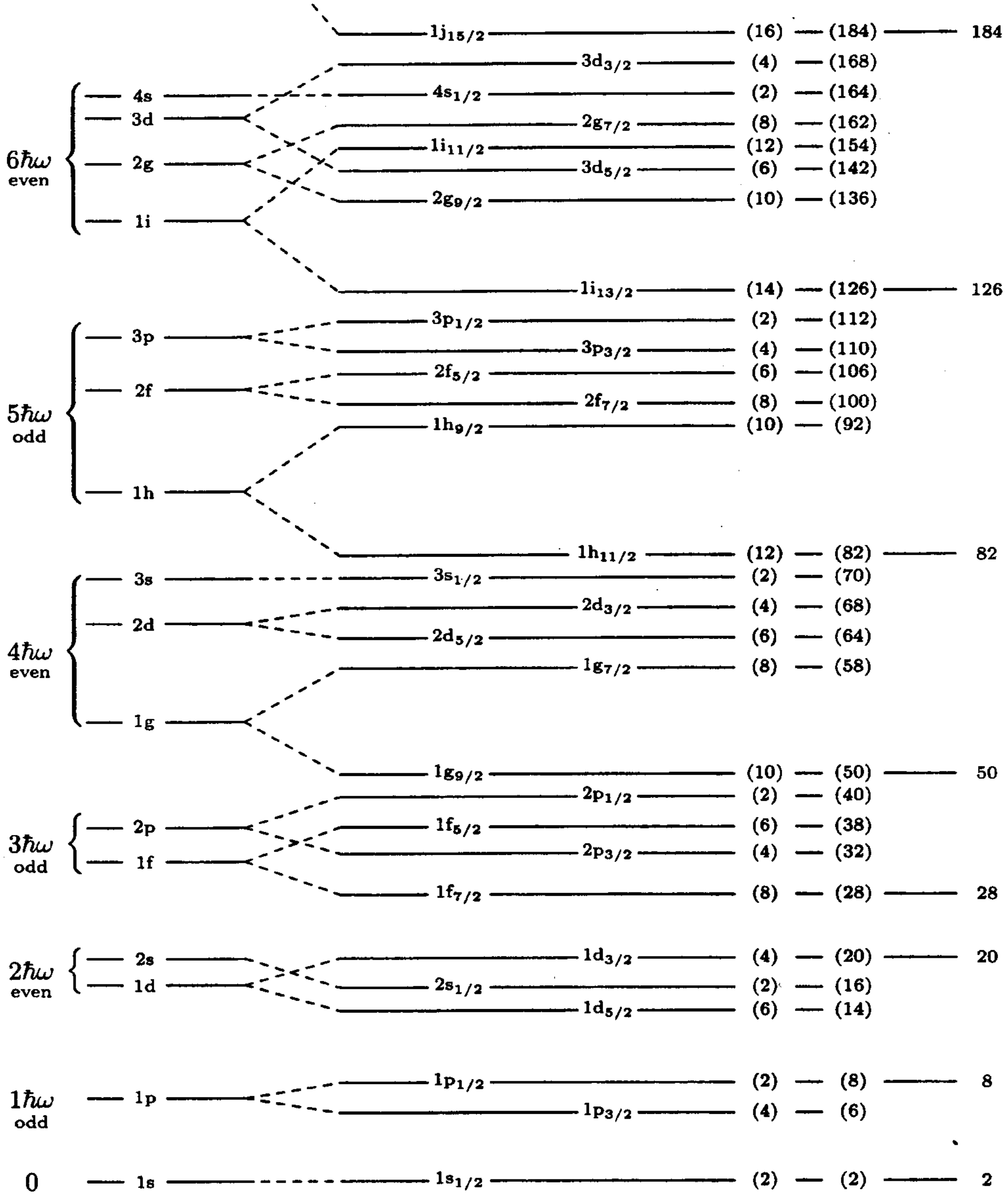
$$a_V = 16 \text{ MeV}, \; a_S = 18.5 \text{ MeV}, \; a_C = 0.72 \text{ MeV}, \; \text{and} \; a_{sym} = 23.4 \text{ MeV}.$$

Differences between observed and predicted B(Z,N) [MeV]



(from Moeller et al. 1994)

Solution: Shell Model and Magic Numbers (e.g. Moeller et al. 94)



- Non-uniform distribution of level by splitting

- Gaps in E

- coupling

-Magic numbers

Nuclear Decay Modes and Stability (closed system)

Necessary Conditions:

1) Energy gain

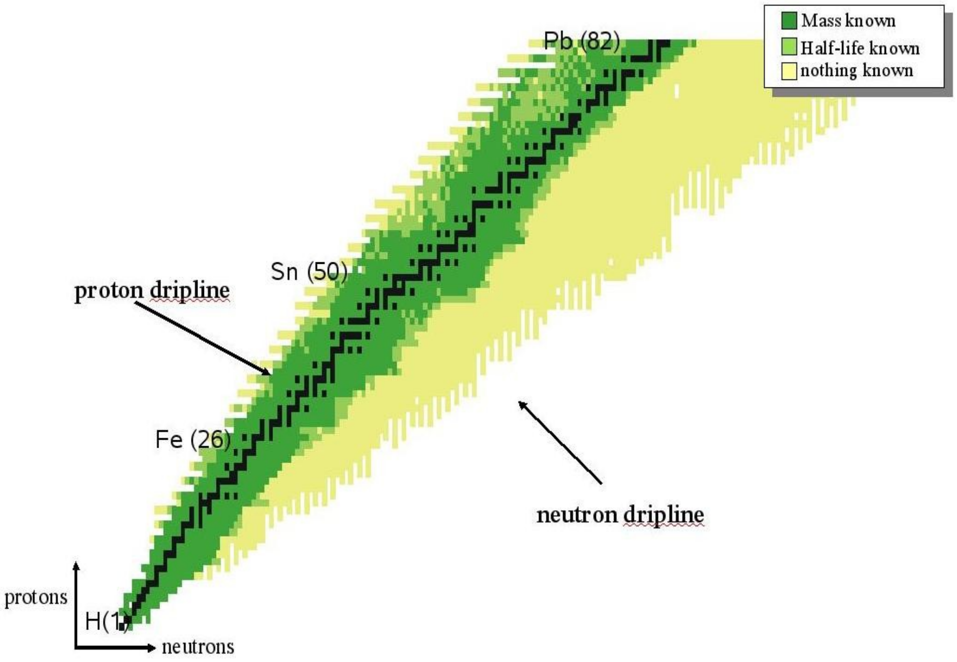
$$Q = \sum_{i=initial} M_i - \sum_{i=final} M_i > 0.$$

Example: p or n emission $(Z,A) \rightarrow (Z,A-1)+n$ or $(Z,A) \rightarrow (Z-1,A-1)+p$

Drip lines of n and p

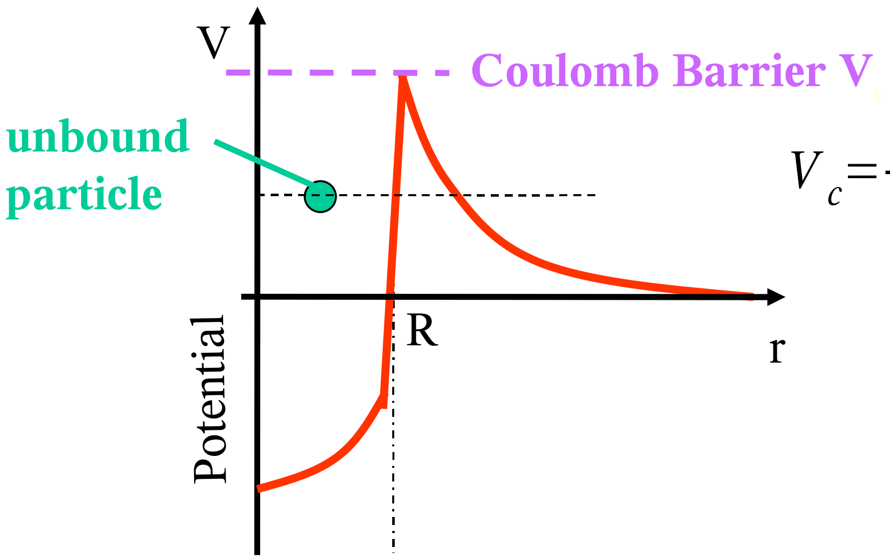
$$S_n(Z,A) = M(Z,A-1) + m_n - M(Z,A)$$

$$S_p(Z,A) = M(Z-1,A-1) + m_p - M(Z,A).$$



2) Transition must exist

The Role of the Coulomb Barrier & Lifetime



$$V_c = \frac{Z_1 Z_2 e^2}{R}$$

Example: for ^{197}Au -> ^{58}Fe + ^{139}I   has Q ~ 100 MeV !

=> Small tunneling probability may cause life
life times of the Universe => Element is stable

Main Decays for Nuclei between Drip Lines I

a) Strong decays (short time scales)

(i) *alpha decay* $(Z, A) \rightarrow (Z-2, A-4) +^4 \mathrm{He}$

$$M_{nuc}(^4\mathrm{He}) + M_{nuc}(Z-2, A-4) < M_{nuc}(Z, A)$$

$$M_{at}(^4\mathrm{He}) + M_{at}(Z-2, A-4) < M_{at}(Z, A)$$

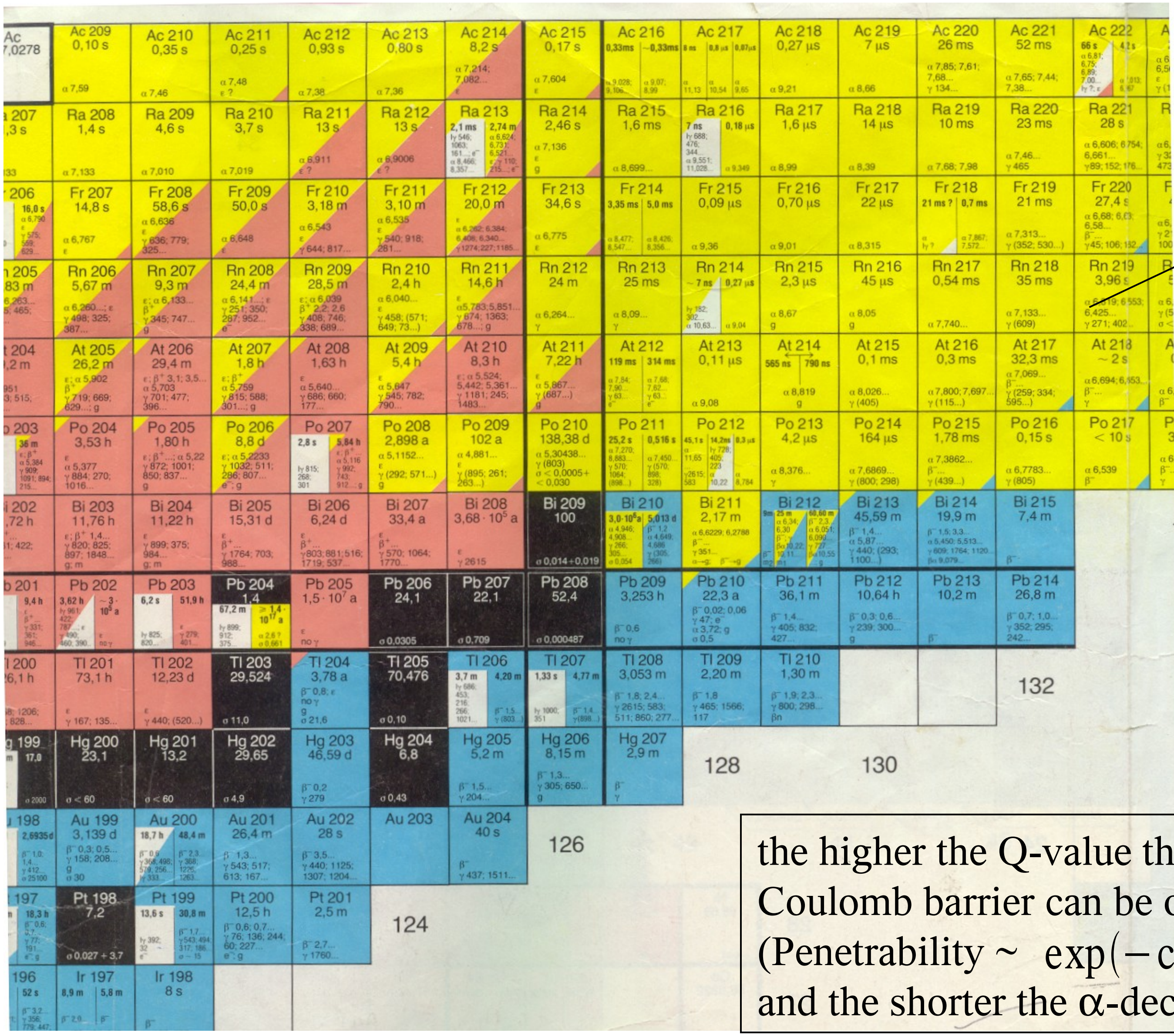
(ii) *fission* $(Z, A) \rightarrow 2(\approx Z/2, \approx A/2)$

$$2 \cdot M_{nuc}(Z/2, A/2) < M_{nuc}(Z, A)$$

$$2 \cdot M_{at}(Z/2, A/2) < M_{at}(Z, A)$$

lightest α emitter: ^{144}Nd (Z=60)   (Qα= 1 .9 MeV but still $T_{1/2}$=2.3 x 10^{15} yr)

beyond Bi α emission ends the valley of stability !

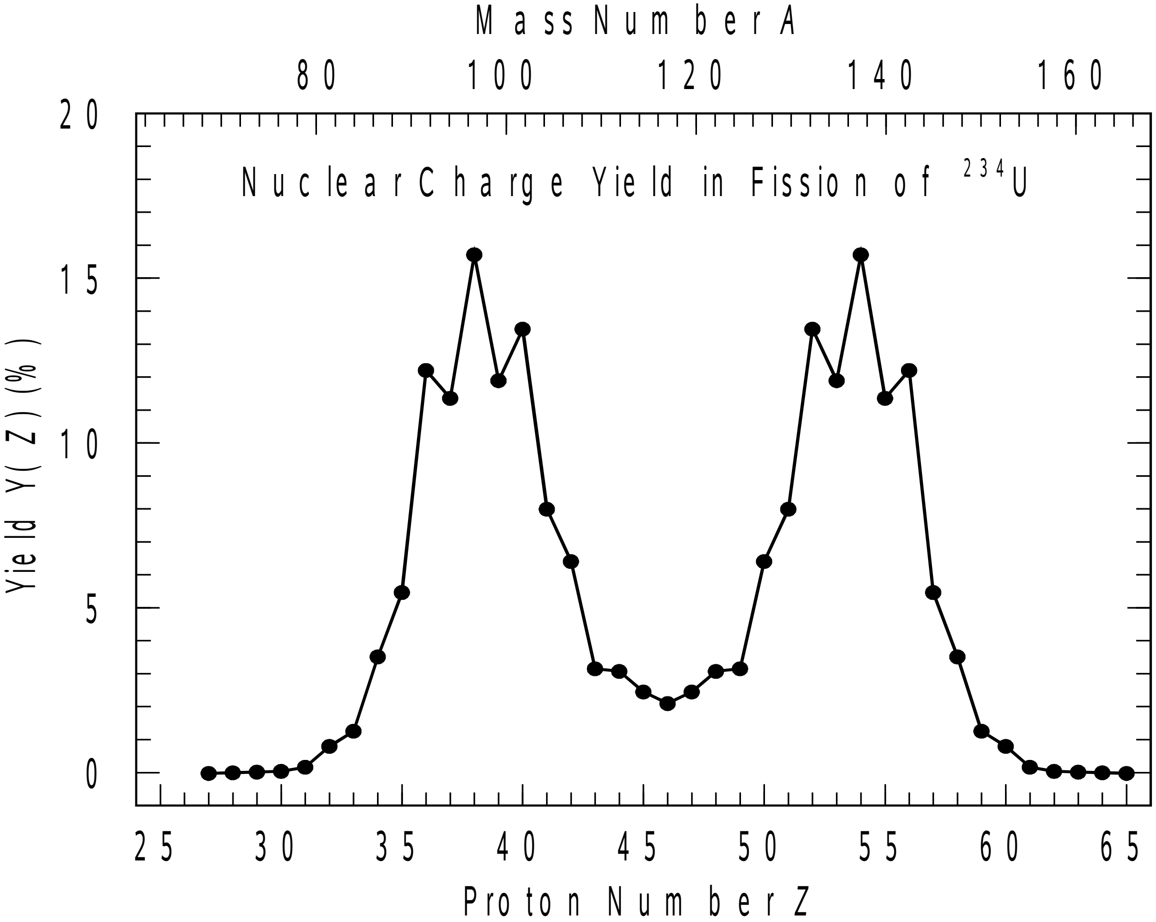


yellow
are α emitter

the higher the Q-value the easier the Coulomb barrier can be overcome (Penetrability ~ $\exp\left(-\mathrm{const}\cdot E^{-1/2}\right)$) and the shorter the α-decay half-lives

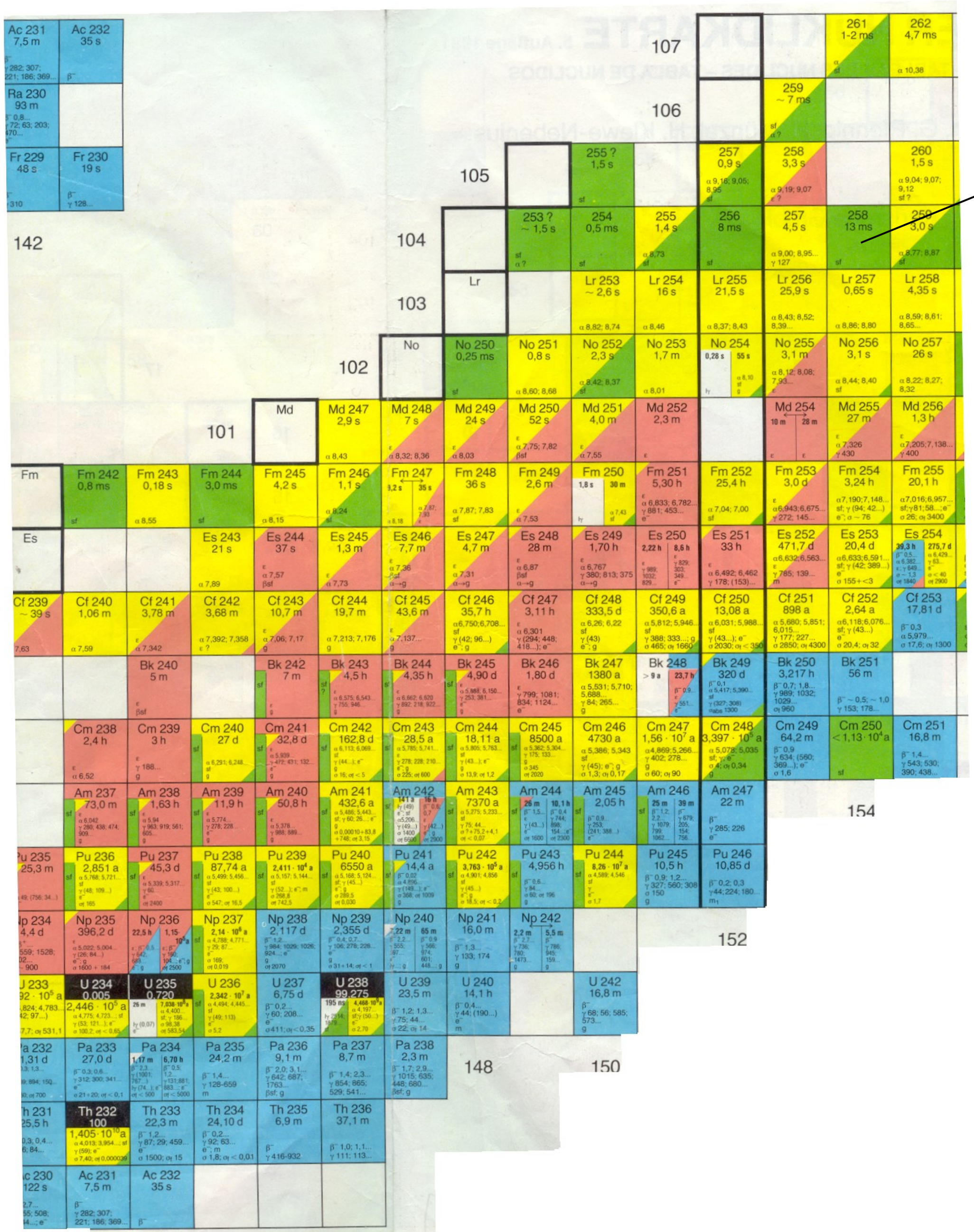
Remark on Fission of Heavy Nuclear (Z>>56):

Example: ^{234}U from Moeller et al. Nature (2001)



- mass formula -> ½ favored
- Shell effects

Fission in the (Z,N) plane



green = spontaneous fission

Main Decays for Nuclei between Drip Lines II

b1) weak Interactions (long time scales)

(iii) β^- $decay\ (Z,A) \rightarrow (Z+1,A) + e^- + \bar{\nu}_e$

$$M_{nuc}(Z,A) > M_{nuc}(Z+1,A) + m_e$$

$$M_{at}(Z,A) > M_{at}(Z+1,A)$$

(iv) β^+ $decay\ (Z,A) \rightarrow (Z-1,A) + e^+ + \nu_e$

$$M_{nuc}(Z,A) > M_{nuc}(Z-1,A) + m_e$$

$$M_{at}(Z,A) > M_{at}(Z-1,A) + 2m_e$$

Main Decays for Nuclei between Drip Lines II

b2) weak Interactions (long time scales)
Electron capture

(v) *electron capture* $(Z,A)+e^- \rightarrow (Z-1,A)+\nu_e$

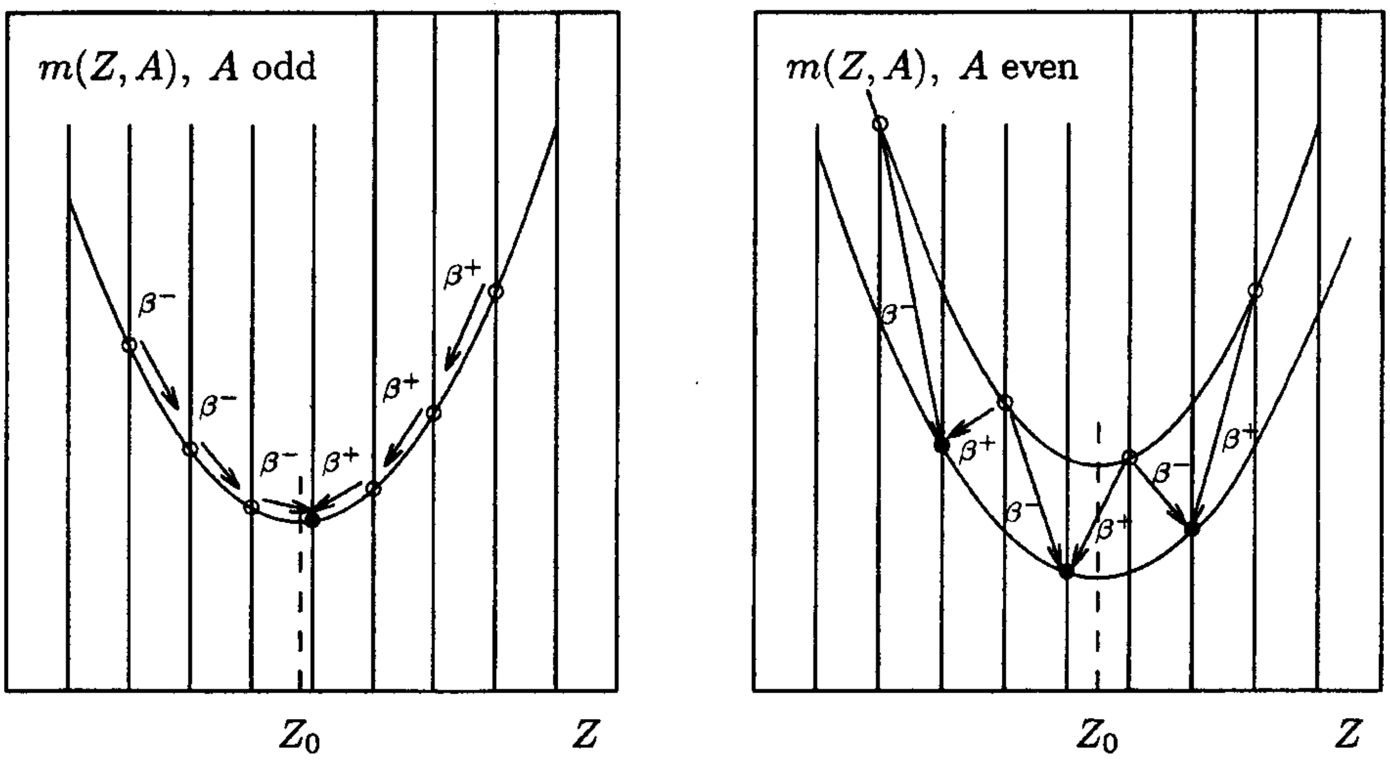
$$M(Z,A)+m_e > M(Z-1,A)$$

$$M_{at}(Z,A) > M_{at}(Z-1,A)$$

Remark: Electron Capture has a Q value of 2 m_e lower than decays => happens mostly in environments with high density and temperatures (T>1MeV , >1E9 K).

The Valley of Stability via Beta Decays

beta decays don't have to overcome Coulomb barrier => always happen (with lifetimes of weak interaction)

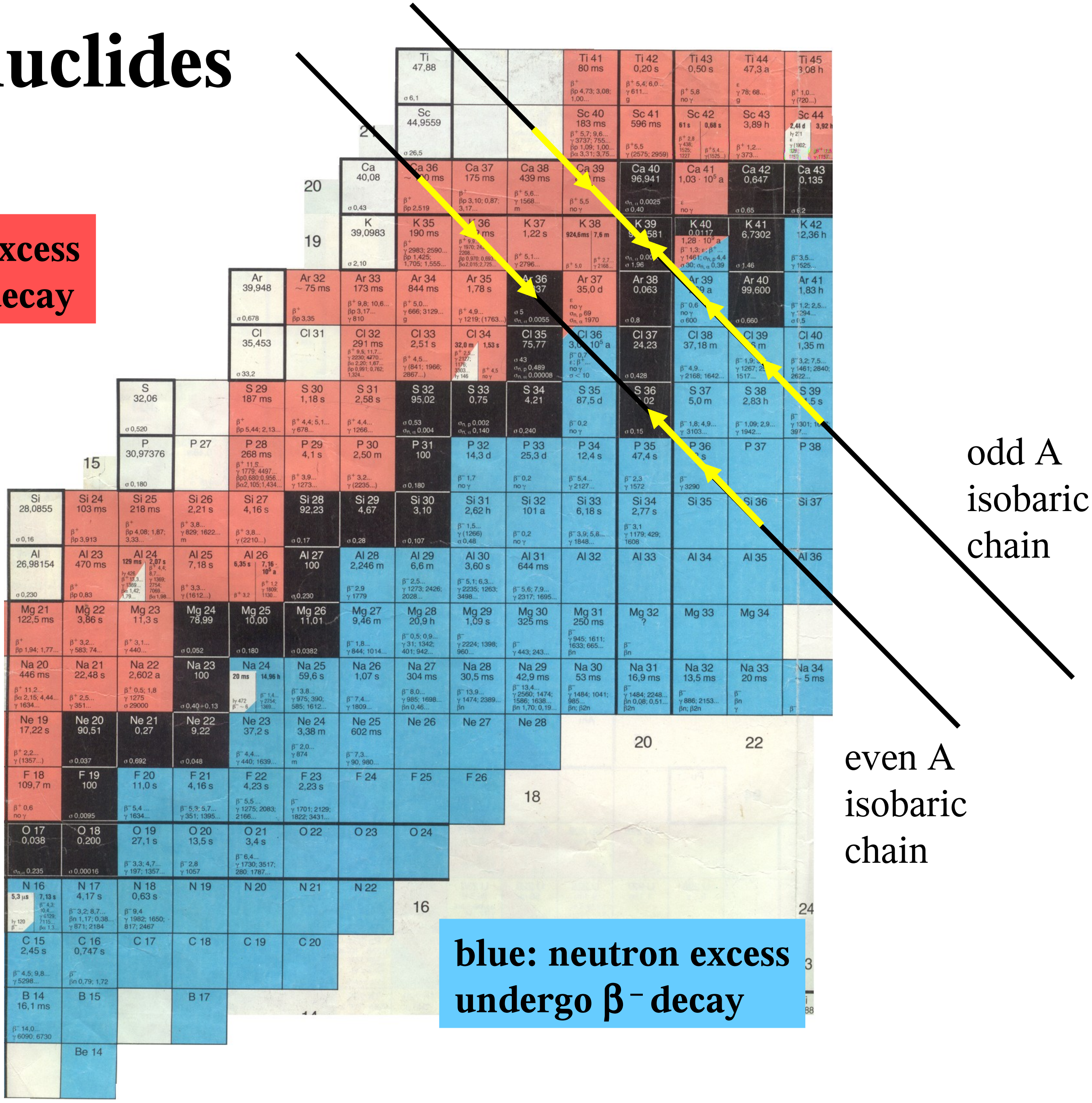


(from Cameron, 1983)

Chart of nuclides

(Lanl)

red: proton excess undergo β⁺ decay



odd A isobaric chain

even A isobaric chain

blue: neutron excess undergo β⁻ decay

Z

N

Mass Formula and Abundances in Nature

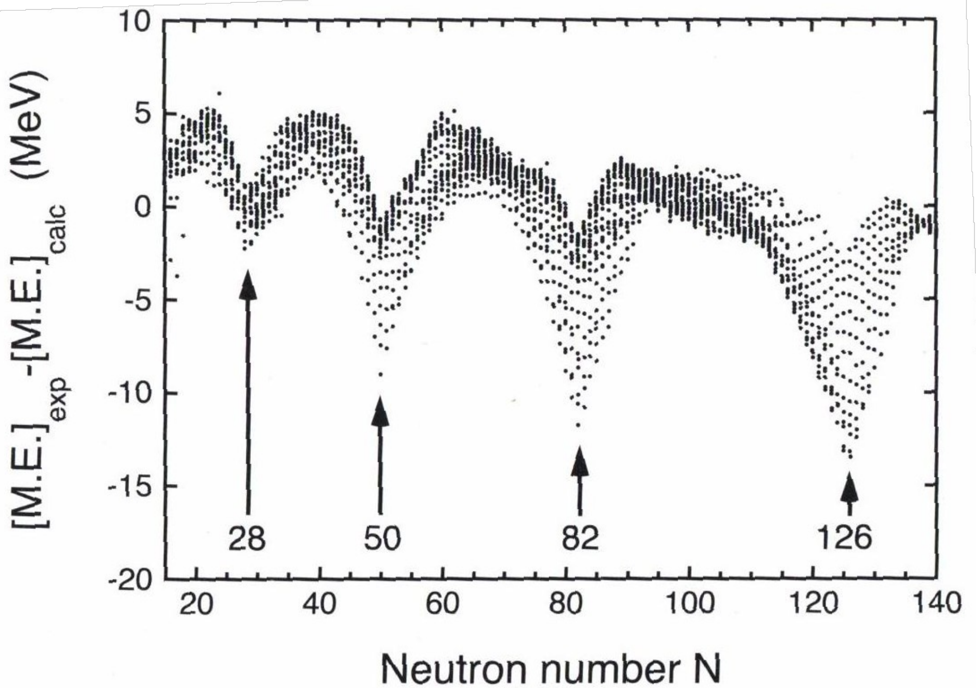


Fig. 1.11 Difference between experimental ground-state atomic mass excess (Audi et al. 2003) and the mass excess predicted by the spherical macroscopic part of the finite-range droplet (FRDM) mass formula (Möller et al. 1995) versus neutron number.