Stellar Structure & Evolution

- Stellar Properties

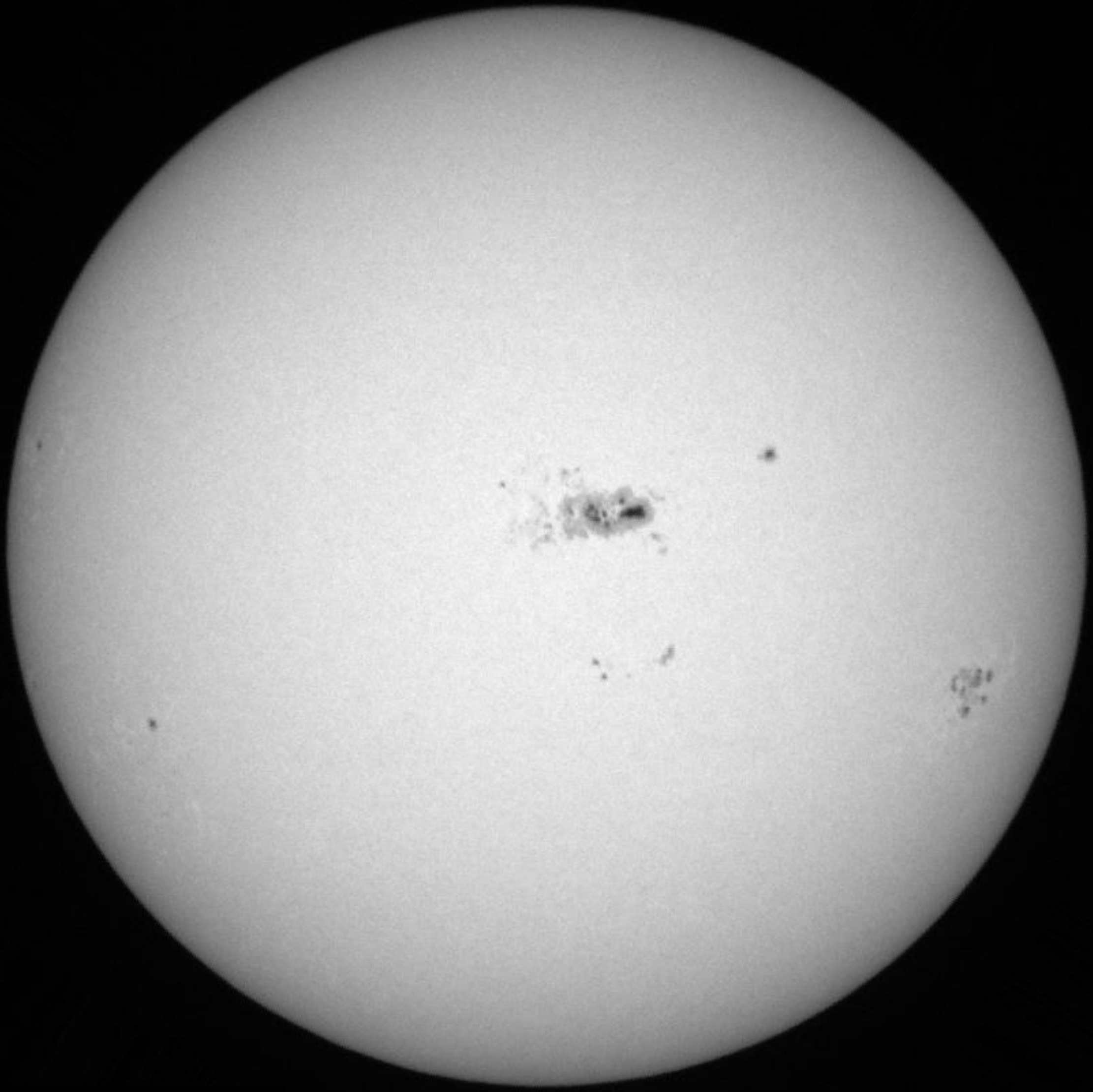
- The Hertzsprung-Russel Diagram

- Stellar Structure

- Stellar Evolution

See Caroll/Oslie: Introduction to Astrophysics
Clayton: Chapters 2.4, 3

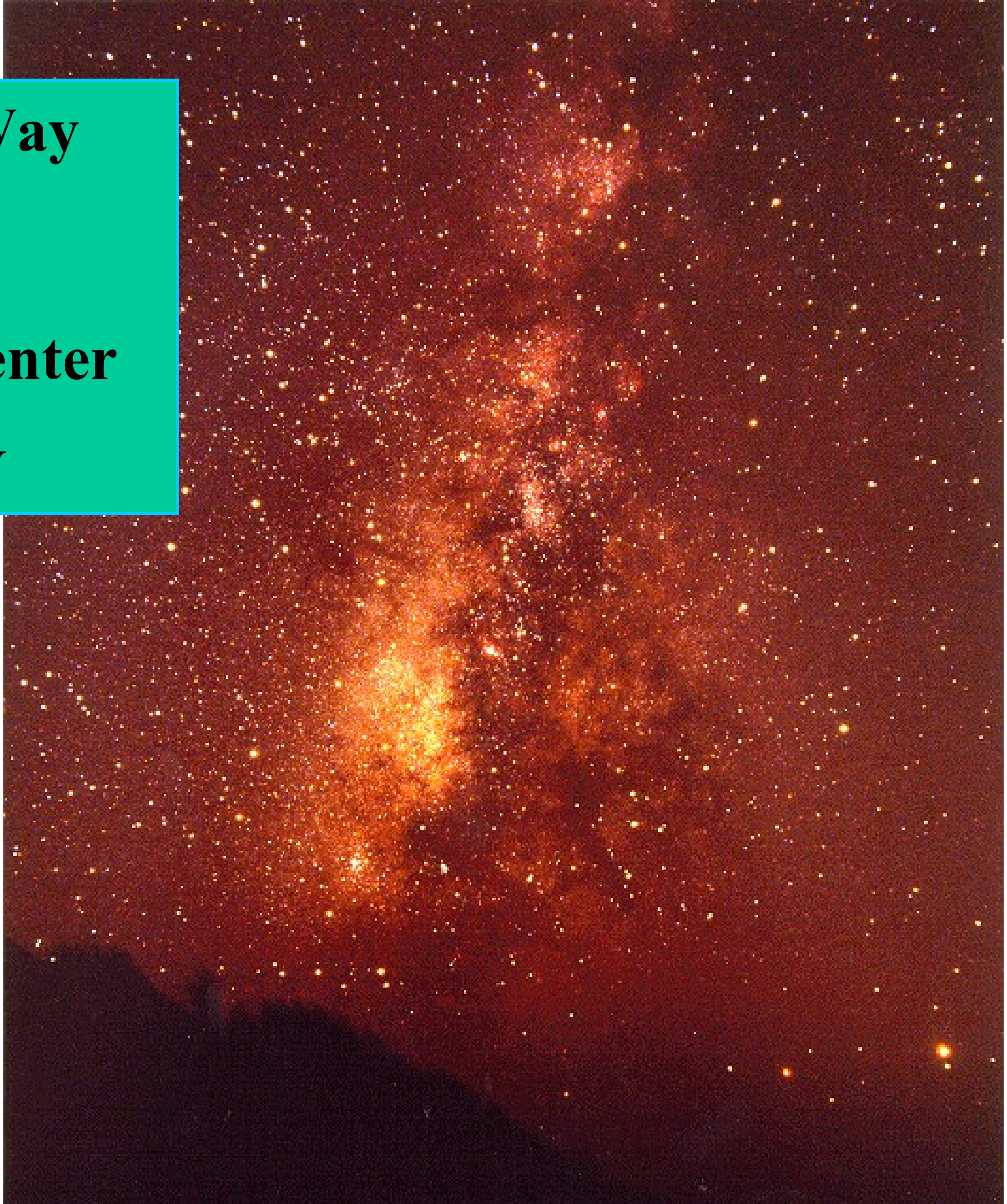
The Sun at Visible Wavelengths



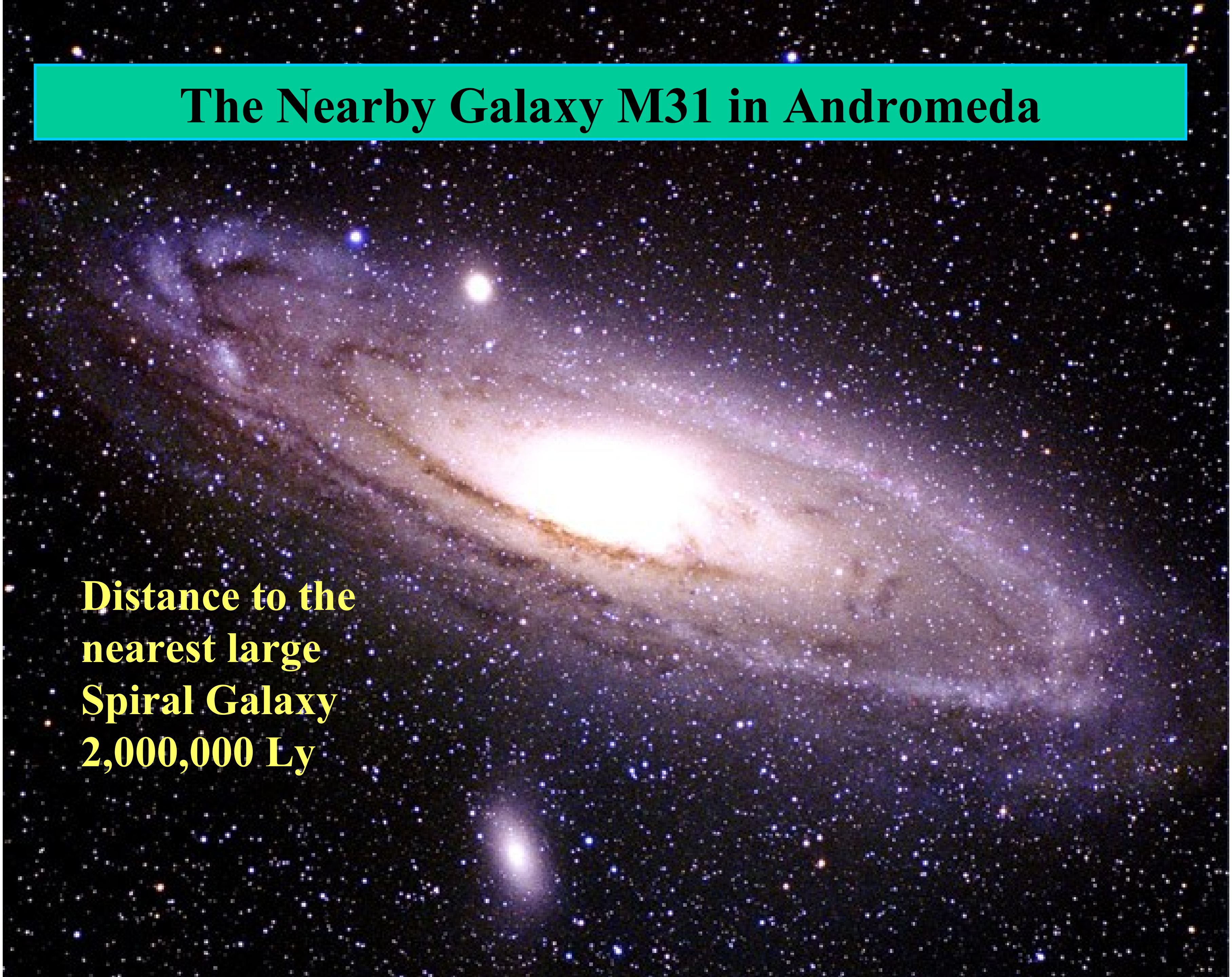
The Milky Way

Distance to Center

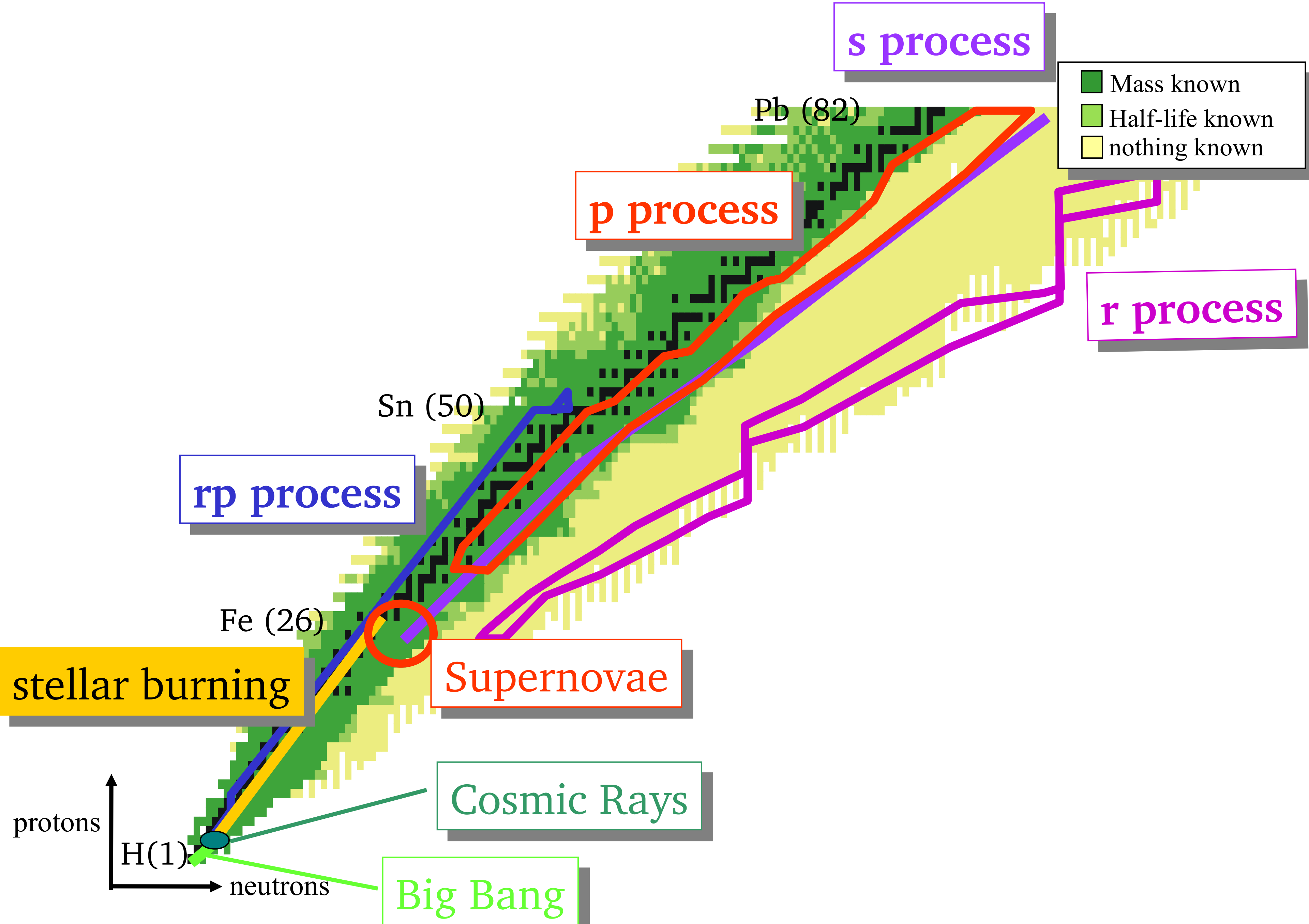
10,000 LY



The Nearby Galaxy M31 in Andromeda



Distance to the
nearest large
Spiral Galaxy
2,000,000 Ly



s process

🟩 Mass known

🟩 Half-life known

🟨 nothing known

Pb (82)

p process

r process

Sn (50)

rp process

Fe (26)

stellar burning

Supernovae

Cosmic Rays

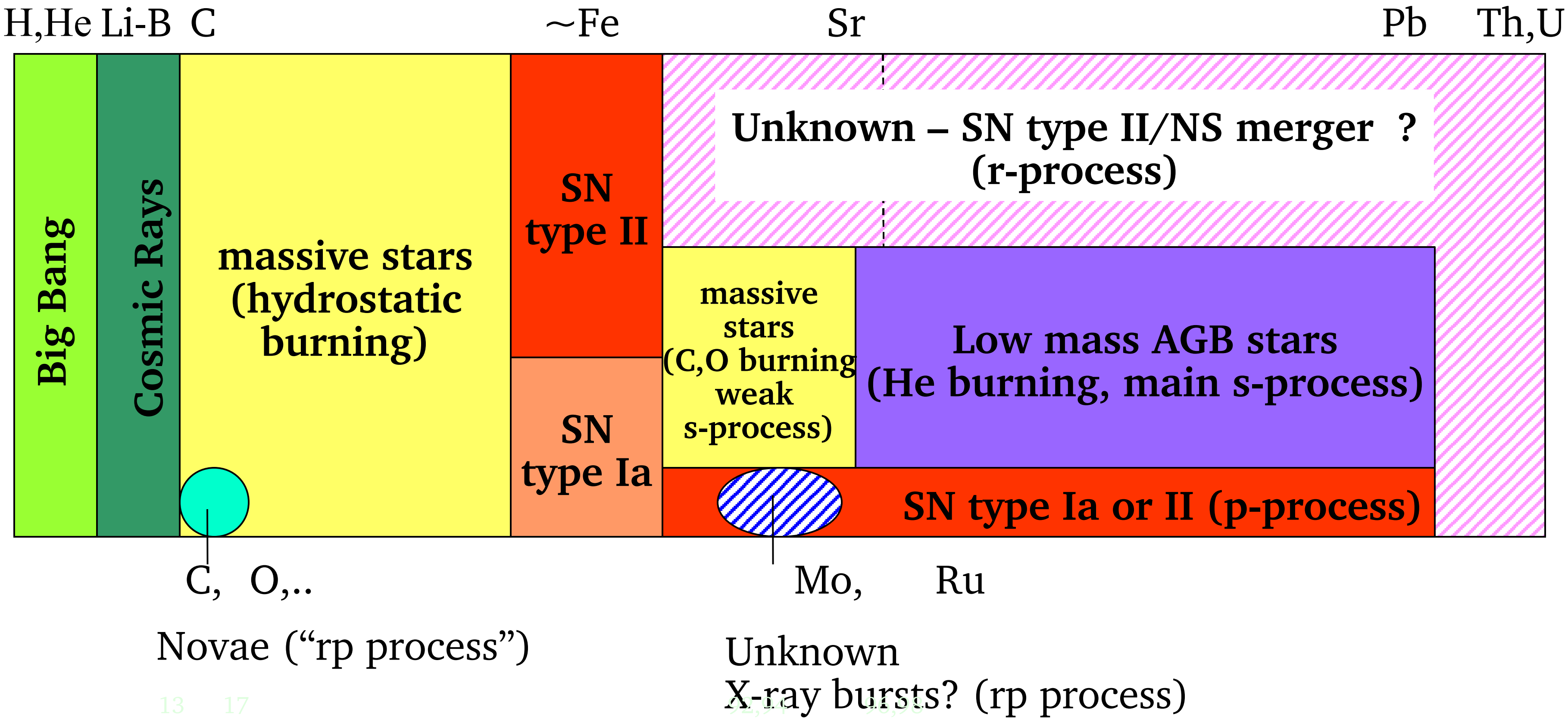
protons

H(1)

neutrons

Big Bang

Where ? Or The Origin of the Elements



H,He Li-B C ~Fe Sr Pb Th,U

Big Bang

Cosmic Rays

massive stars (hydrostatic burning)

SN type II

SN type Ia

Unknown – SN type II/NS merger ?
(r-process)

massive stars (C,O burning weak s-process)

Low mass AGB stars (He burning, main s-process)

SN type Ia or II (p-process)

^{13}C, ^{17}O,..

Novae (“rp process”)

^{92,94}Mo, ^{96,98}Ru

Unknown
X-ray bursts? (rp process)

Possible type II SN (ν -process) contribution to ….

Note: yellow-red all related to massive stars (>8-12 solar masses at ZAMS)

M16

Star-Birth Clouds · M16

HST · WFPC2

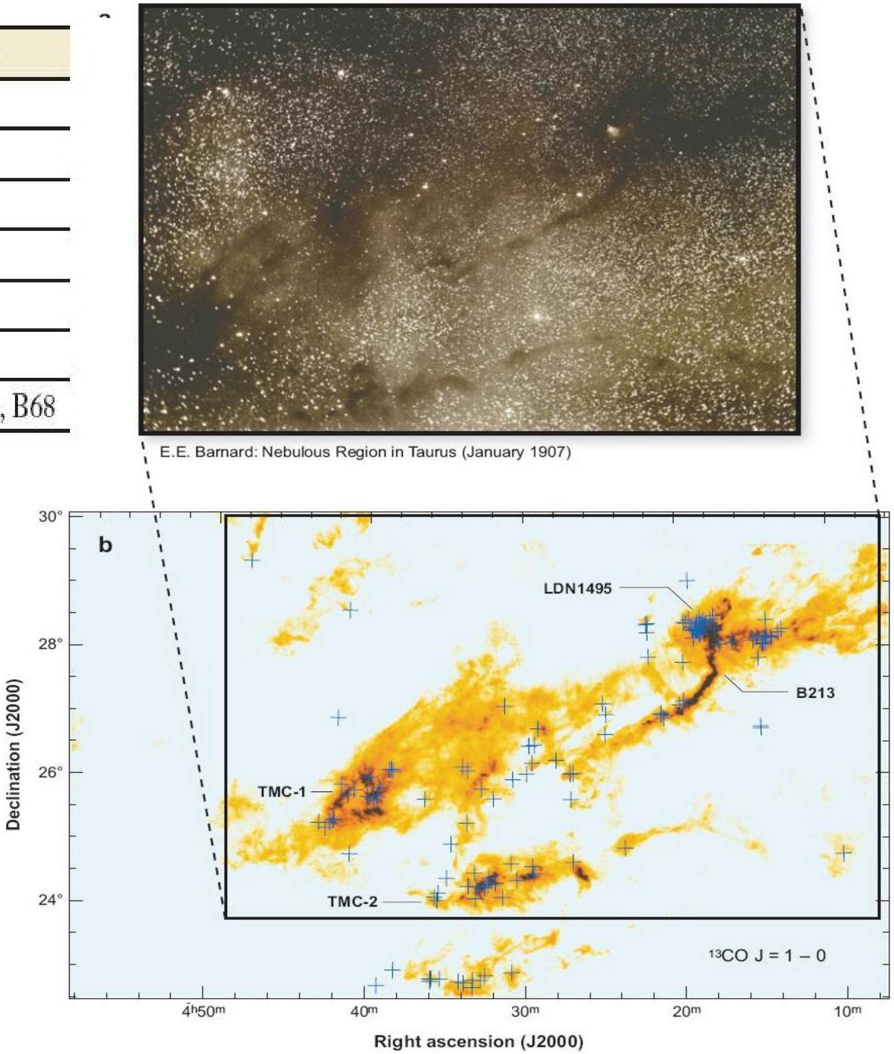
PRC95-44b · ST ScI OPO · November 2, 1995

J. Hester and P. Scowen (AZ State Univ.), NASA

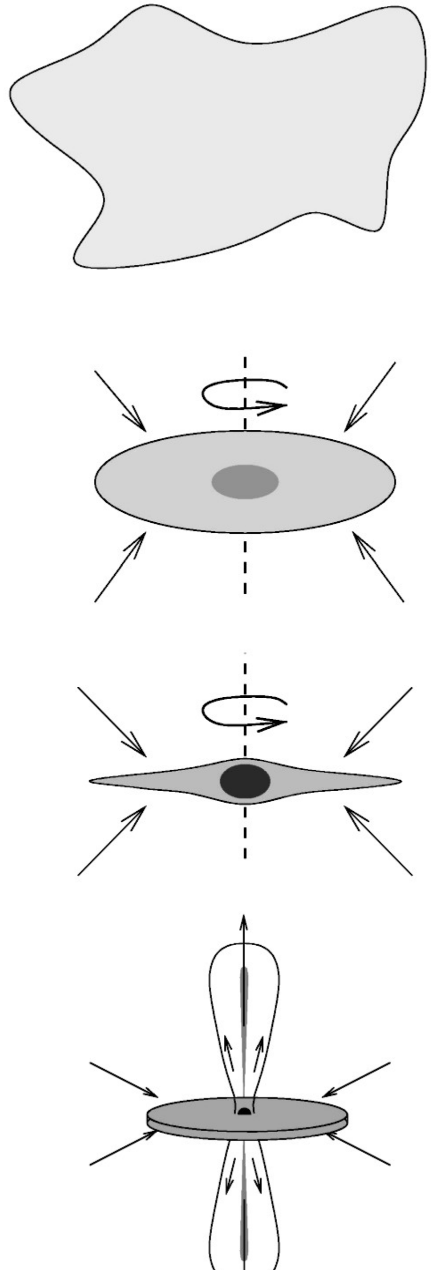
Molecular Clouds & Star Formation

	Clouds[a]	Clumps[b]	Cores[c]
Mass ($M_\odot$)	$10^3 - 10^4$	50–500	0.5–5
Size (pc)	2–15	0.3–3	0.03–0.2
Mean density (cm^{-3})	50–500	10^3–10^4	10^4–10^5
Velocity extent ($km\ s^{-1}$)	2–5	0.3–3	0.1–0.3
Crossing time (Myr)	2–4	≈1	0.5–1
Gas temperature (K)	≈10	10–20	8–12
Examples	Taurus, Oph, Musca	B213, L1709	L1544, L1498, B68

Complex systems with
various components including
- HII regions
- molecular (CO2) regions
- dark clouds


E.E. Barnard: Nebulous Region in Taurus (January 1907)

The Formation of Stars: Collapse from an Interstellar Cloud



0 years

A dense cloud of
cold interstellar gas
rotating slowly

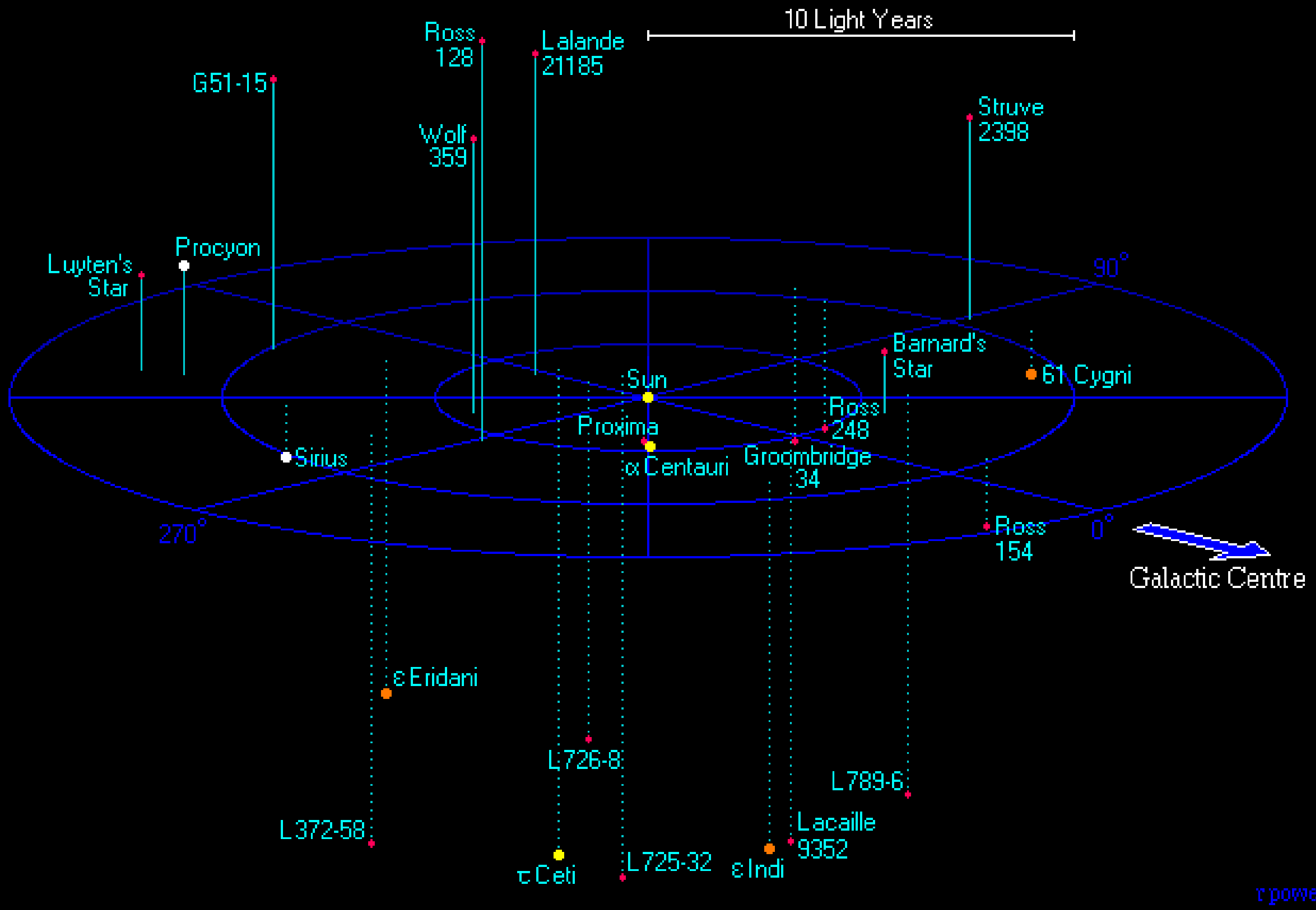
Free-fall collapse.
Speed of rotation
increases.

10^5 yrs

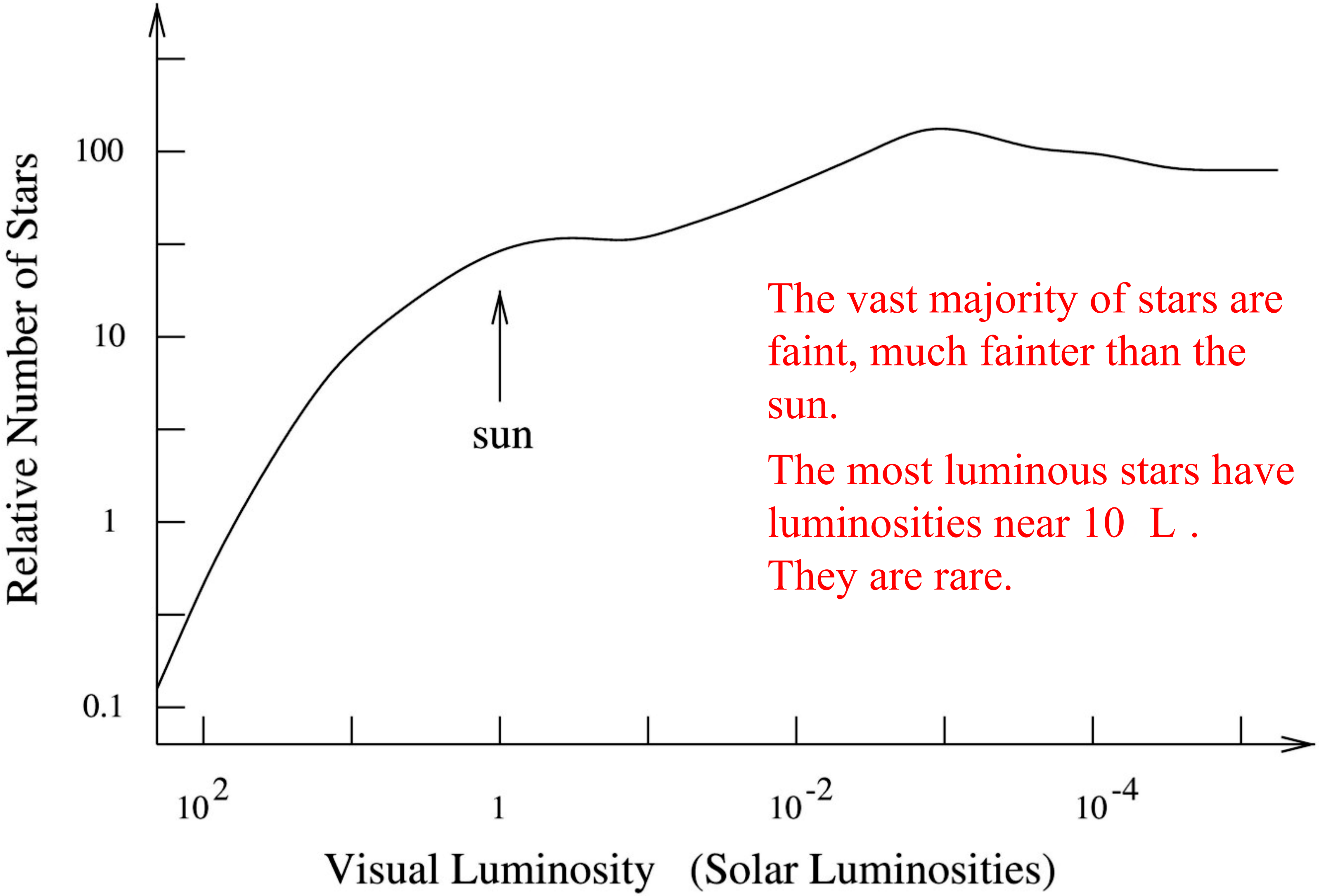
Core becomes hot,
free-fall collapse
ends, protostar forms.

Slow contraction,
protoplanetary disk
forms, mass loss in
polar winds and jets

Stars Within 4 Parsecs of the Sun



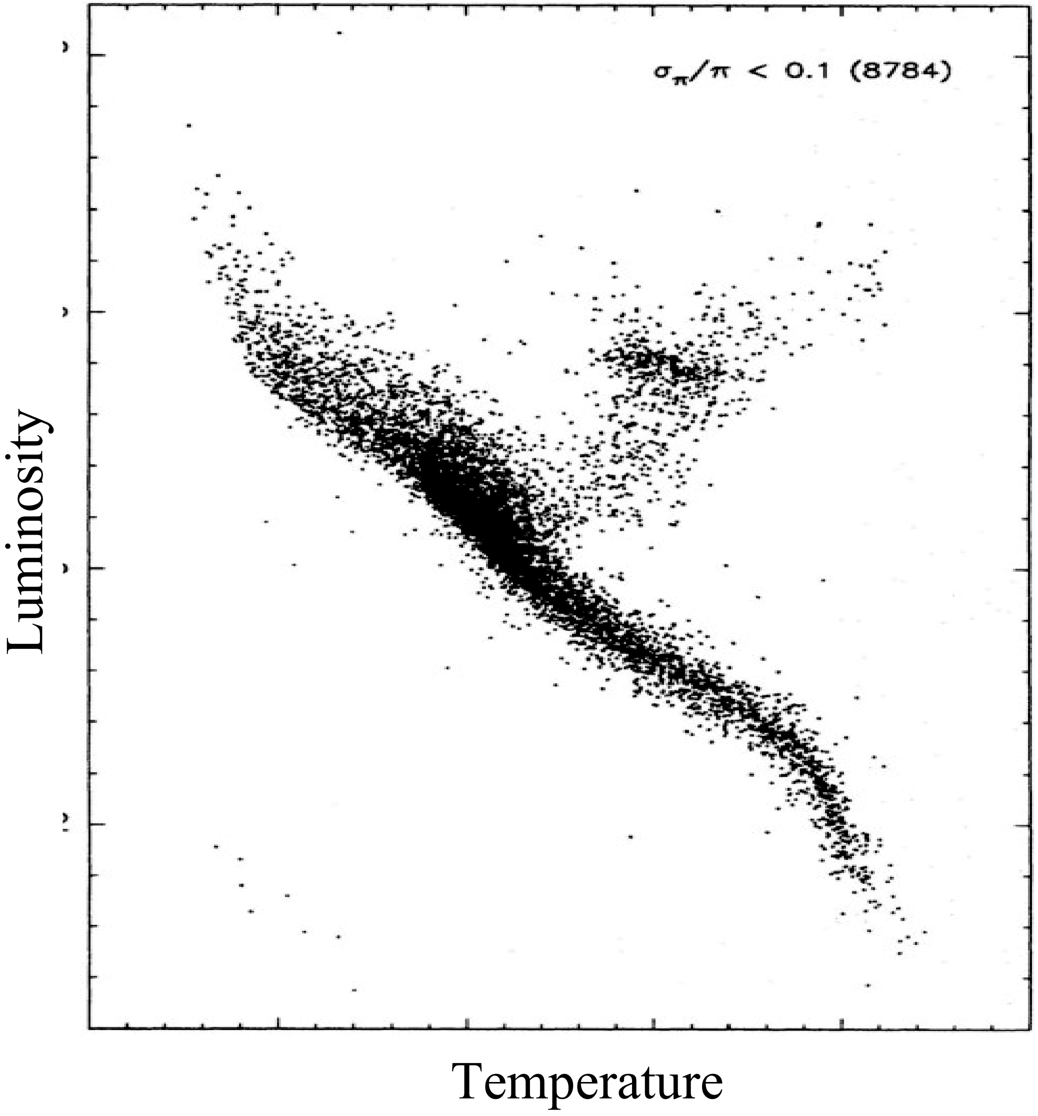
The Luminosity Function



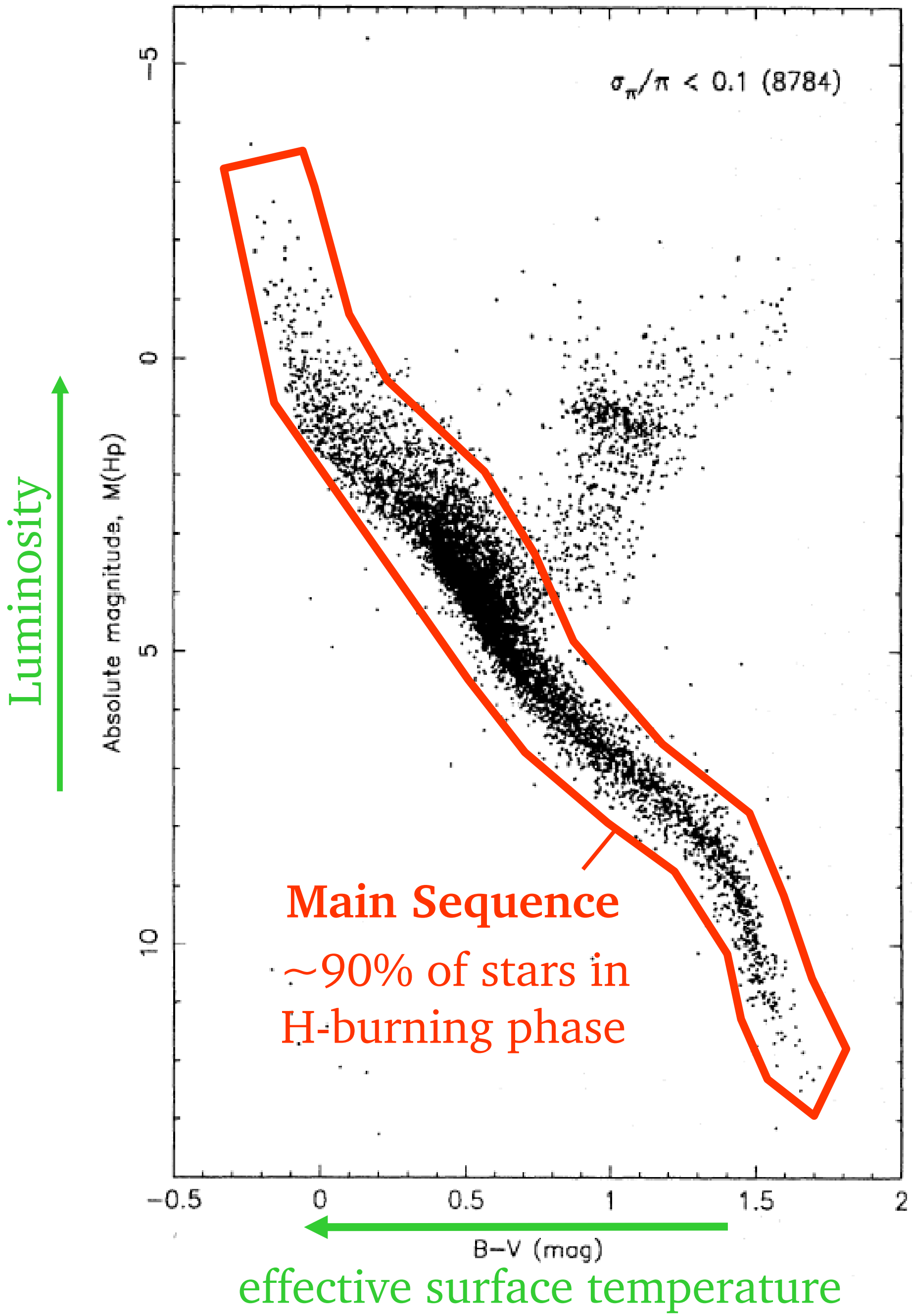
The vast majority of stars are faint, much fainter than the sun.

The most luminous stars have luminosities near 10 L .
They are rare.

The Real H-R Diagram for Stars near the Sun



Observational Hertzsprung-Russell diagram



Main Sequence

~90% of stars in H-burning phase

Perryman et al. A&A 304 (1995) 69

HIPPARCOS distance measurements

Magnitude:

Measure of stars brightness

Def: difference in magnitudes m from ratio of brightnesses b:

$$m_2 - m_1 = 2.5 \log \frac{b_1}{b_2}$$

(star that is x100 **brighter** has by 5 **lower** magnitude)

absolute scale historically defined
(Sirius: -1.5, Sun: -26.72
naked eye easy: <0, limit: <4)

absolute magnitude is measure of luminosity = magnitude that star would have at 10 pc distance

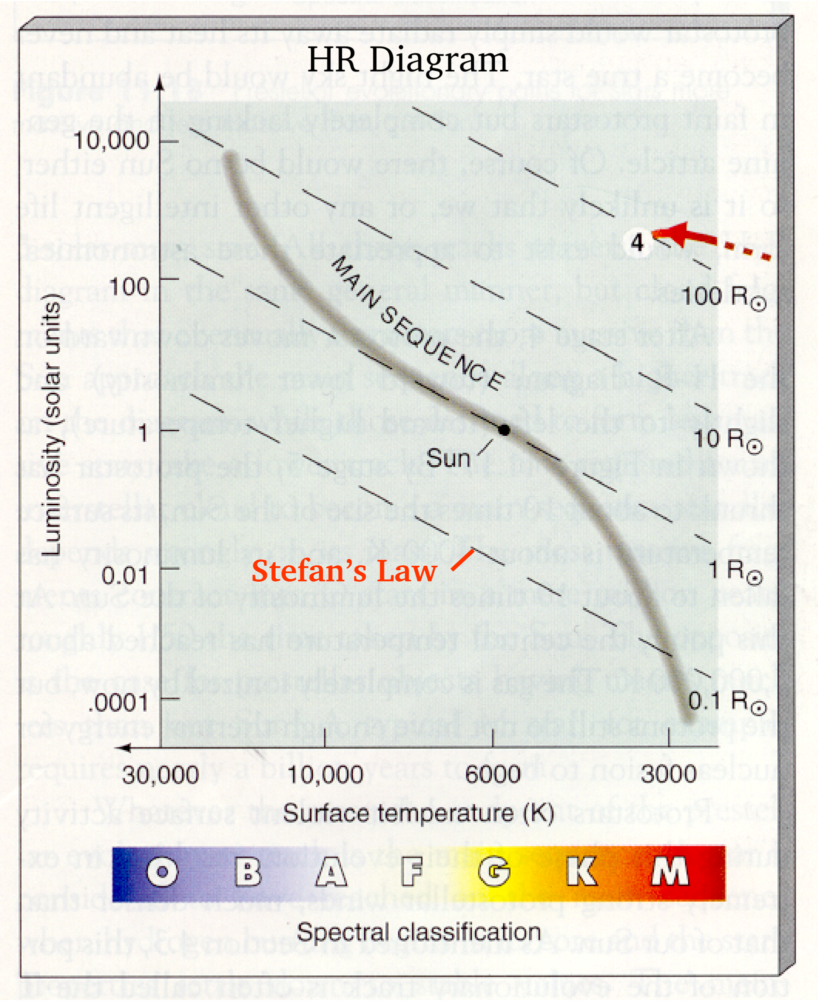
Sun: + 4.85

Temperature,Luminosity, Mass relation on the Main Sequence:

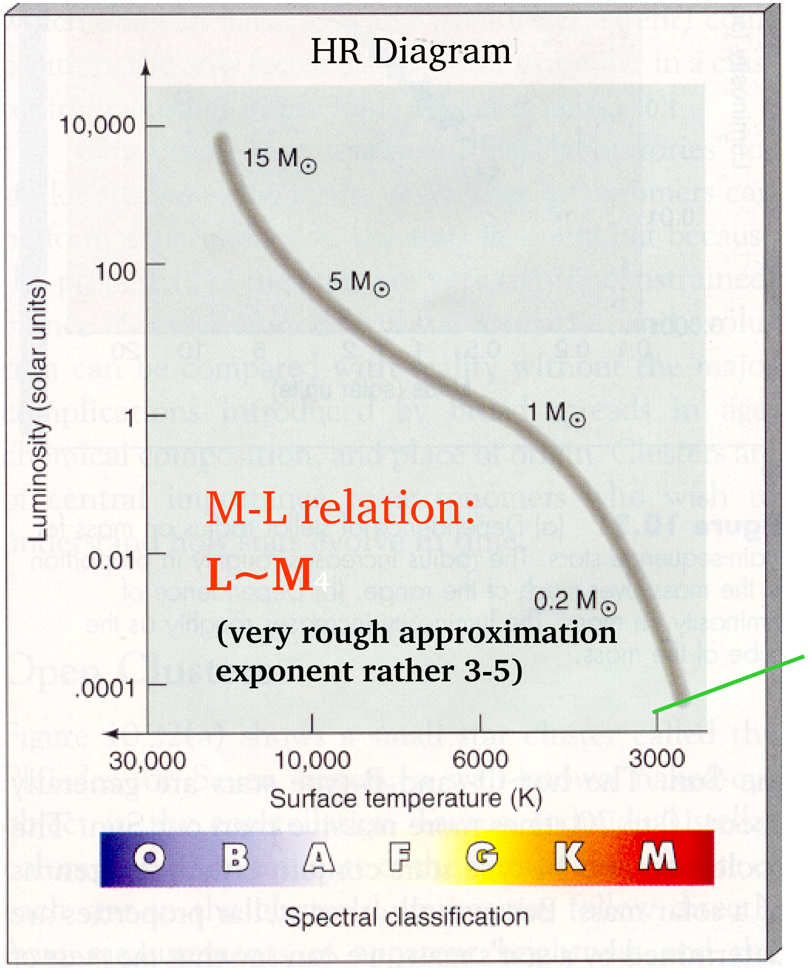
It turns out that as a function of mass there is a rather unique relationship between

- surface temperature (can be measured from contineous spectrum)
- luminosity (can be measured if distance is known)

(recall Stefan's Law L~R T , why does Main Sequence deviate from it ?)



Stefan's Law



M-L relation:

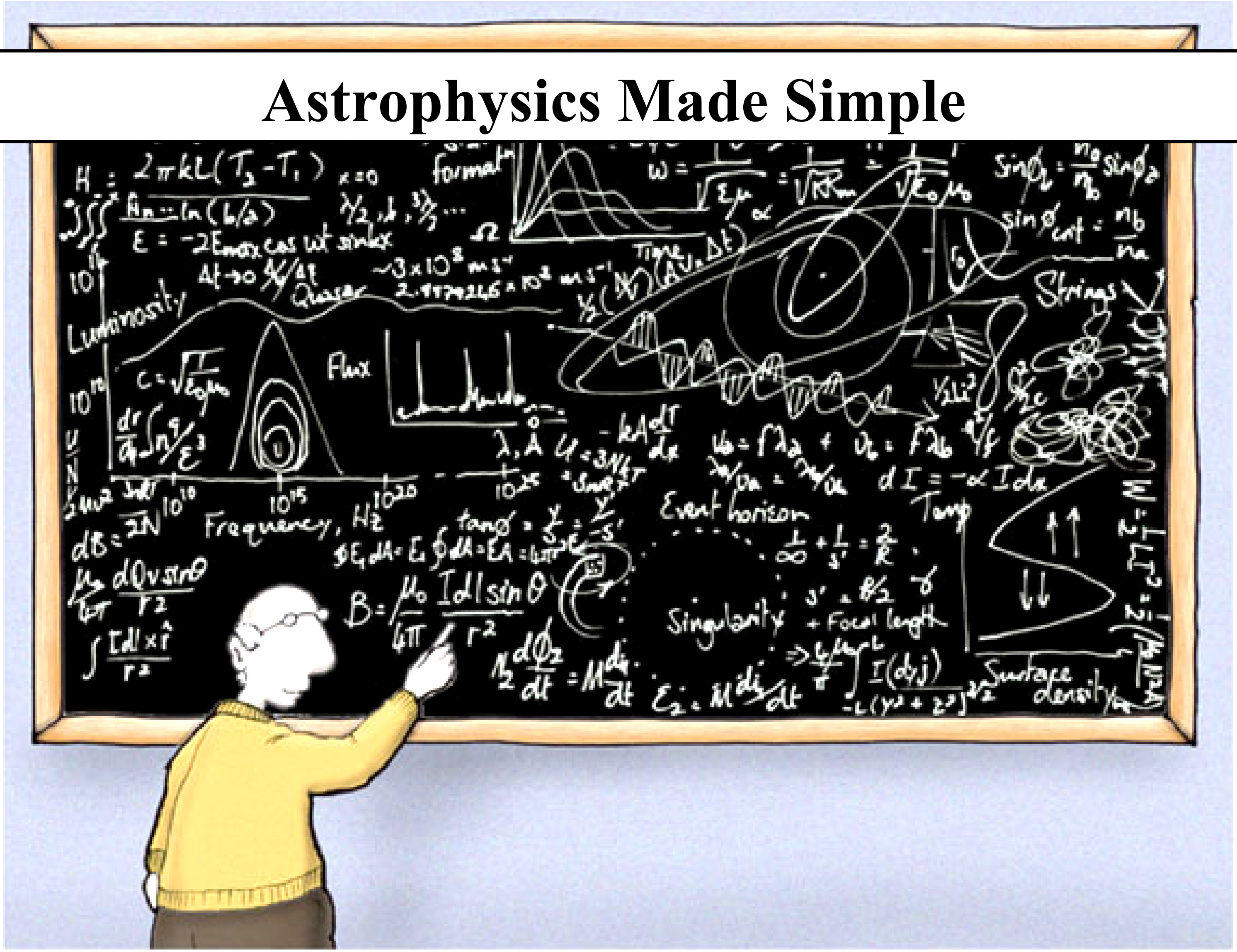
$L\sim M_4$

(very rough approximation exponent rather 3-5)

cutoff at ~0.08 M

(from Chaisson McMillan)

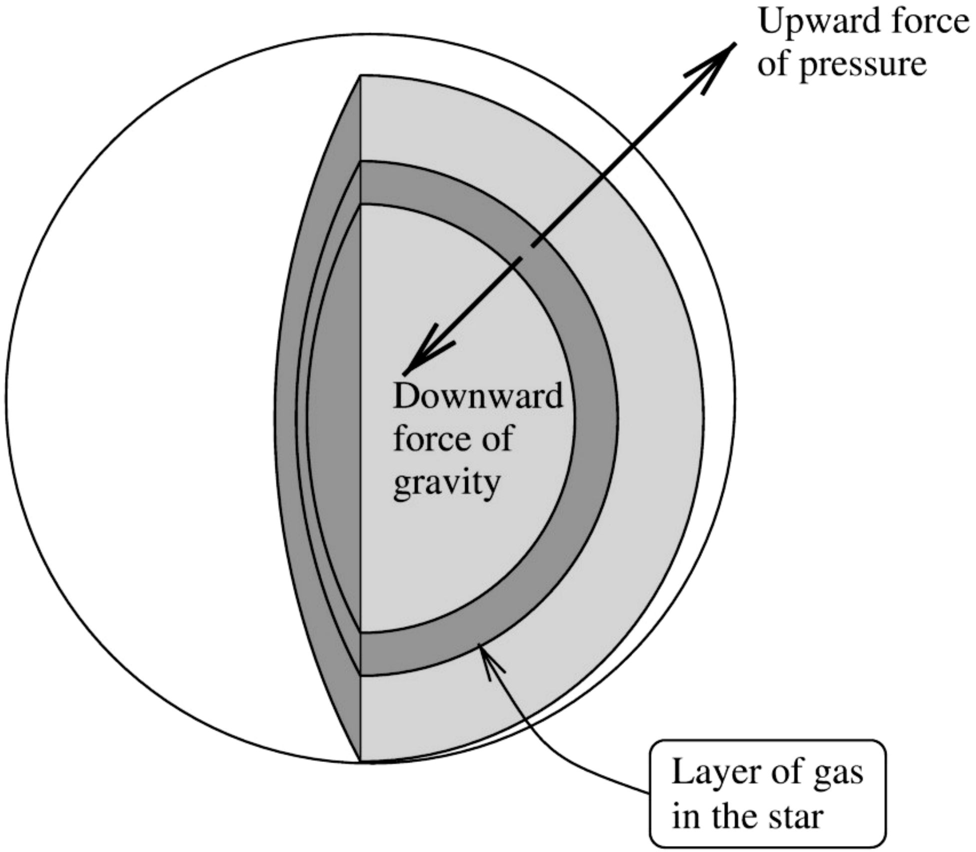
Astrophysics Made Simple



Astrophysics made simple

Hydrostatic Equilibrium: The Balance of Pressure against Gravity

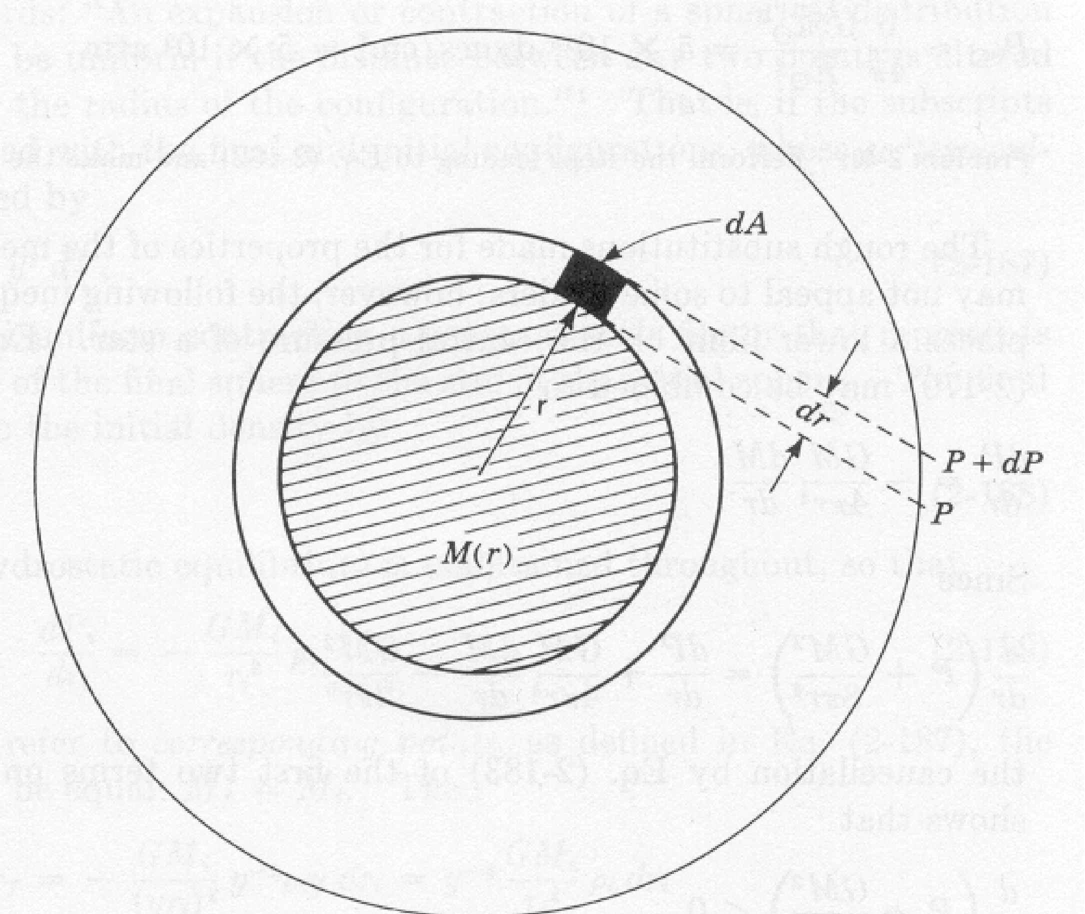
Cut away view of the interior of a star:



Pressure and gravity must balance or the star will expand or contract.

Hydrostatic equilibrium:

a fluid element is “held in place” by a pressure gradient that balances gravity



Clayton Fig. 2-14

Force from pressure:

$$F_p = PdA - (P + dP)dA$$

$$= -dPdA$$

Force from gravity:

$$F_G = -GM(r)\rho(r)\,dAdr\,/\,r^2$$

For balance: $F_G = F_P$ need:

$$\boxed{\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}}$$

Simple Estimates for Stellar Structure

(a) Pressure equilibrium

$$\frac{dP_r}{dr} = -\rho_r \frac{GM_r}{r^2}$$

(b) Conservation of mass

$$\frac{dM_r}{dr} = 4\pi\, r^2 \rho_r$$

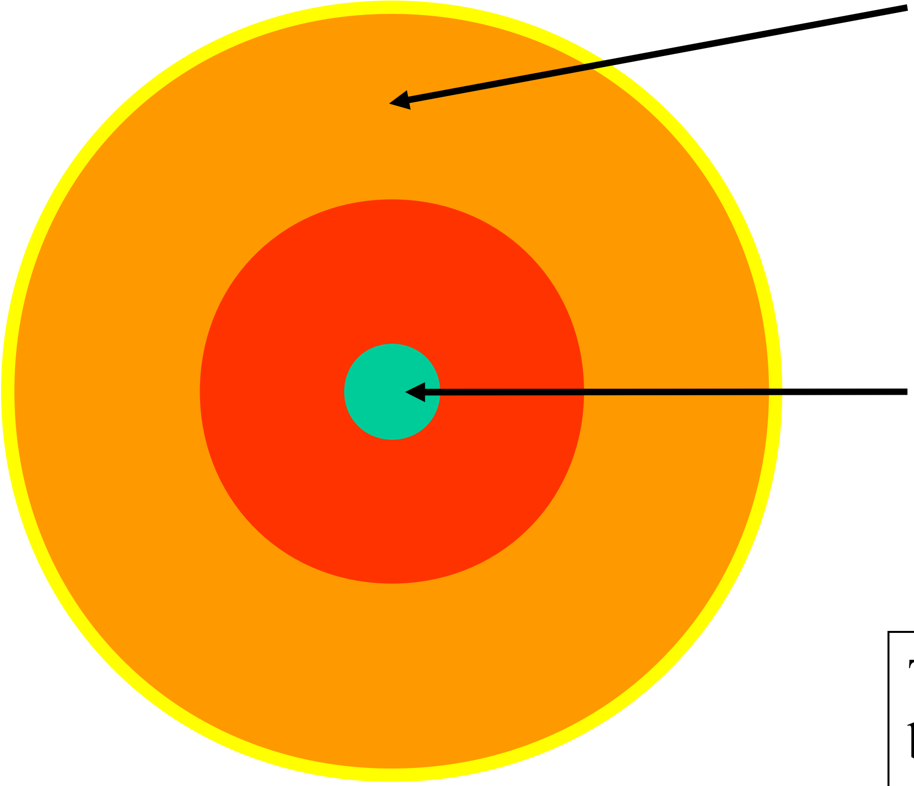
Example: dP=P(central)-P(surface), and mean M(r)=M/2 and r=R/2 plus ideal gas for **sun**

P(central) = 7E15 dyn/cm

T(central) < 3E7 K (real value 1.6E7 K)

2) Fully convective star (Protostar, Red Giant) because T cancels in global equations (see later).

The Distribution of Mass Inside a Star



Low density outer regions. Not much mass.

High density inner regions. Lots of mass.

The mass in a star tends to be concentrated towards the center.

The origin of pressure: equation of state

Under the simplest assumption of an ideal gas: $P = \rho N_A RT / \mu_I$

$\longrightarrow$ need high temperature !

Keeping the star hot:

The star cools at the surface - energy loss is luminosity L

To keep the temperature constant everywhere luminosity must be generated

In general, for the luminosity of a spherical shell at radius r in the star:
(assuming steady state dS/dt = 0)

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \varepsilon \qquad \text{(energy equation)}$$

where ε is the energy generation rate (sum of all energy sources and losses) per g and s

Luminosity is generated in the center region of the star (L(r) rises) by nuclear reactions and then transported to the surface (L(r)=const)

A Fundamental Principle of Energy Flow

If no external forces are acting,

Energy flows from hotter to cooler regions.

Example: House air conditioner.

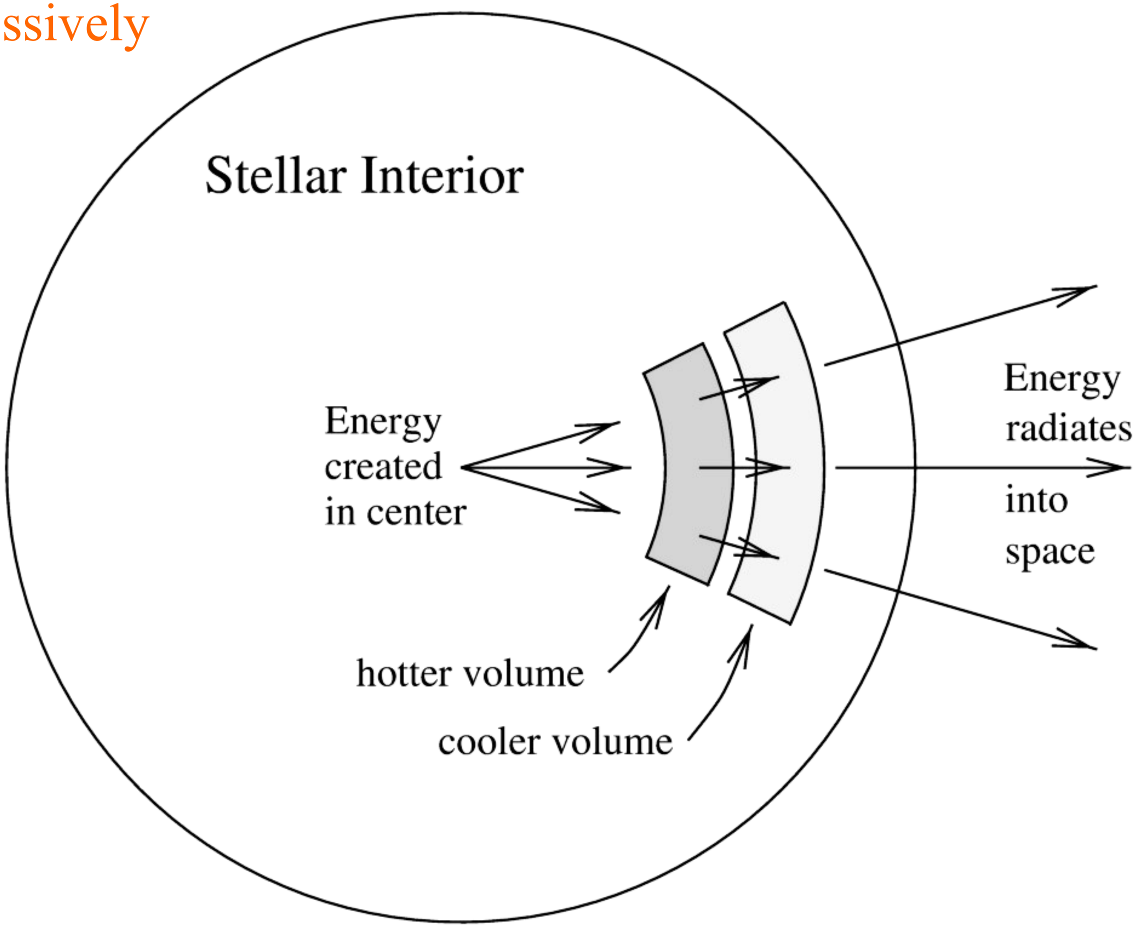
Heat is a form of energy.

If the air conditioner is turned off, heat leaks into the house (from hot outside to cool inside).

Heat moves from inside the house to outside only when the air conditioner is on and doing lots of work with lots of expensive electricity.

The Transport of Energy inside a Star

The temperature inside a star must increase progressively towards its center to make energy flow from the center to the surface.



Energy transport to the surface - cooling:

The star will settle into a hydrostatic and thermal equilibrium, where cooling is balanced by nuclear energy generation and there is no time dependence of any state variables.

The generated heat will then exactly match the outgoing energy flow (luminosity) at any point in the star.

Heat flows from hot to cold

→ temperature gradient is required to carry the luminosity outward:

Therefore T(r) and P(r) drop towards the surface → ρ (r) also drops

Possible mechanisms of heat transport:

1. Conduction (not important at low densities in normal stars)
2. Radiative diffusion
3. Convection

Radiative energy transport:

Effectiveness depends on opacity κ :
unit cm^2/g – could call it specific cross section,
for example luminosity L in a layer r gets attenuated by photon absorption with a cross section σ :

$$L = L_0 e^{-\sigma n r} = L_0 e^{-\kappa \rho r}$$

Photon mean free path l: $l = \frac{1}{\kappa\rho}$ (about 1cm in the sun)

Required temperature gradient:

Luminosity per cm

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4acT^3}\boxed{\frac{L(r)}{4\pi r^2}}$$

a: radiation density constant
=7.56591e-15 erg/cm /K

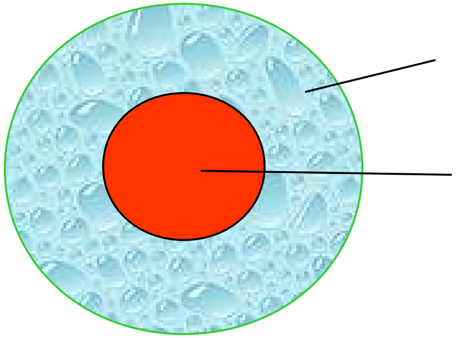
→ Large gradients needed for

- large luminosity at small r (large L/cm)
- large opacity

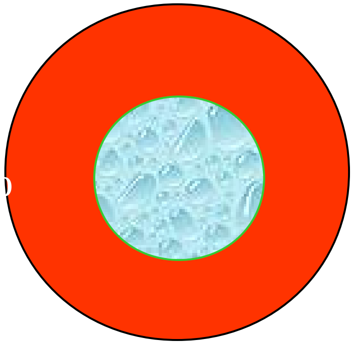
Convective energy transport:

takes over when necessary temperature gradient is too steep
hot gas moves up, cool gas moves down, within convective zone
fluid elements move adiabatically (adiabatic temperature gradient) driven by temperature dependent bouyancy

Stars with $M<1.2$ M have radiative core and convective outer layer (like the sun):

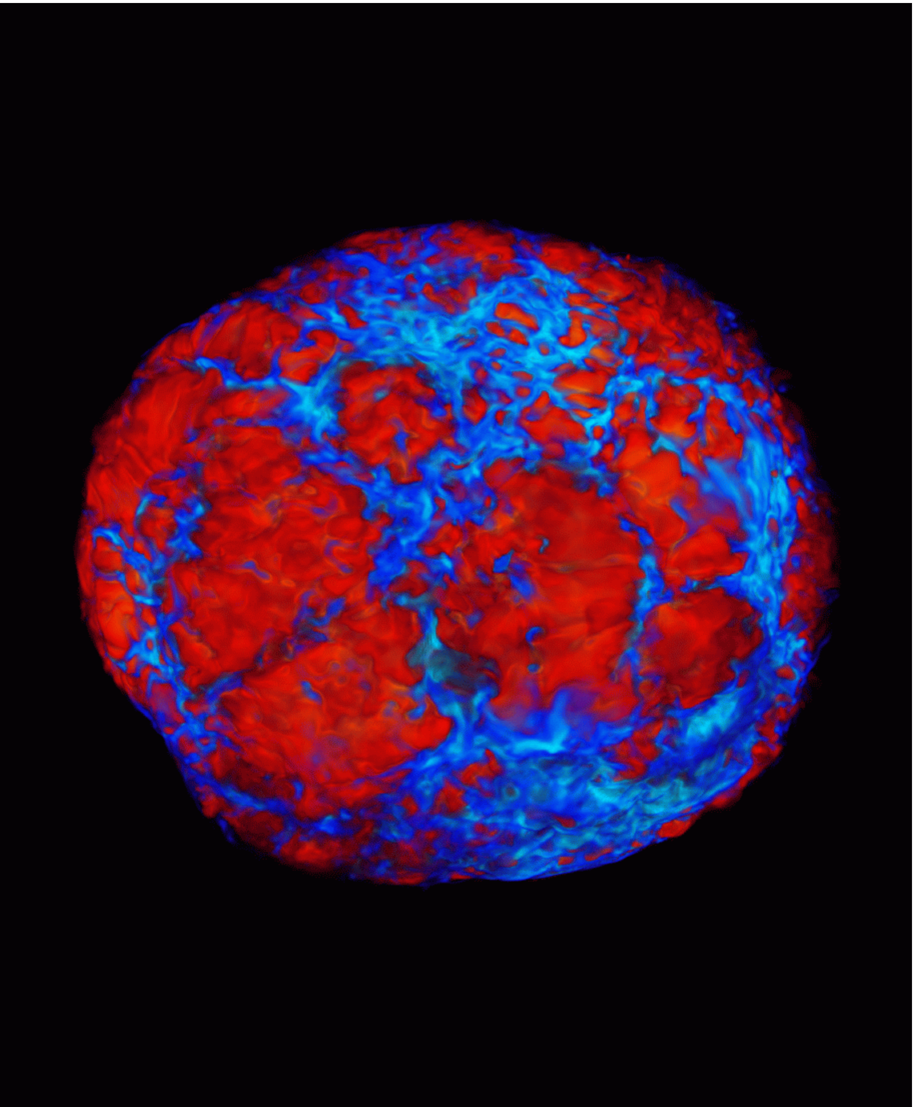


Stars with $M>1.2$ M have convective core and radiative outer layer:



(convective core about 50% of mass for 15M star)

Energy Transport by Convection in Red Giant



Mixing length theory: mean distance. Free parameter is a typical length scale.

Works for fully developed convection because the solution depends on 'compressibility' only.

Buss words: Overshooting, chemical mixing, etc.

Direct simulation:

Single star, rotating red giant (Woodward/ LLNL)

The Equations of Stellar Structure

1. Four structure equations

(a) Pressure equilibrium

$$\frac{dP_r}{dr} = -\rho_r \frac{GM_r}{r^2}$$

(c) Energy generation

$$\frac{dL_r}{dr} = 4\pi r^2 \rho_r \varepsilon$$

(b) Conservation of mass

$$\frac{dM_r}{dr} = 4\pi\ r^2 \rho_r$$

(d) Energy transport (convection)

$$\frac{dT_r}{dr} = \left(1 - \frac{1}{\gamma}\right) \frac{T_r}{P_r} \frac{dP_r}{dr}$$

$$\frac{dT}{dr} = -\frac{3}{32\pi\sigma c} \frac{\kappa\rho L}{r^2 T^3}$$

with L: luminosity, σ Stephan-Boltzman constant

2. Subsidiary equations, such Equation of State for gas

Ideal Boltzmann: Fermi:

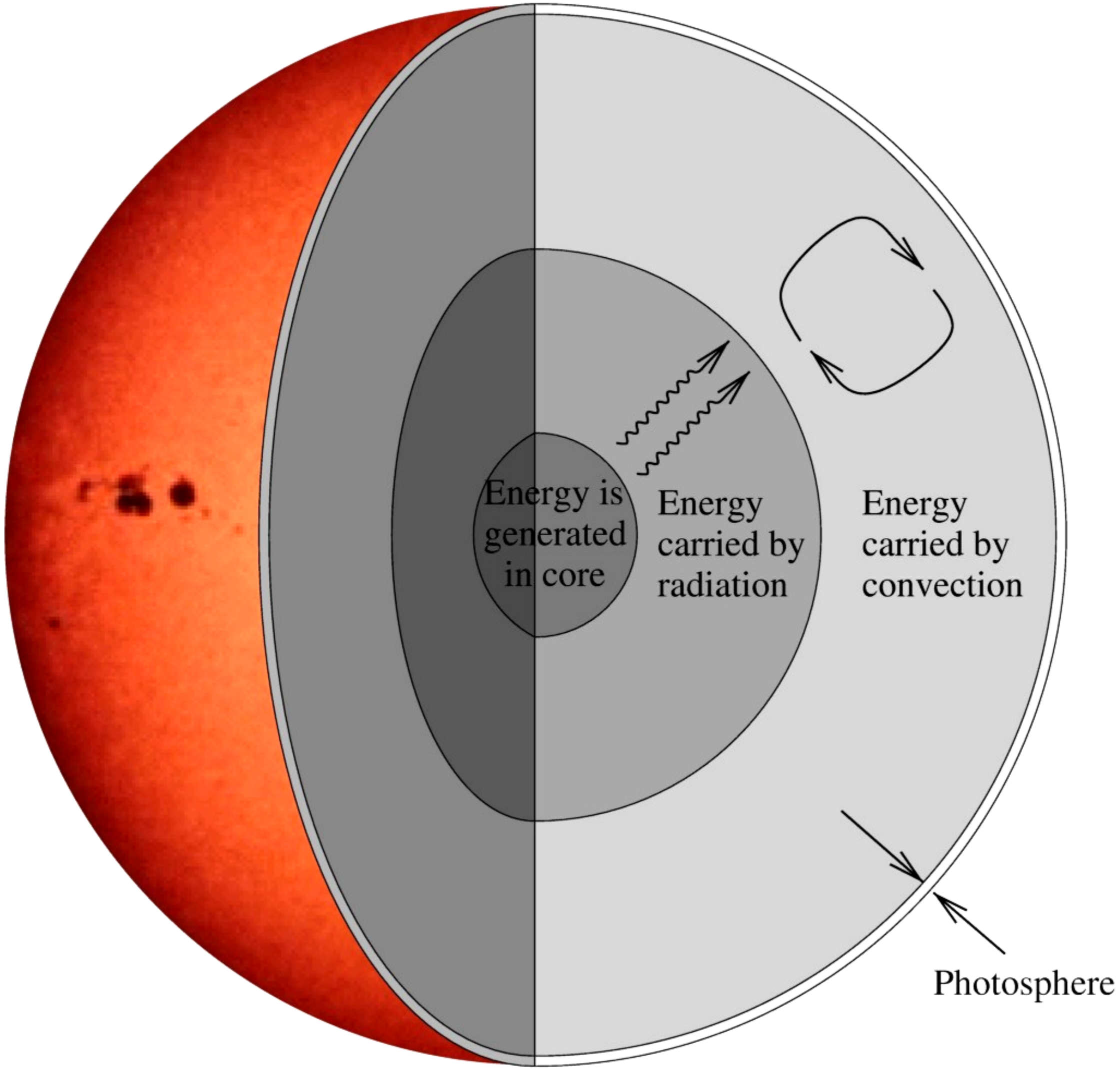
$$P = \frac{\rho k T}{\mu\ m_H}$$

$$P = \rho^\gamma$$

with $\gamma = 5/3$ in the limit of non-relativistic degenerate gas

with $\gamma = 4/3$ in the limit of relativistic degenerate gas

The Interior of the Sun



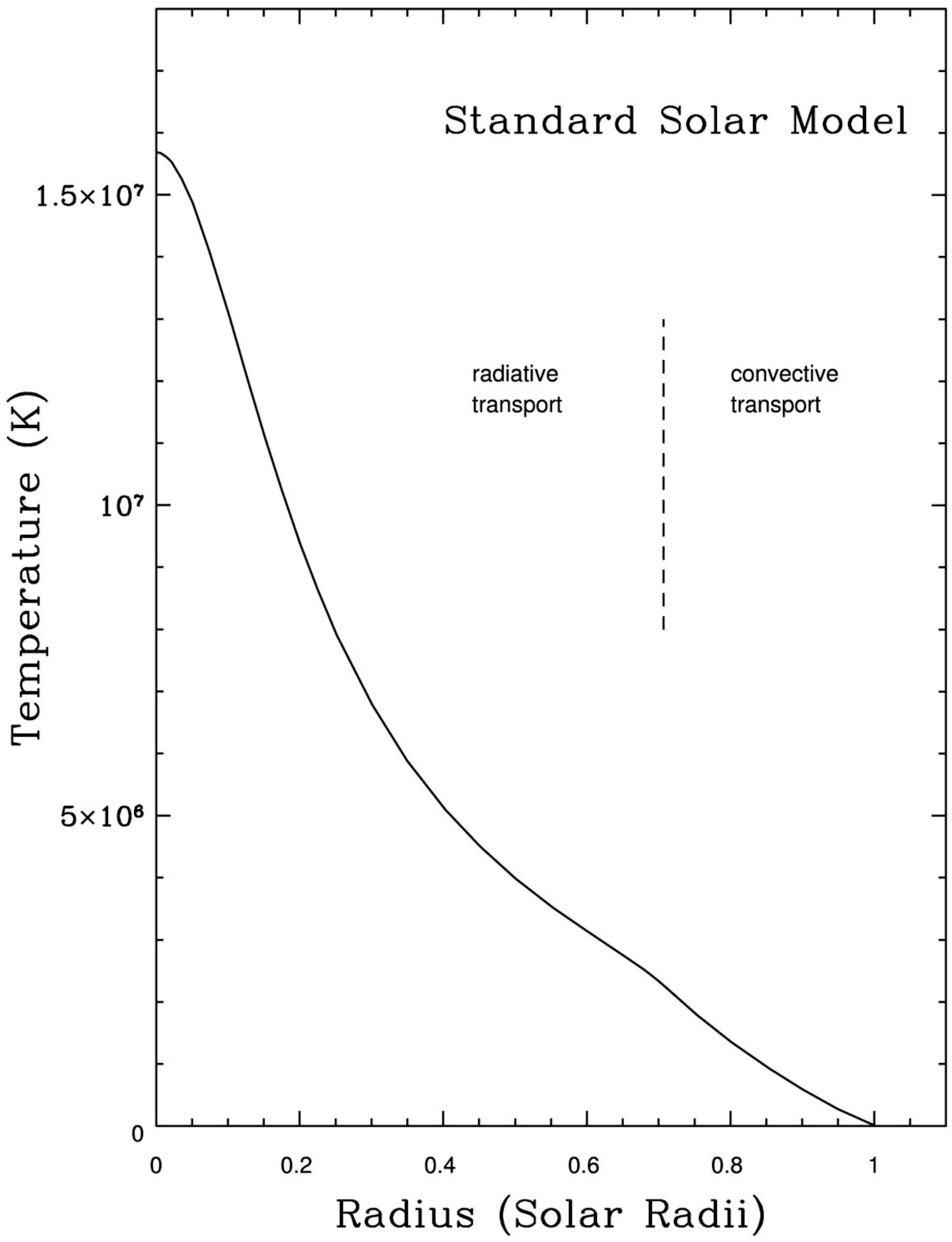
Energy is generated in core

Energy carried by radiation

Energy carried by convection

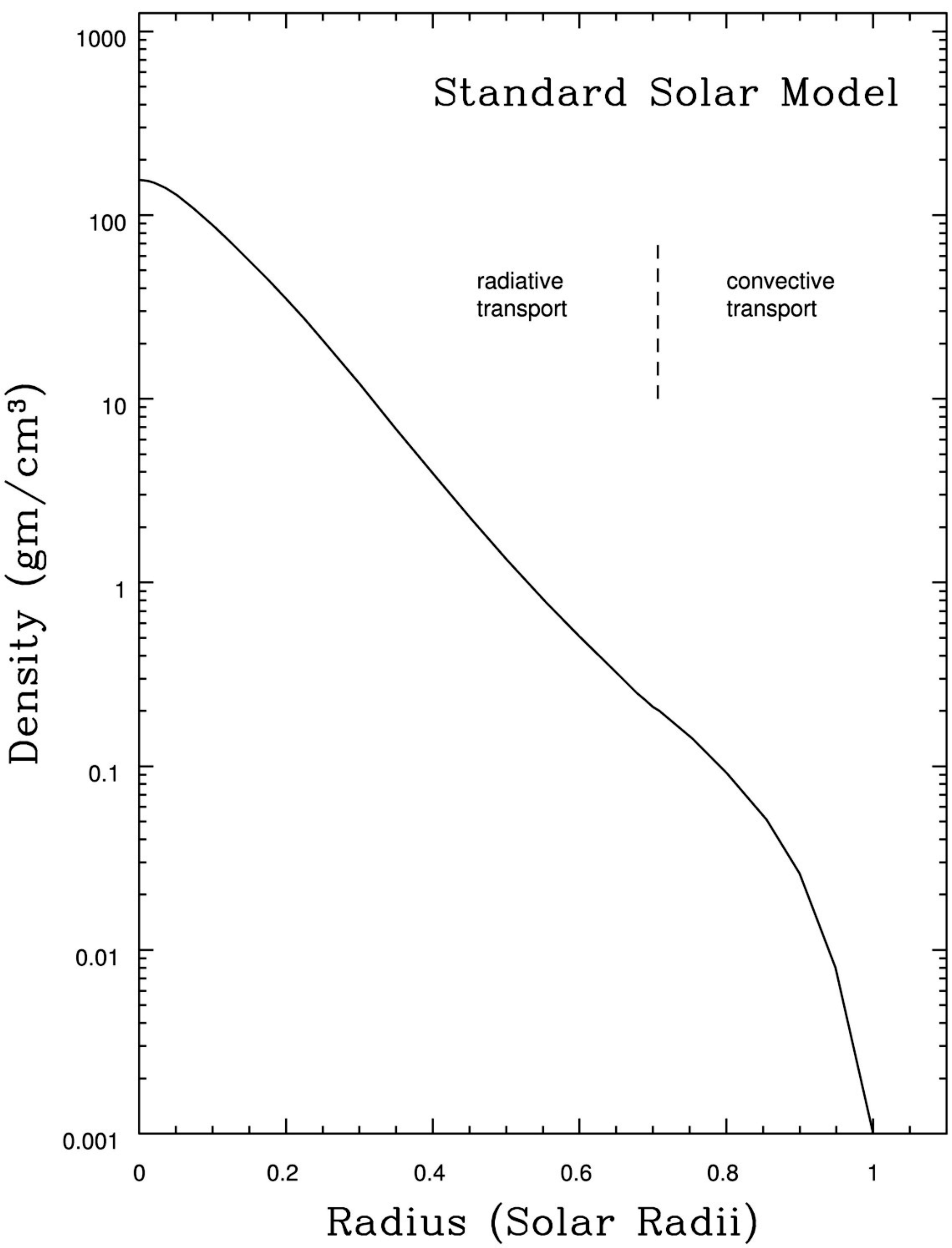
Photosphere

The Temperature Inside the Sun



The temperature at the center of the sun is a bit more than 15 million K.

The Density Inside the Sun

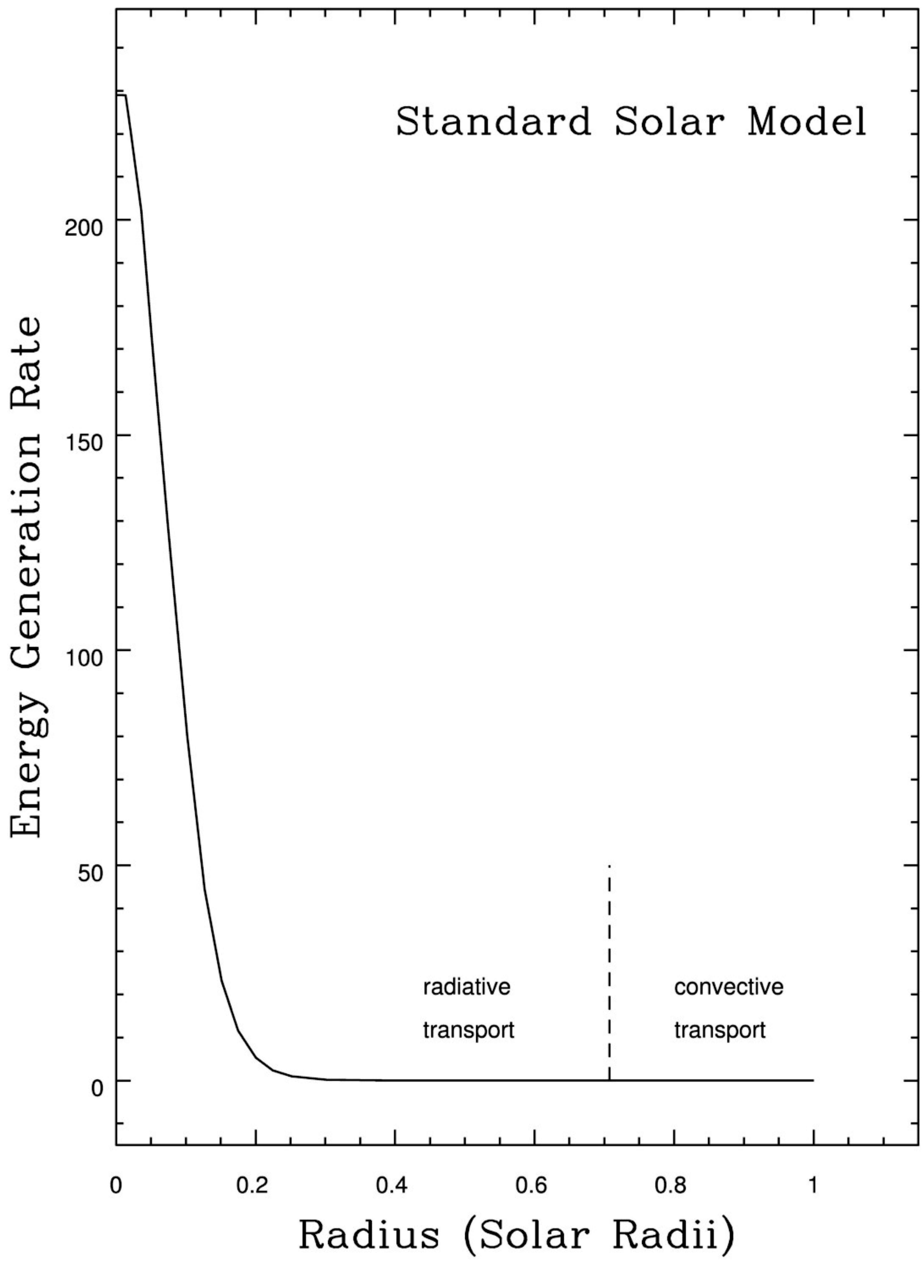


- The density at the center of the sun is a bit more than 150 gm/cm .

  This is about 10 times denser than lead!

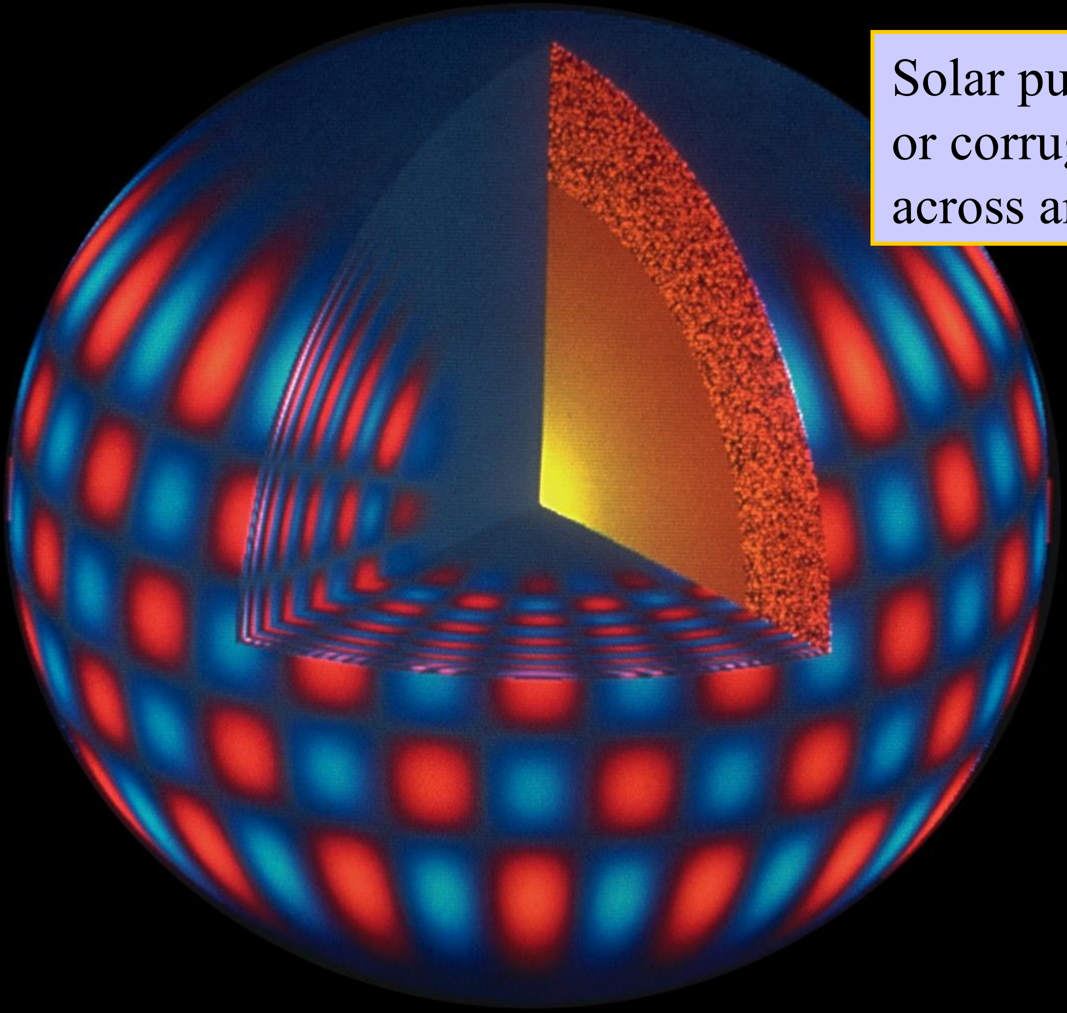
- It its visible surface the density of the sun is 10,000 times lower than the Earth’s atmosphere.

The Energy Generation Rate Inside the Sun

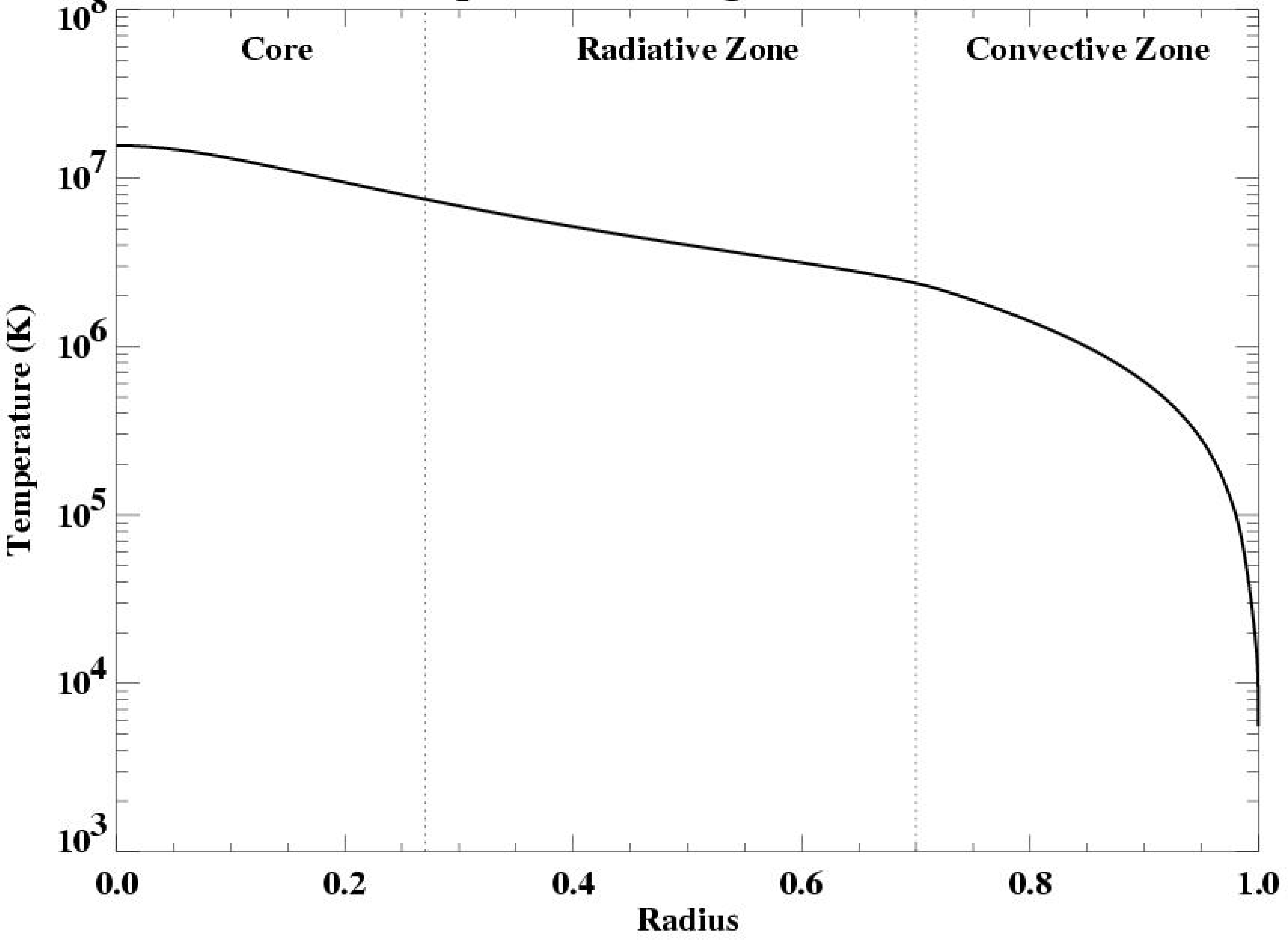


All the nuclear energy in the sun is produced inside 0.2 R_s.

A Computer Model of Solar Pulsations

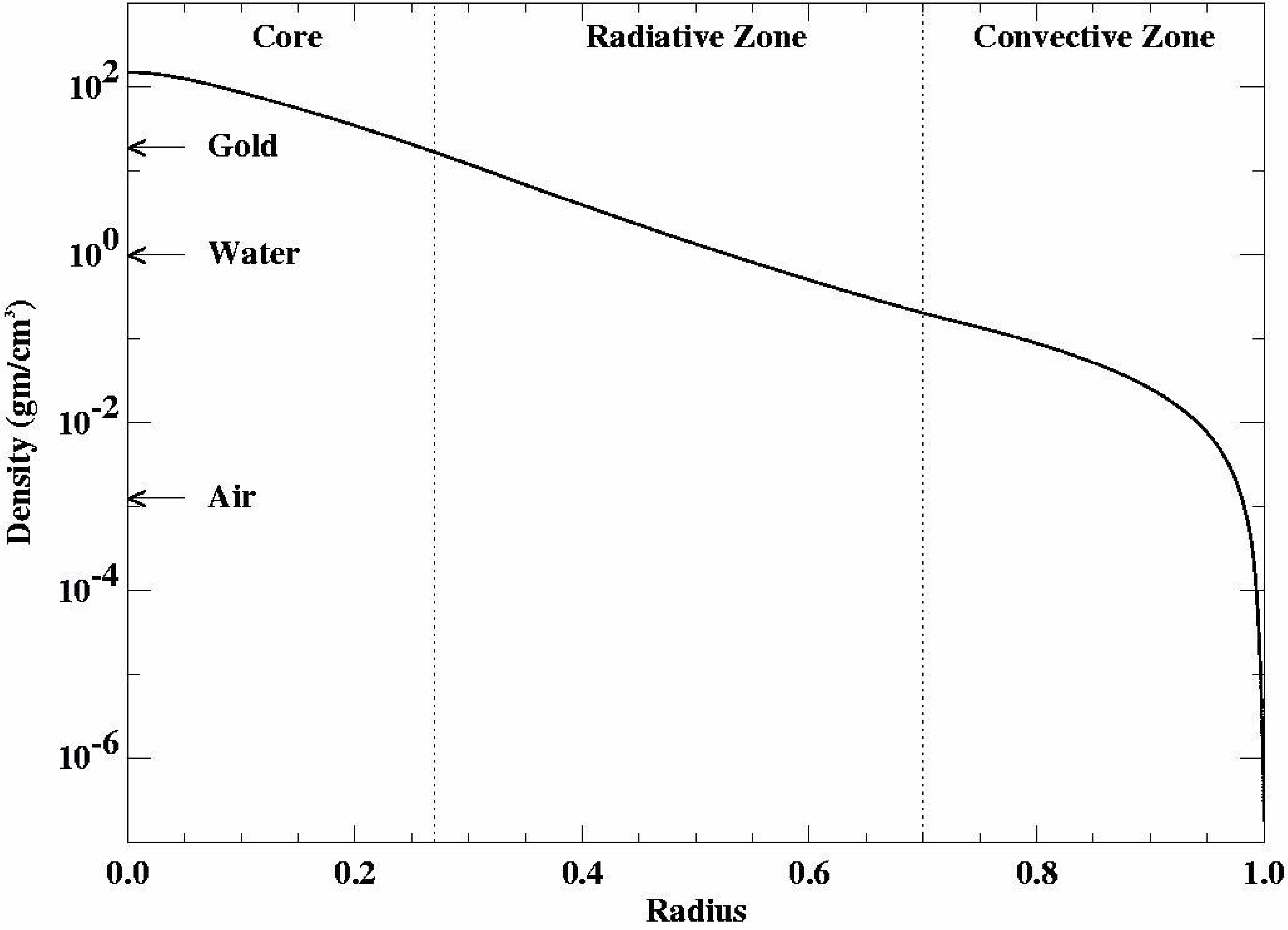


Solar pulsations are waves or corrugations that travel across and through the sun.

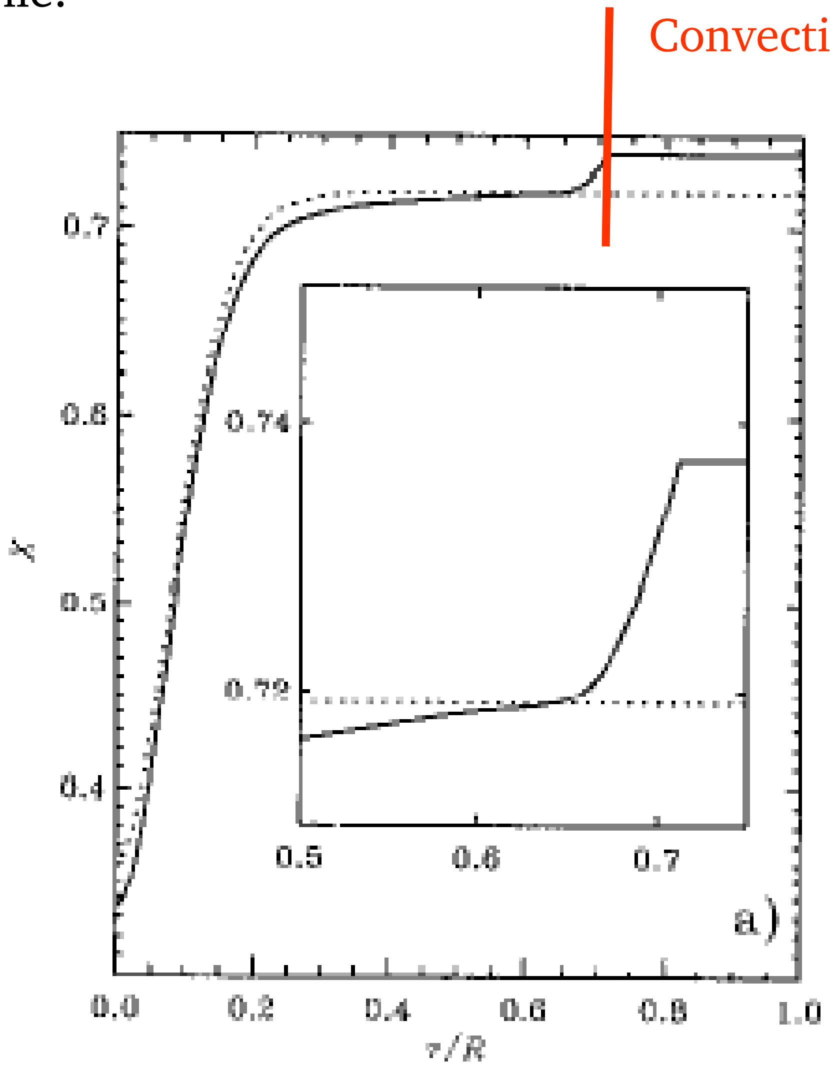


Temperature (Dalsgaard Model 1)

Density (Dalsgaard Model 1)



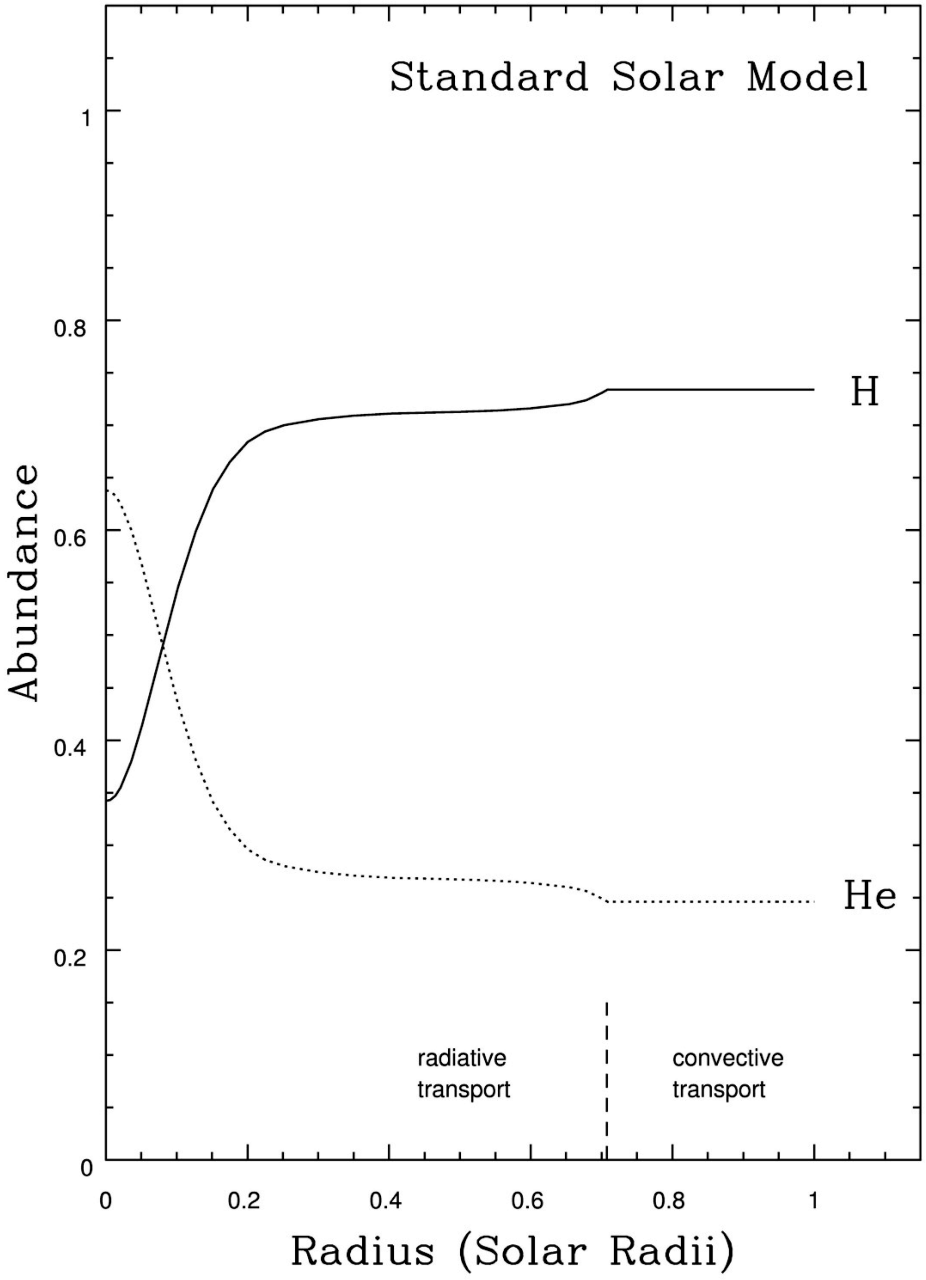
Hydrogen profile:



Convective zone (const abundances)

Christensen-Dalsgaard, Space Science Review 85 (1998) 19

The Abundance of Hydrogen and Helium in the Sun



- The models show that the sun has used ½ of the original hydrogen in its core.
- It takes 4.7 × 10 years for the sun to use up ½ of the hydrogen in its core.
- So, the sun is 4.7 × 10 years old.

Mass – Radius relation:

In solar units: R ~ M (10 M_sol: 6.3 x R_sol, 100 M_sol: 40 x R_sol)

Main Sequence evolution:

Main sequence lifetime:

H Fuel reservoir F~M
Luminosity L~M

$$\longrightarrow \quad \text{lifetime} \quad \tau_{\mathrm{MS}} = \frac{F}{L} \propto M^{-3}$$

Recall from Homework: H-burning lifetime of sun ~ 10 years

$$\boxed{\tau_{\mathrm{MS}} \approx \left(\frac{M}{M_{\oplus}}\right)^{-3} 10^{10} \text{ years}}$$

note: very approximate
exponent is really
between 2 and 3

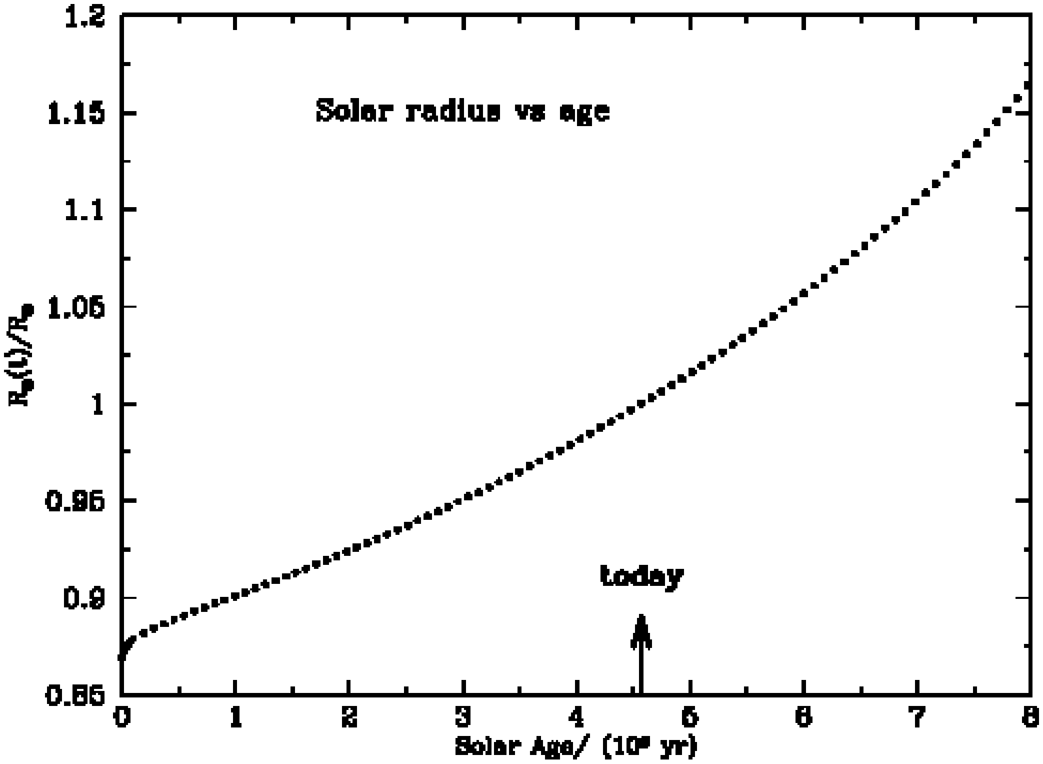
so a 10 solar mass star lives only for 10-100 Mio years
a 100 solar mass star only for 10-100 thousand years !

Changes during Main Sequence evolution:

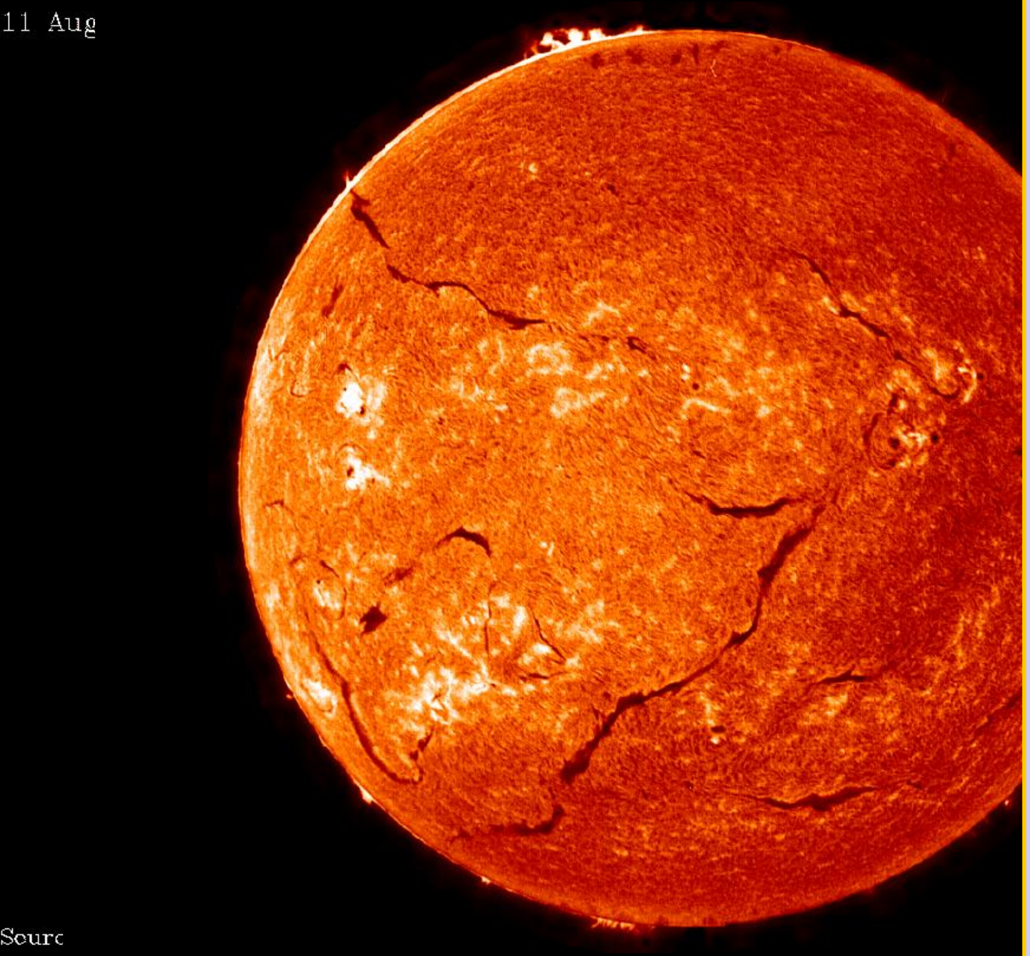
With the growing He abundance in the center of the star slight changes occur (star gets somewhat cooler and bigger) and the stars moves in the HR diagram slightly

→ main sequence is a band with a certain width

For example, predicted radius change of the sun according to Bahcall et al. ApJ555(2001)990



The Observed Surface Properties of the Sun



L = 3.86×10^{33} ergs/sec

L

M = 1.99×10^{33} gm

R = 6.96×10^{10} cm

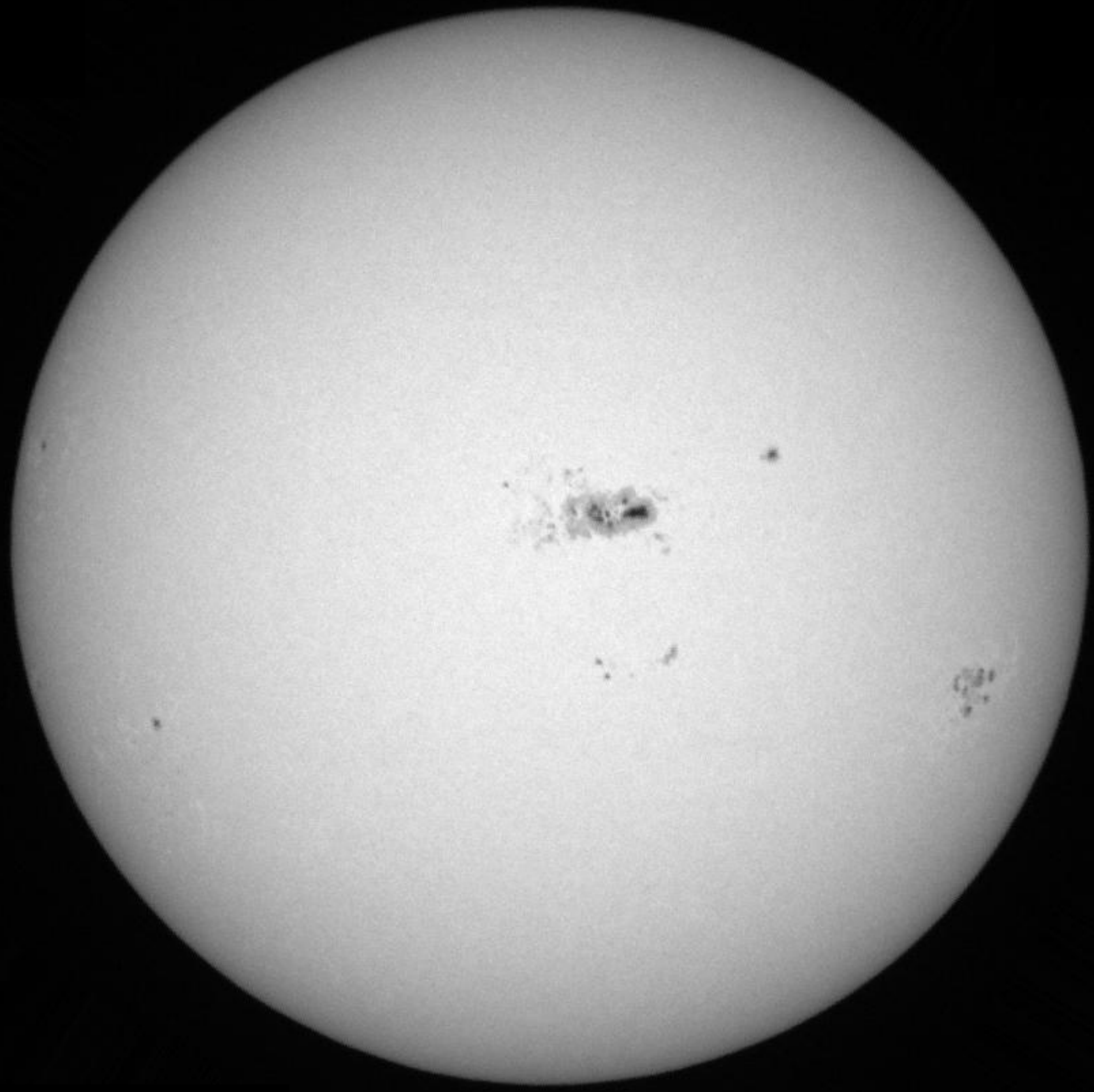
$\Rightarrow$ density = 1.4 gm/cm^3

T = 5780 K (at surface)

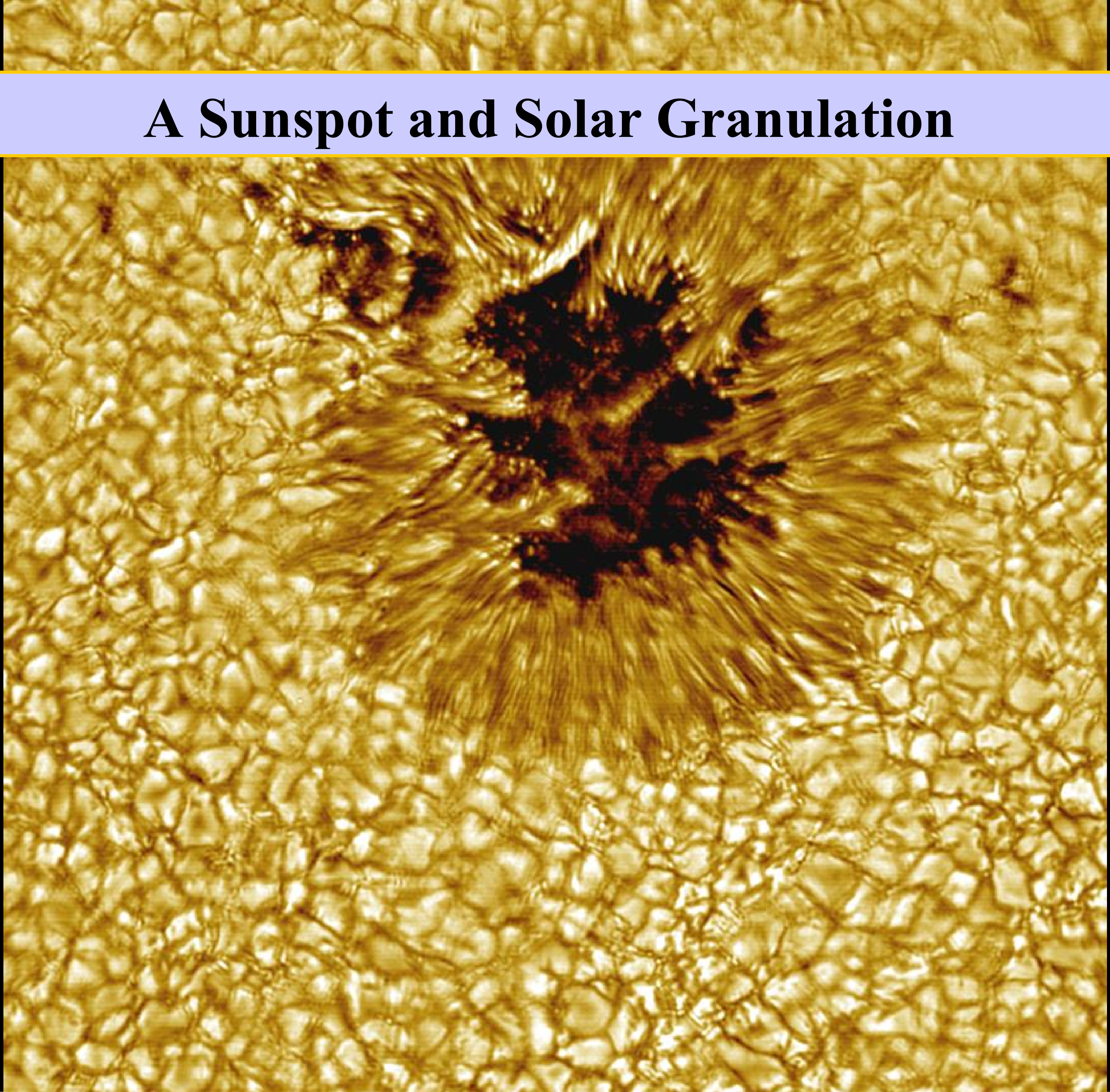
Three Tests of Models for the Solar Interior

1. Solar Granulation: Test of convection at the surface of the sun.
2. Solar Pulsations: Test of convection in the solar interior.
3. Solar Neutrinos: Test of calculations of energy generation in the sun.

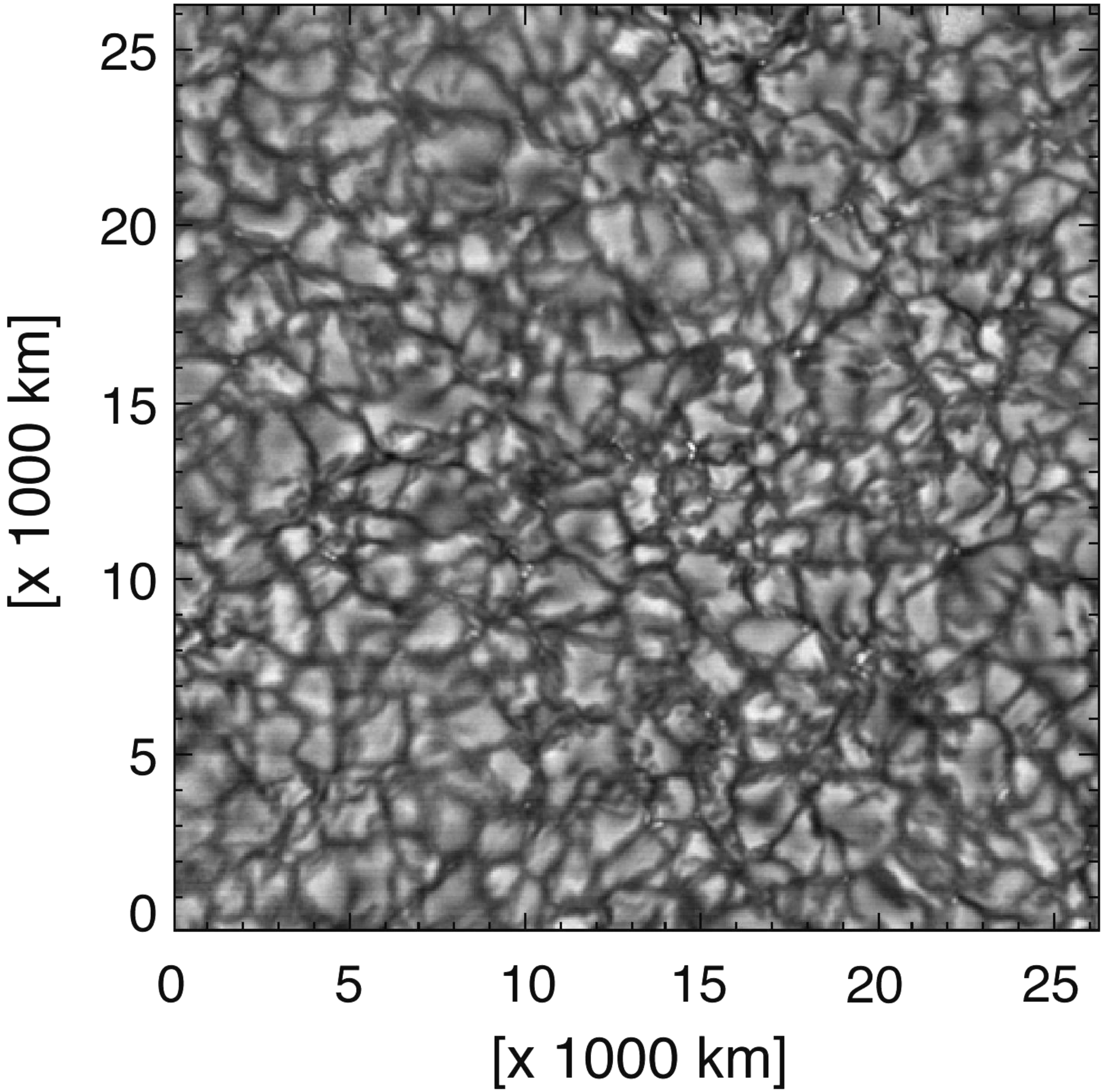
The Sun at Visible Wavelengths



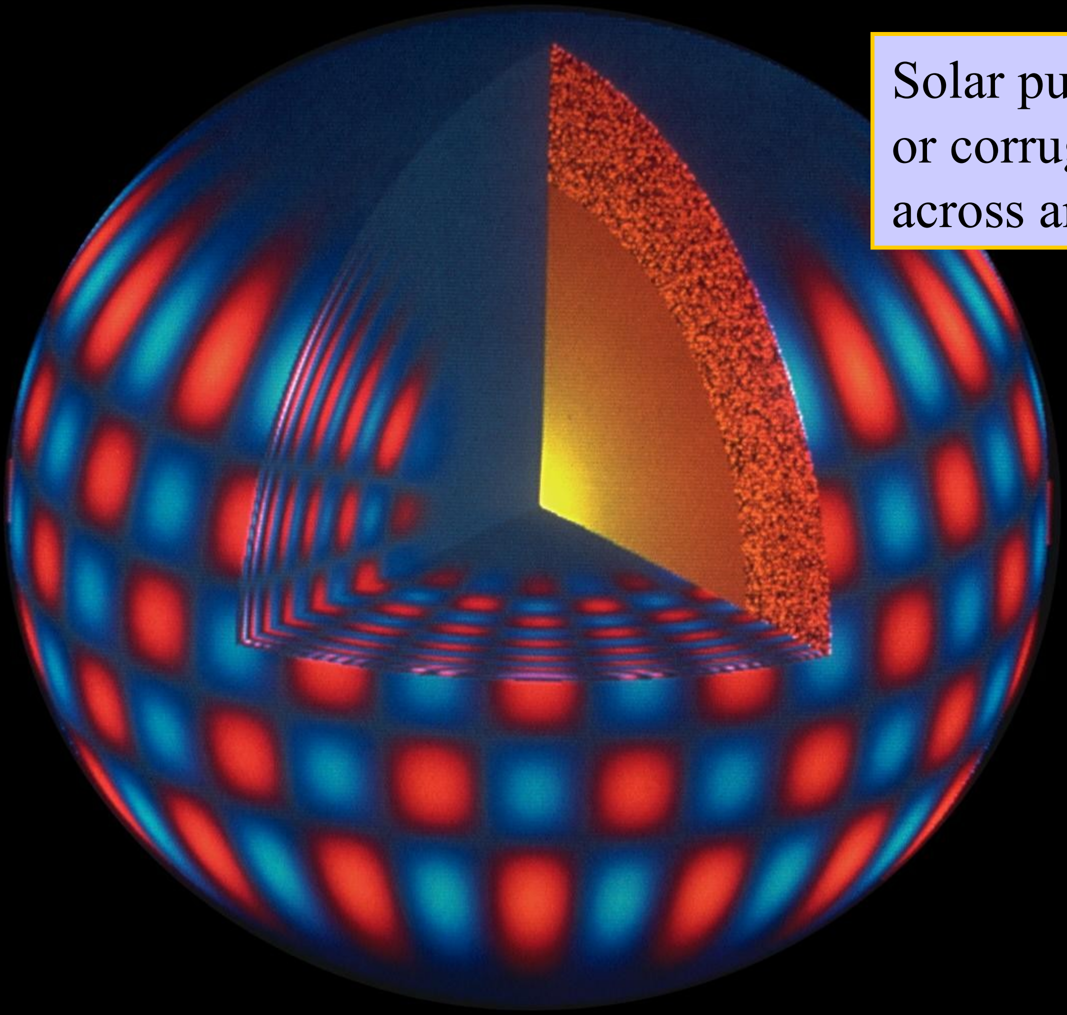
A Sunspot and Solar Granulation



Solar Granulation



A Computer Model of Solar Pulsations



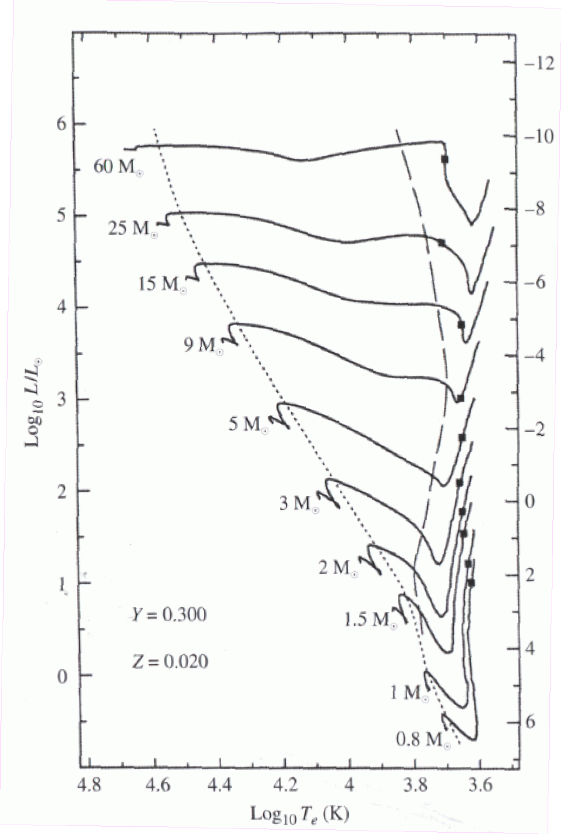
Solar pulsations are waves or corrugations that travel across and through the sun.

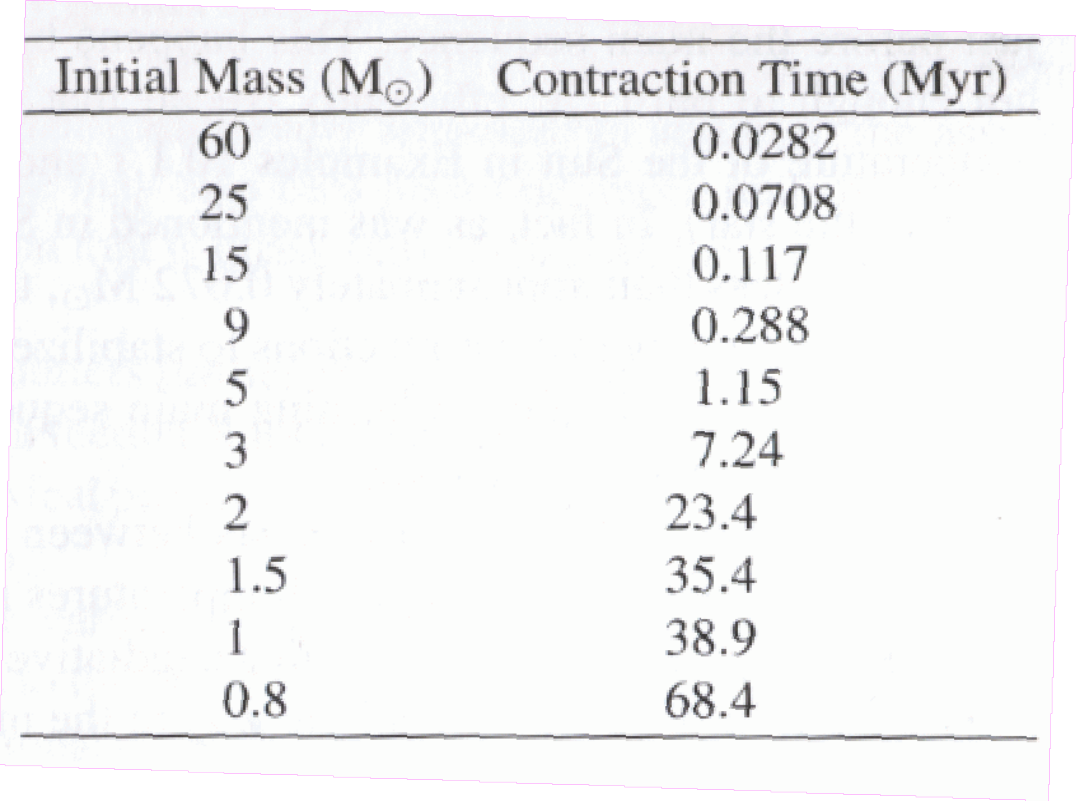
Results from Observations of Solar Pulsations

- Energy transport in the sun switches from radiative to convective transport at a radius of 0.71 R . This agrees with models of the solar interior to within 1%.
- Theory predicts a small but significant decrease in the hydrogen abundance at the same place because heavier helium atoms have sunk slightly in the sun. Observations agree on the location and amount of the decrease.

2) Quasi-Hydrostatic Evolution

1) Initial Collapse of an optically thick cloud
2) Optically thick, hydrostatic cloud. Energy comes from contraction,
thus, time scale is given by L/E(potential) (adiabatic time scale)
Star us fully convective (Hayashi Track).

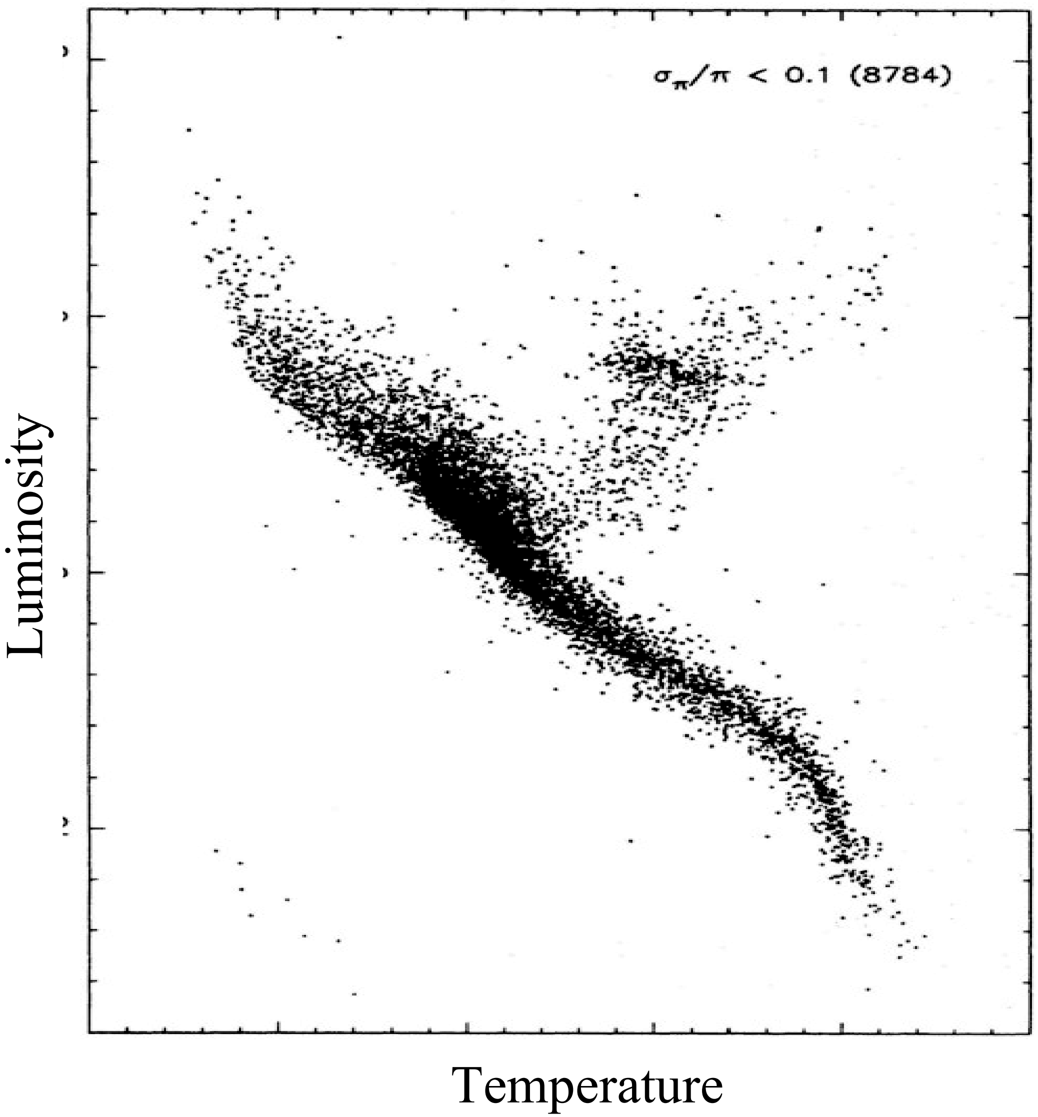




Initial Mass ($M_\odot$)	Contraction Time (Myr)
60	0.0282
25	0.0708
15	0.117
9	0.288
5	1.15
3	7.24
2	23.4
1.5	35.4
1	38.9
0.8	68.4

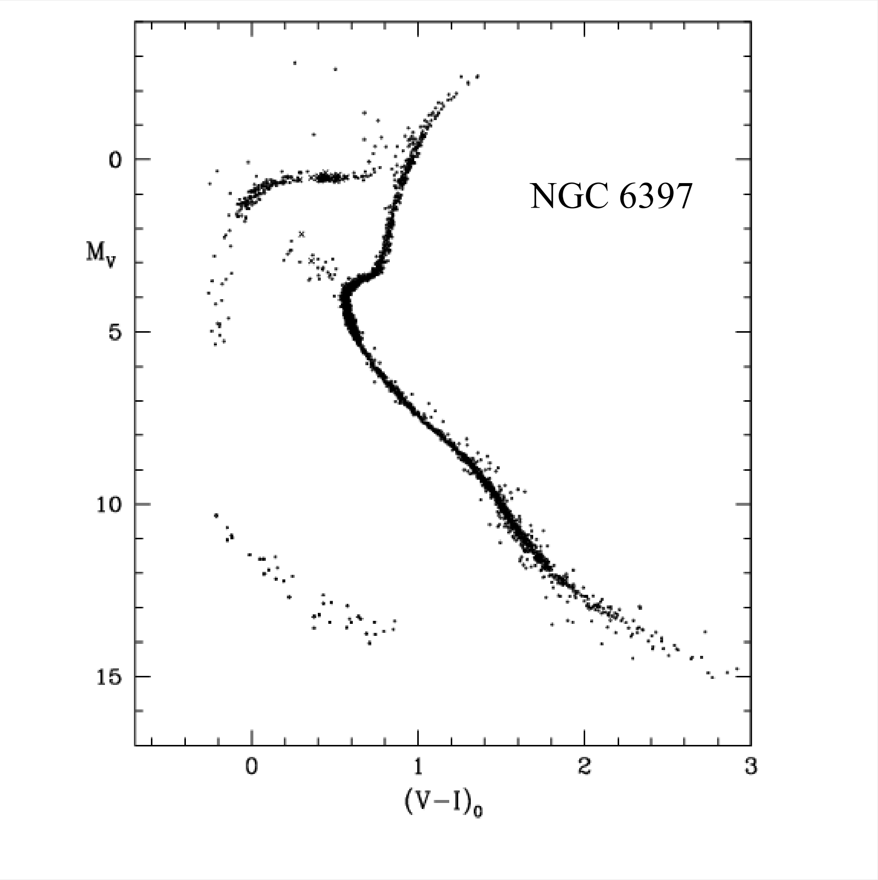
3) Last phase before hydrogen ignition: Lithium burning

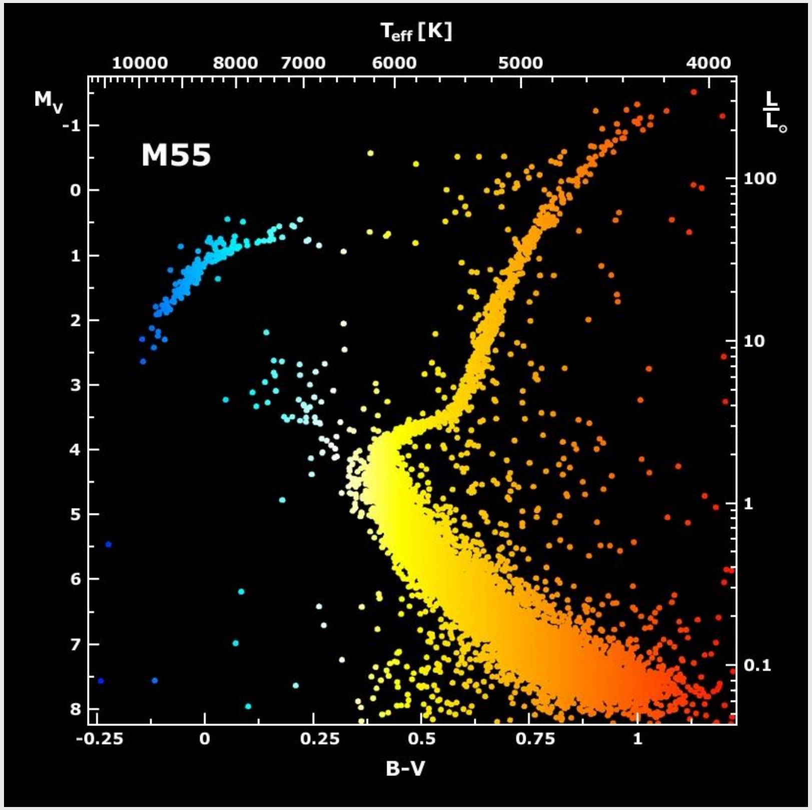
The Real H-R Diagram for Stars near the Sun

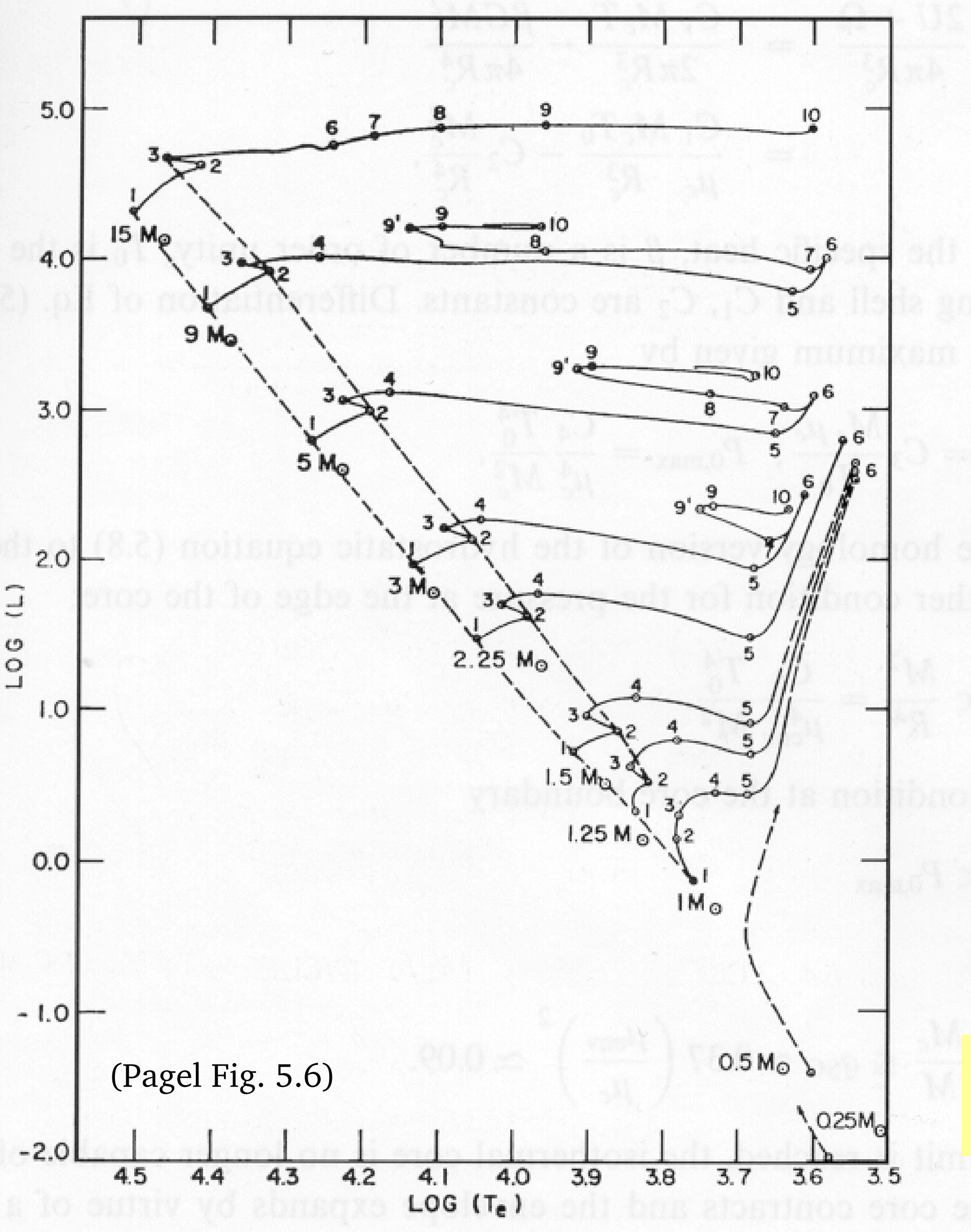


Real H-R Diagrams of Globular Clusters

Real H-R diagrams are messy!







(Pagel Fig. 5.6)

Zero Age Main Sequence (ZAMS): "1"

End of Main Sequence: "2"

Stellar masses are usually given in ZAMS mass !