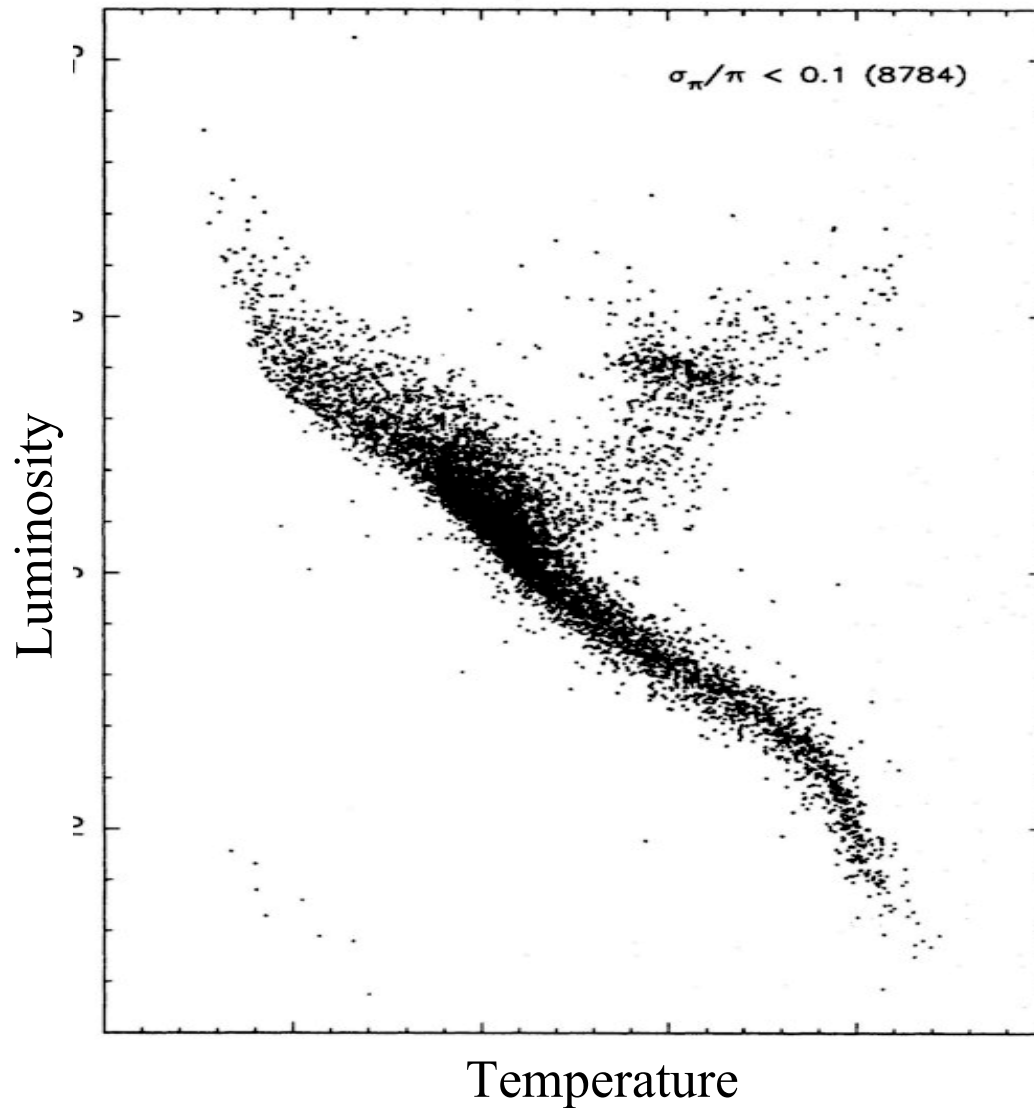


Post-Main-Sequence Evolution: He-burning

- Post-Main sequence Evolution
- The triple-alpha Process

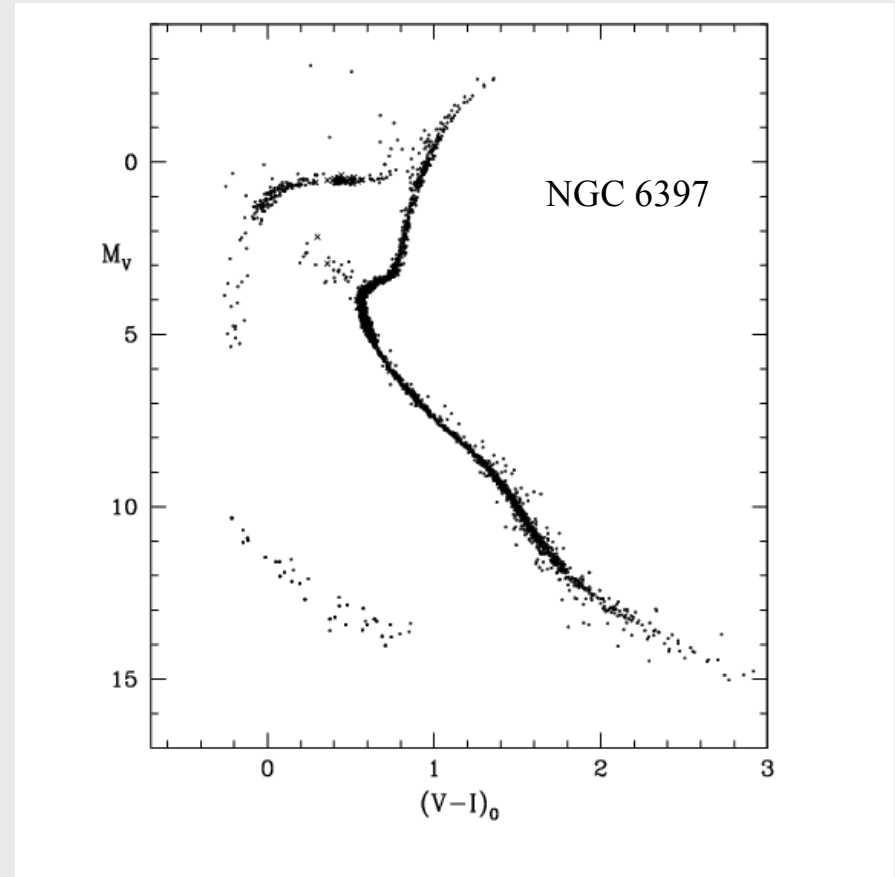
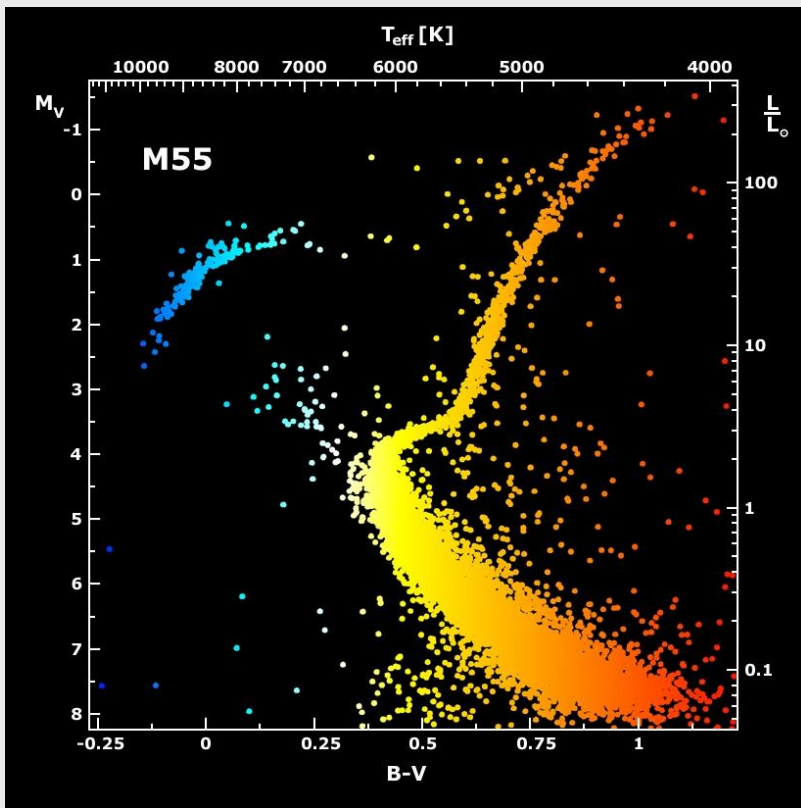
Literature: Iliadis, Chapt. 5.3

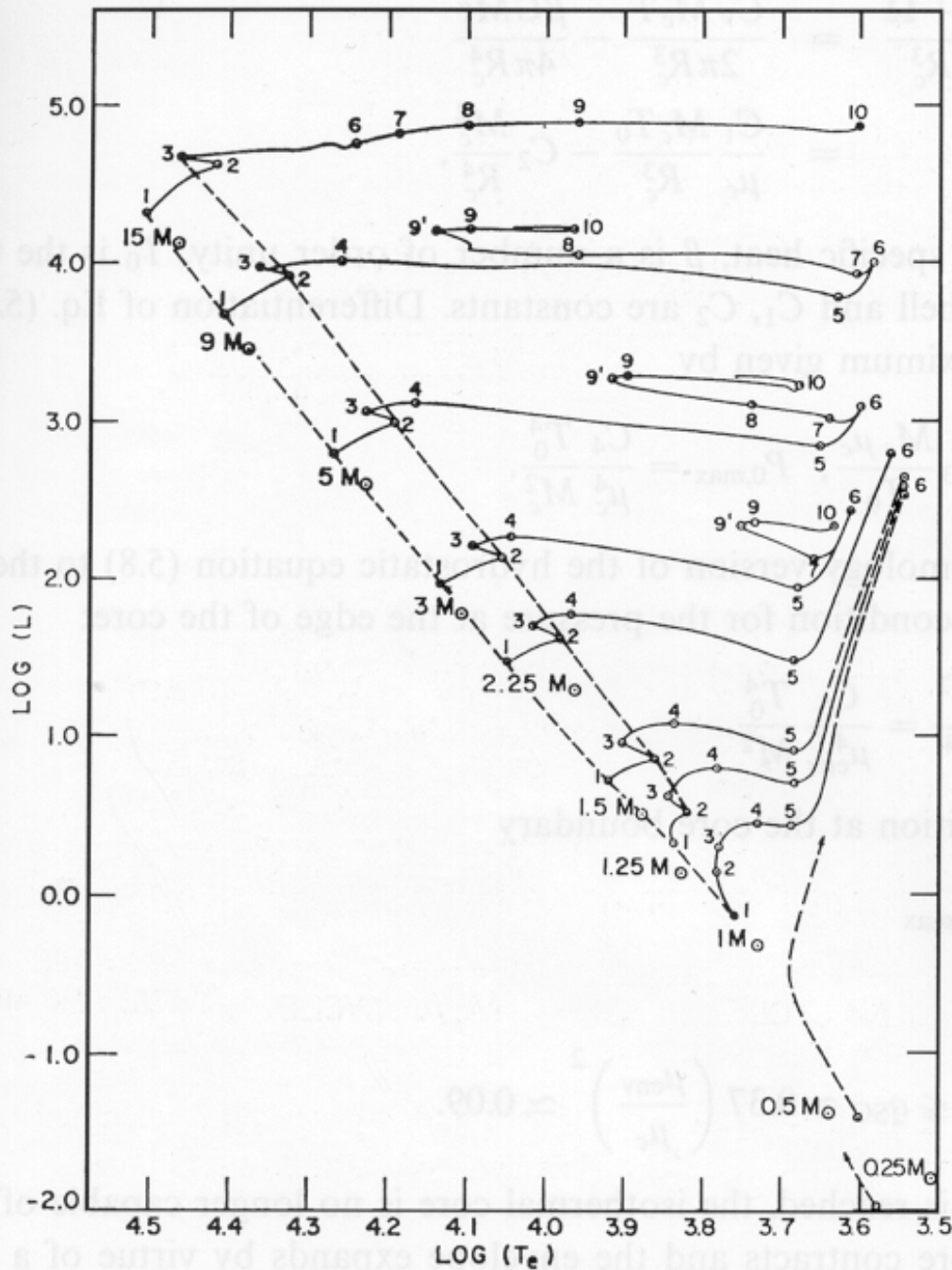
The Real H-R Diagram for Stars near the Sun



Real H-R Diagrams of Globular Clusters

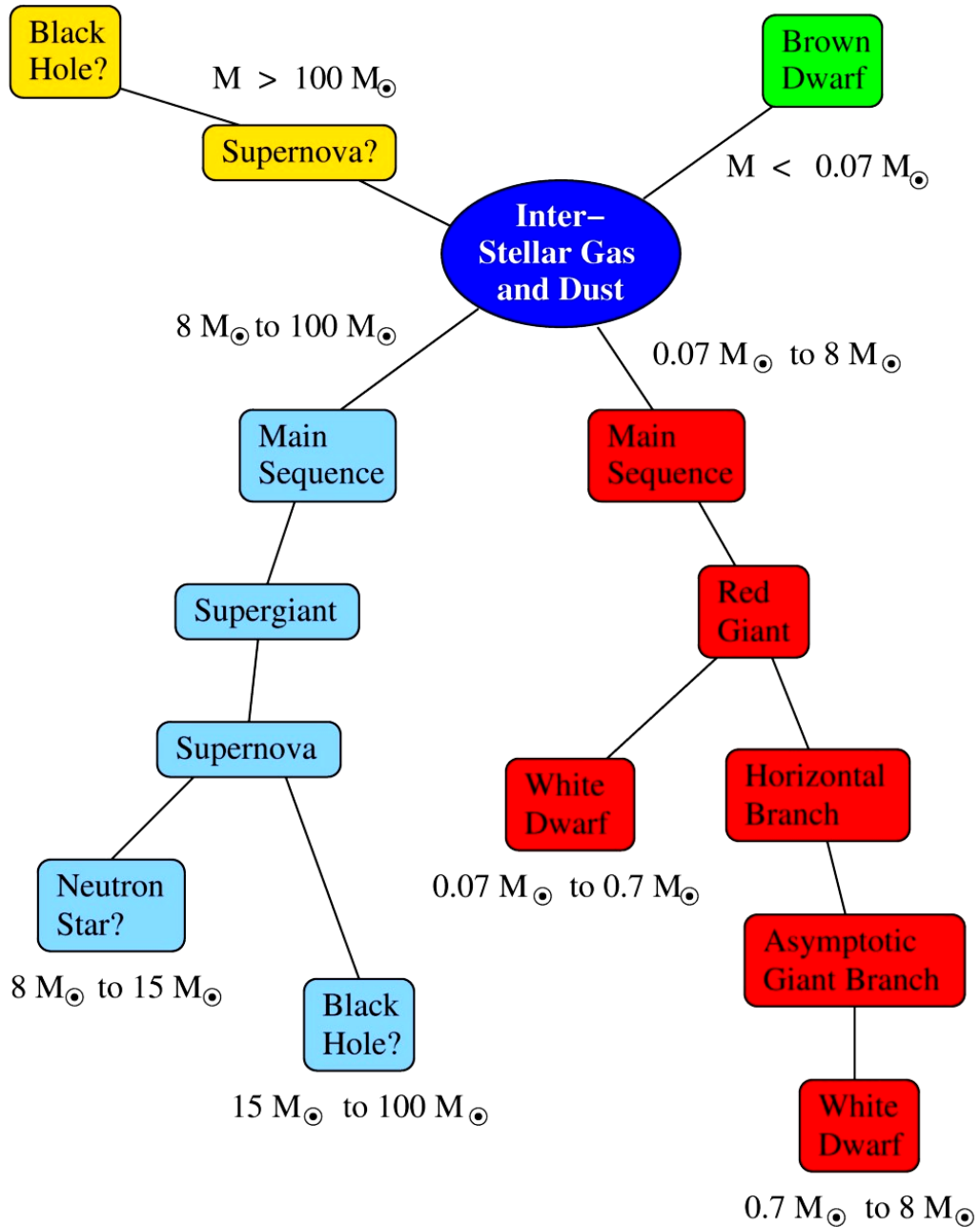
Real H-R diagrams are messy!





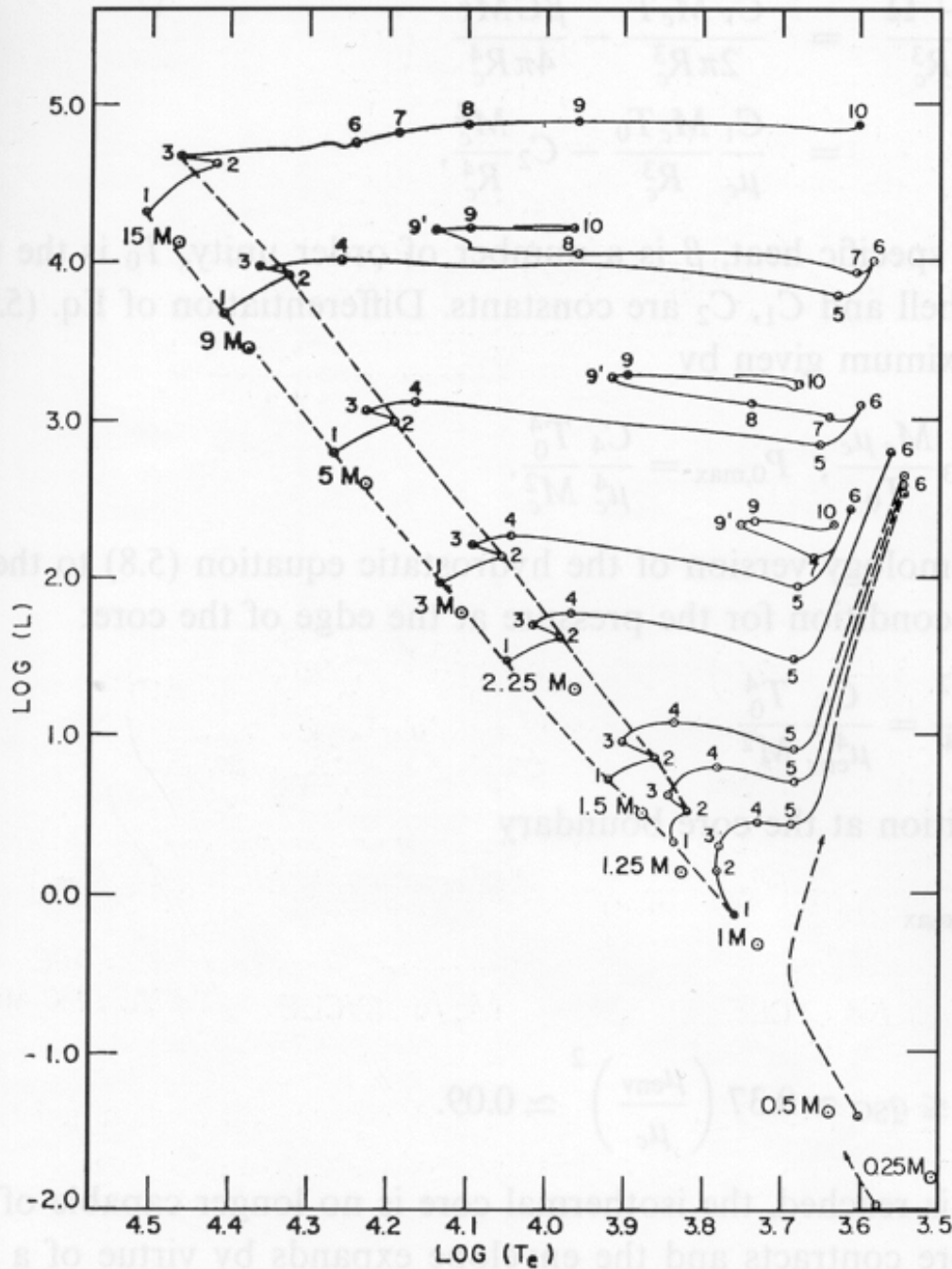
As hydrogen is exhausted in the
 (convective) core of a star
 (point 2)
 it moves away from the main
 sequence (point 3)
 becomes a fully convective structure
 a red giant (5-6)

A Summary of Stellar Evolution



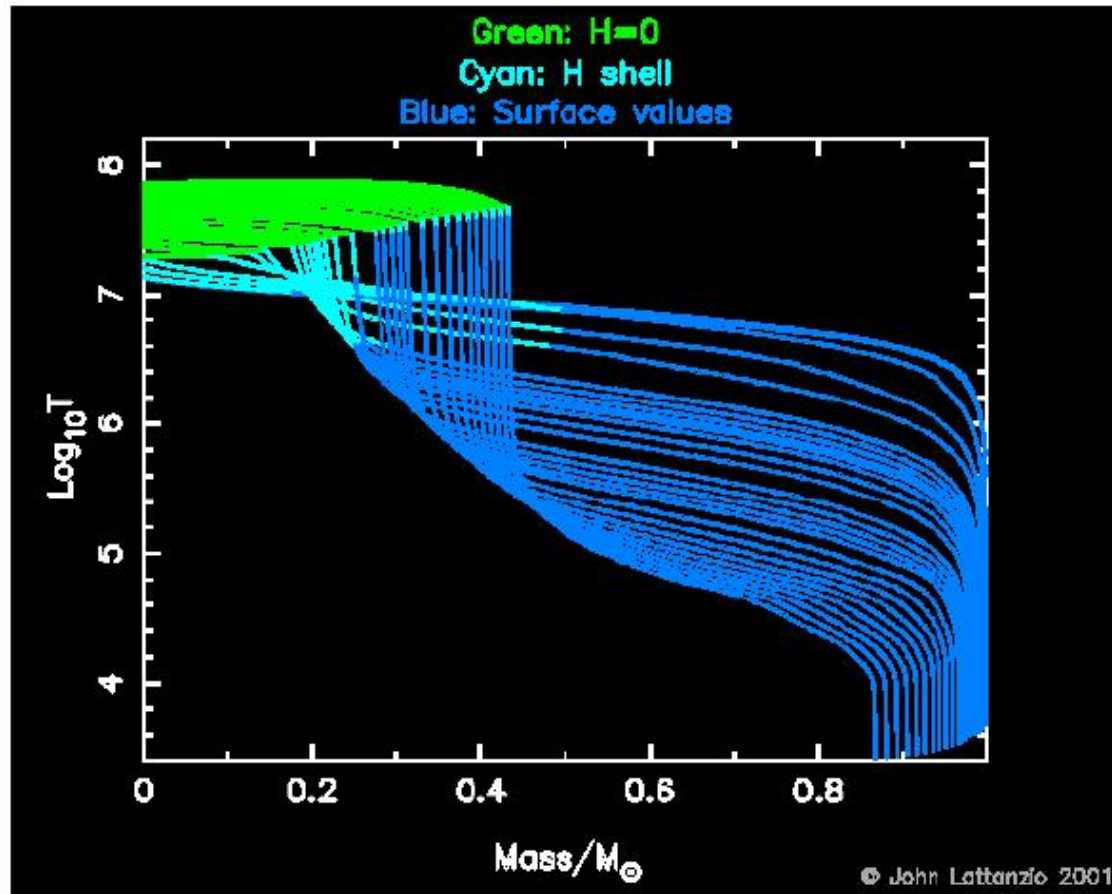
Zero Age Main Sequence (ZAMS): "1"

End of Main Sequence: "2"
(from Kippenhahn & Weigelt, Stellar Structure & Evolution)



Stellar masses are usually given in ZAMS mass !

Temperature Evolution during H-shell Burn.



- H depletes with time → isothermal core

T-rises to support pressure

Steep gradient at outer edge of burning zone

The Isothermal Core

- $L(r) = 0$ without energy production

(Chandrasekhar Limit for core)

- Maximum central T can be reached

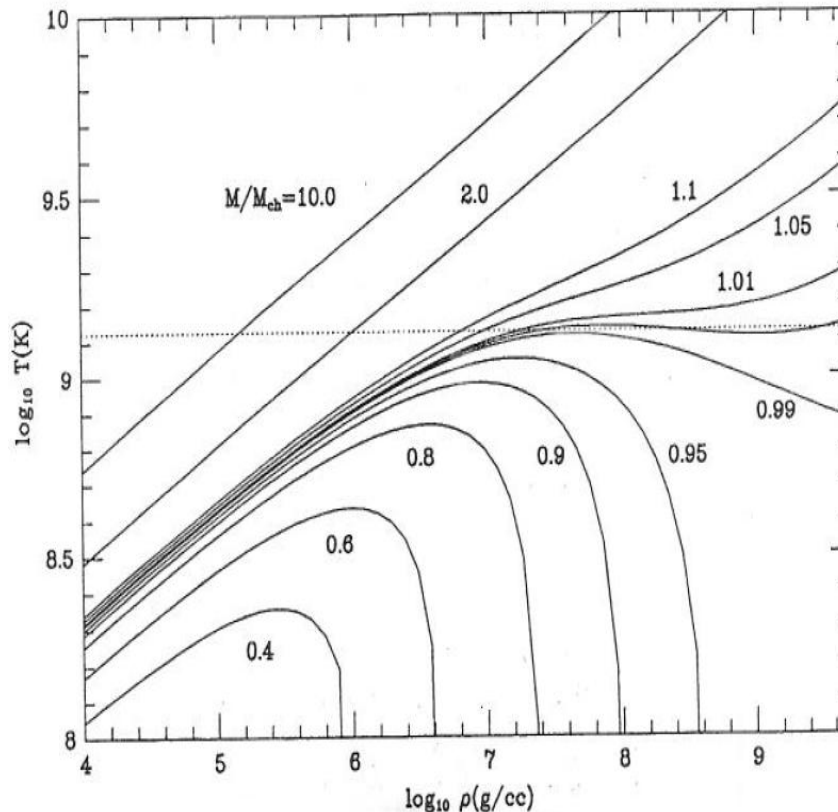
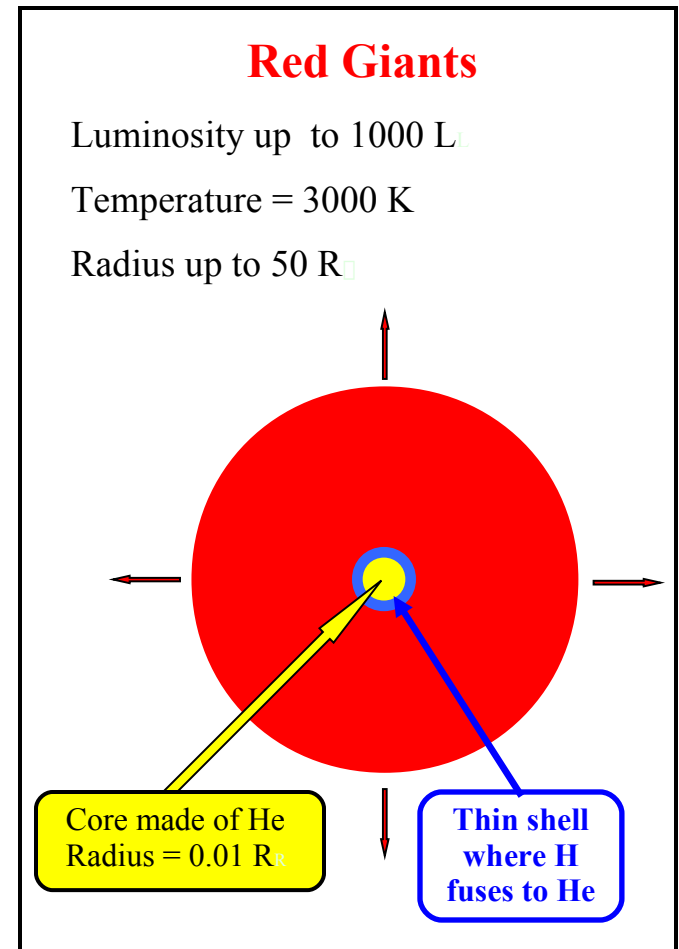
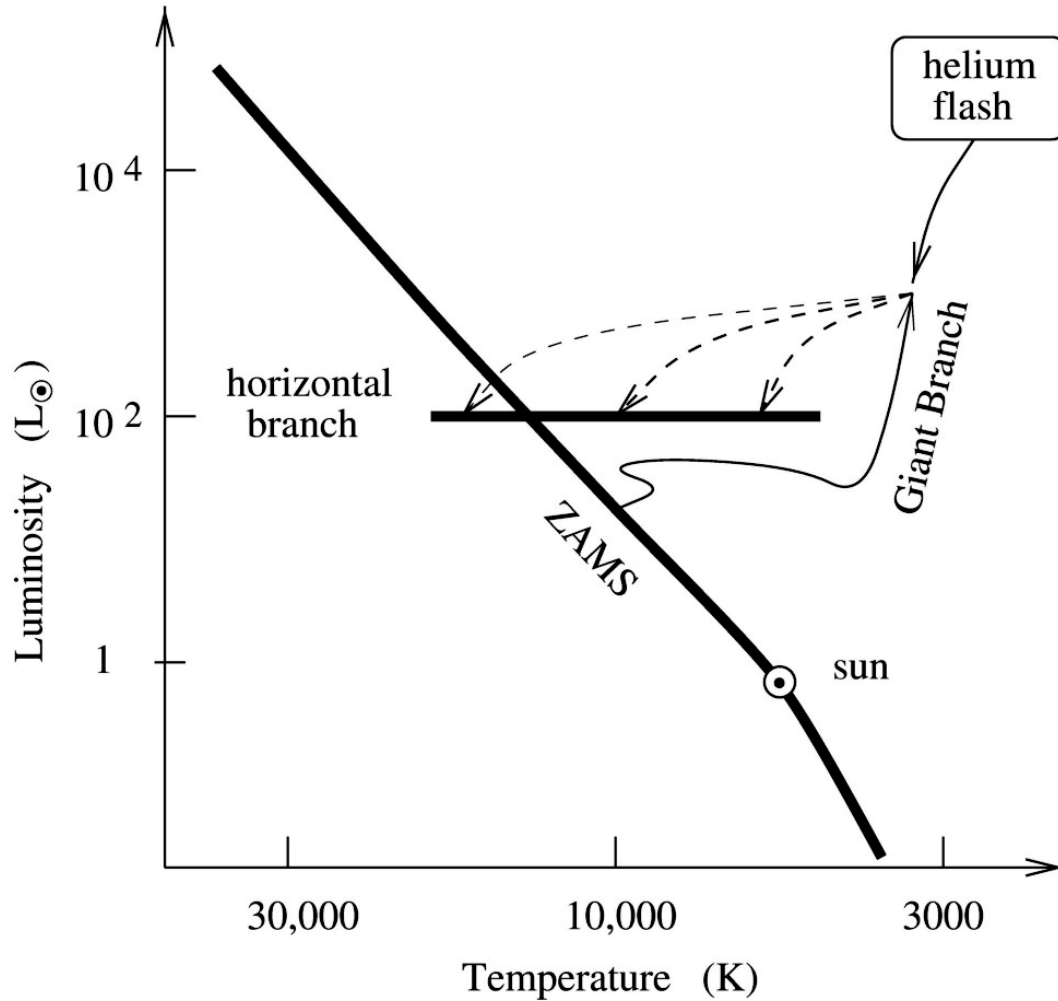


Fig. 6.3. $\rho - T$ for Ignition Masses

mass [M_{Ch}]	T_{max} [$10^9 K$]	$Y_{e\rho}$ [$g\ cm^{-3}$]
0.100	0.0392	1.79×10^4
0.316	0.195	2.88×10^5
0.501	0.399	1.12×10^6
0.631	0.586	2.77×10^6
0.794	0.929	1.07×10^7
0.891	1.250	3.65×10^7
0.931	1.530	9.66×10^7
0.966	2.170	2.84×10^8
0.983	3.060	8.08×10^8
0.986	3.430	1.13×10^9

Evolution of a $2 M_{\odot}$ Star to the Horizontal Branch



The Internal Structure of Stars on the Horizontal Branch

Radius = $10 R_{\odot}$

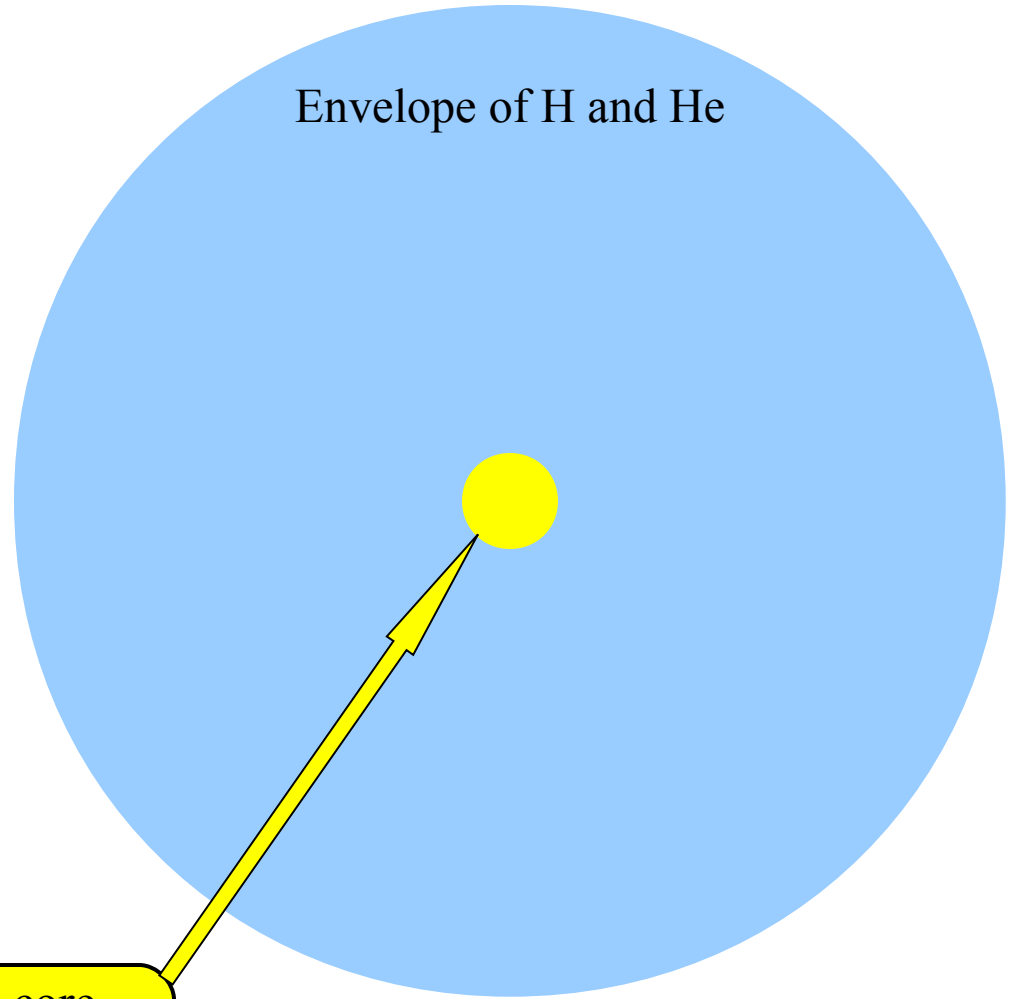
Luminosity = $100 L_{\odot}$

Temperature = 20,000 K
to 5000 K

Helium core
fusing He to C
Radius = $1 R_{\odot}$

Envelope of H and He

And H in a very thin shell



The Structure of a Star on the Asymptotic Giant Branch

Radius up to $1000 R_{\odot}$

Luminosity up to $10^5 L_{\odot}$

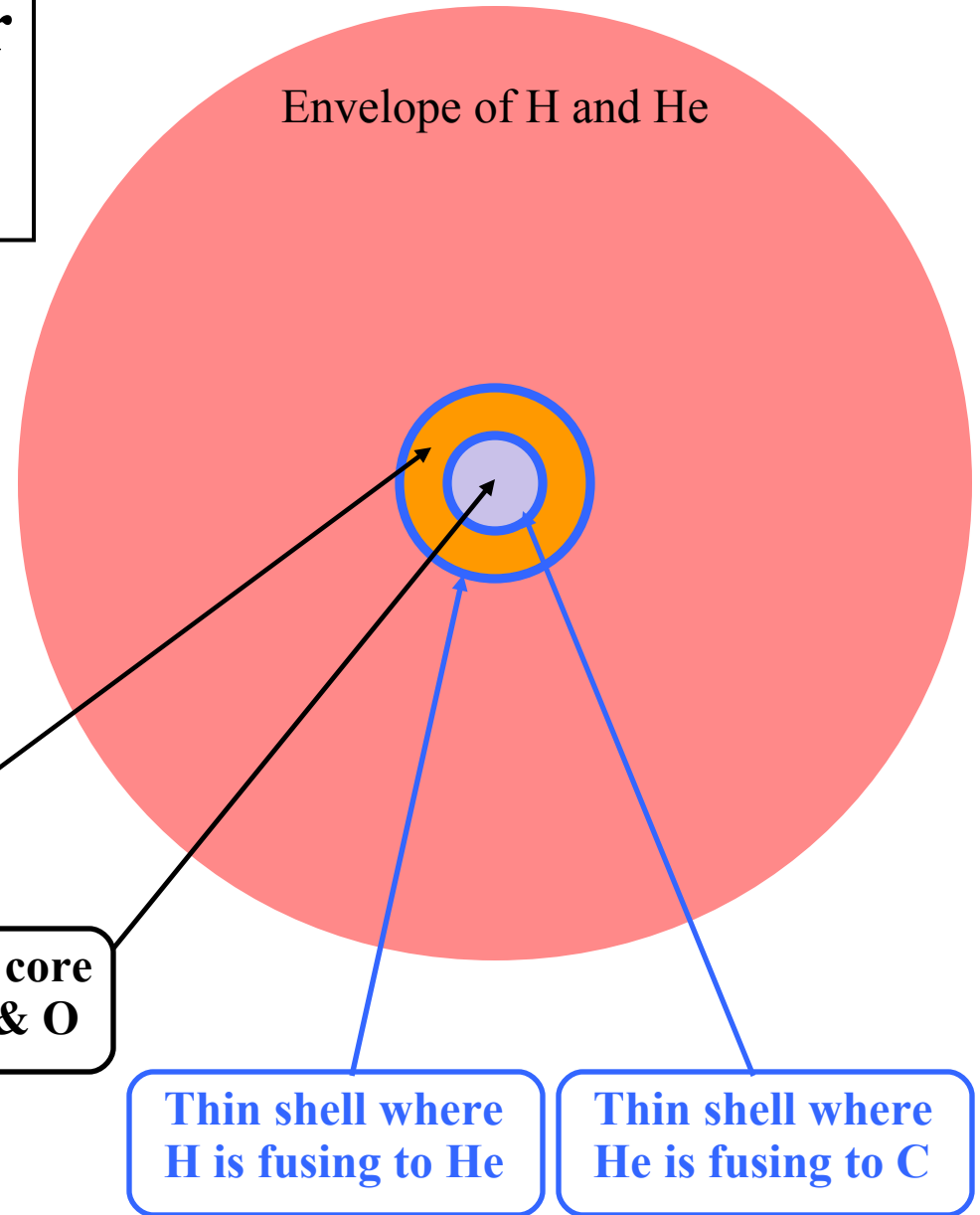
Temperature near 3000 K

Outer core
of He

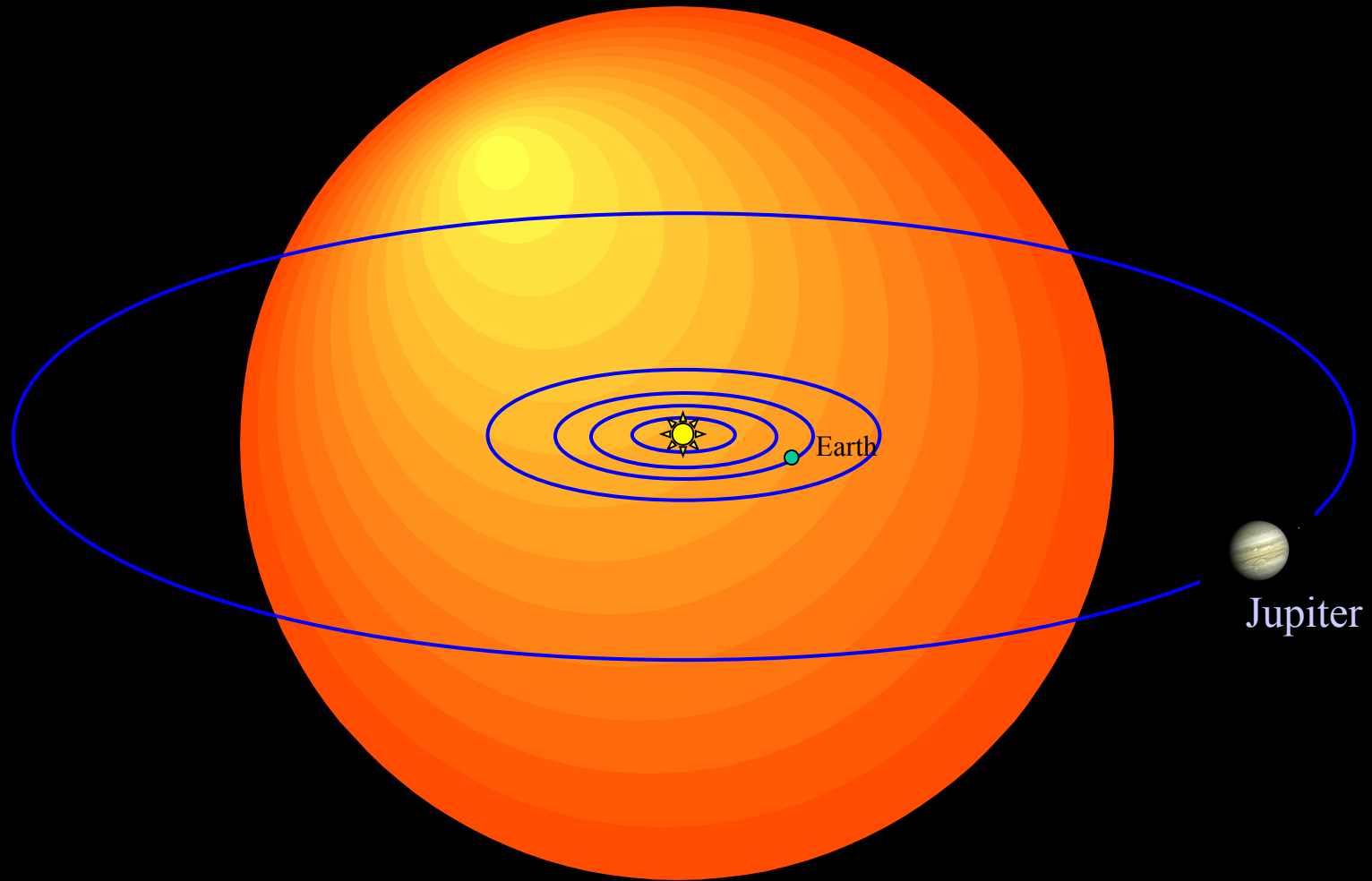
Inner core
of C & O

The radius of the core is
only $0.01 R_{\odot}$.

It is not shown to scale!

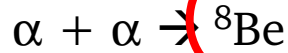


AGB Star Compared to the Solar System



Helium burning 1 – the 3α process

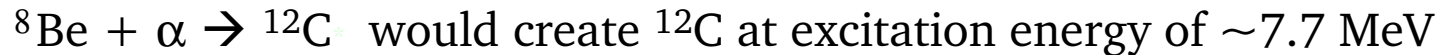
Step 1:



unbound by ~ 92 keV – decays to 2α in $2.6 \cdot 10^{-16}$ s

=> small equilibrium abundance of ${}^8\text{Be}$ is established, nevertheless

Step 2:

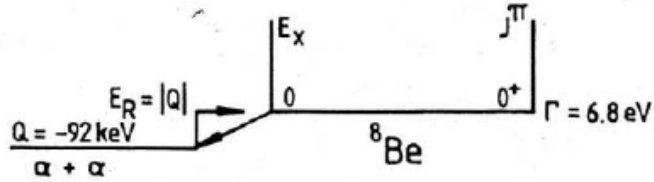
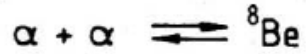


1954 Fred Hoyle realized that the fact that there is carbon in the universe requires a resonance in ${}^{12}\text{C}$ at ~ 7.7 MeV excitation energy

1957 discovered by Cook, Fowler, Lauritsen and Lauritsen at CALTECH with the correct properties (at 7.654 MeV)

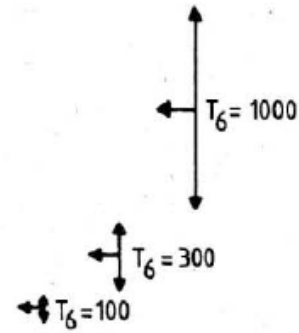
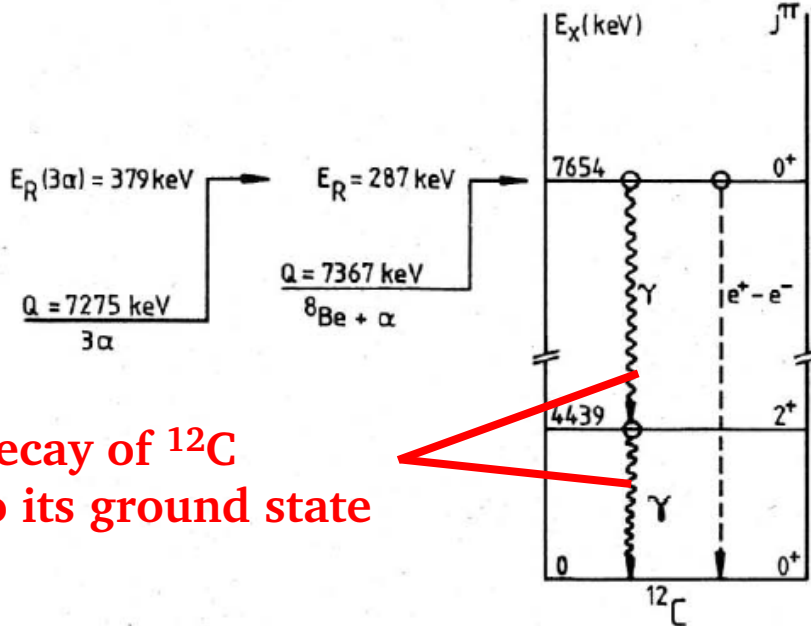
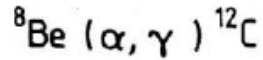
Step 3: Gamma-decay

FIRST STEP :



${}^8\text{Be}$ ground state is in a 92 keV resonance for the $\alpha + \alpha$ reaction

SECOND STEP :

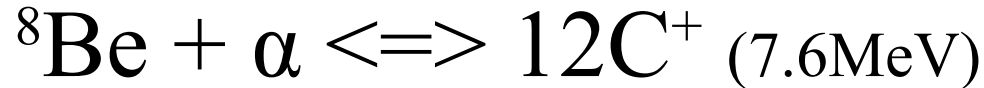
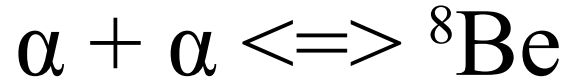


γ decay of ${}^{12}\text{C}$ into its ground state



Total Reaction Rate in Triple α /Part I:

Equilibrium reactions during first steps (fast and small energies)



C (7.6 MeV) is given by Equilibrium Equation:

$$Y_{12\text{C}(7.6\text{MeV})} \left(\frac{2\pi\hbar^2}{m_{12\text{C}} kT} \right)^{3/2} = Y_{4\text{He}}^3 \rho^2 N_A^2 \left(\frac{2\pi\hbar^2}{m_{4\text{He}} kT} \right)^{9/2} e^{-Q/kT}$$

with $Q/c^2 = m_{12\text{C}(7.7)} - 3m_\alpha$ and $m(12\text{C}) \approx 3 m(\alpha)$

$$Y_{12\text{C}(7.6\text{MeV})} = 3^{3/2} Y_{4\text{He}}^3 \rho^2 N_A^2 \left(\frac{2\pi\hbar^2}{m_{4\text{He}} kT} \right)^3 e^{-Q/kT}$$

Total Reaction Rate in Triple α /Part II:

Gamma-decay-reaction determines speed/ total reaction rate !!!

$$r = Y_{12C(7.6\text{MeV})} \rho N_A \frac{\Gamma_\gamma}{\hbar}$$

Rem:: r is dominated by resonance

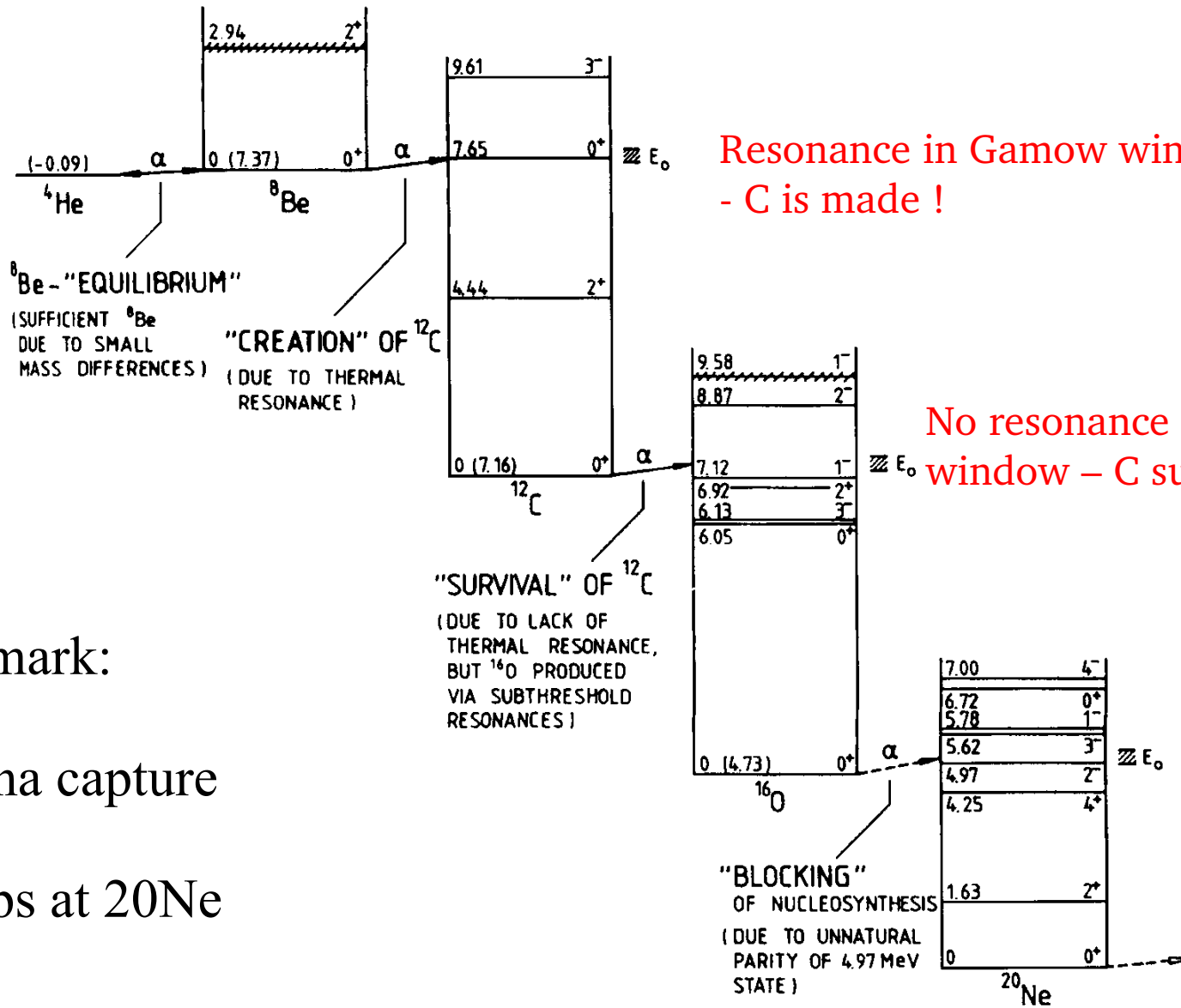
Reaction rate of Helium burning in stellar environment:

$$r(3\alpha) = 1/6 Y_\alpha^2 \rho^2 N_A^2 \langle \alpha\alpha\alpha \rangle$$

$$N_A^2 \langle \alpha\alpha\alpha \rangle = 6 \cdot 3^{3/2} \rho^2 N_A^2 \left(\frac{2\pi \hbar^2}{m_{4\text{He}} kT} \right)^3 \frac{\Gamma_\gamma}{\hbar} e^{-Q/kT}$$

Energy generation is dominated by this transition¹⁶

Helium burning/Part 2: $^{12}\text{C}(\alpha,\gamma)$



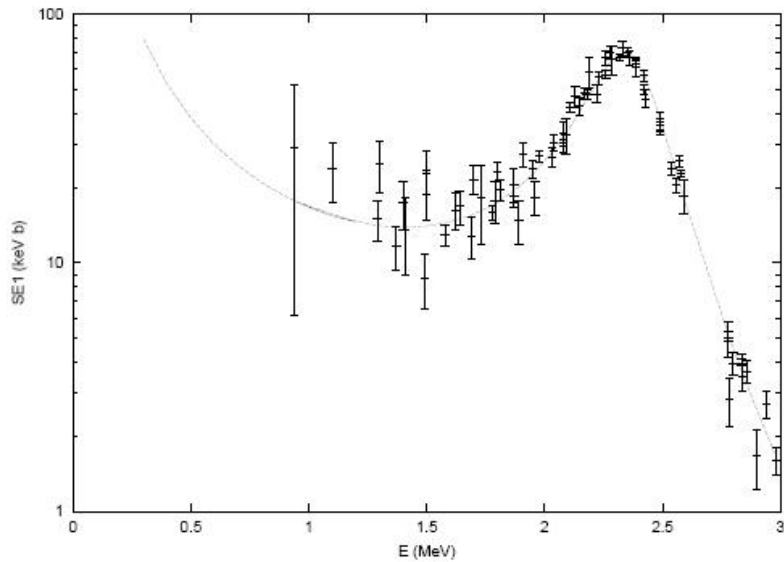
Remark:

alpha capture

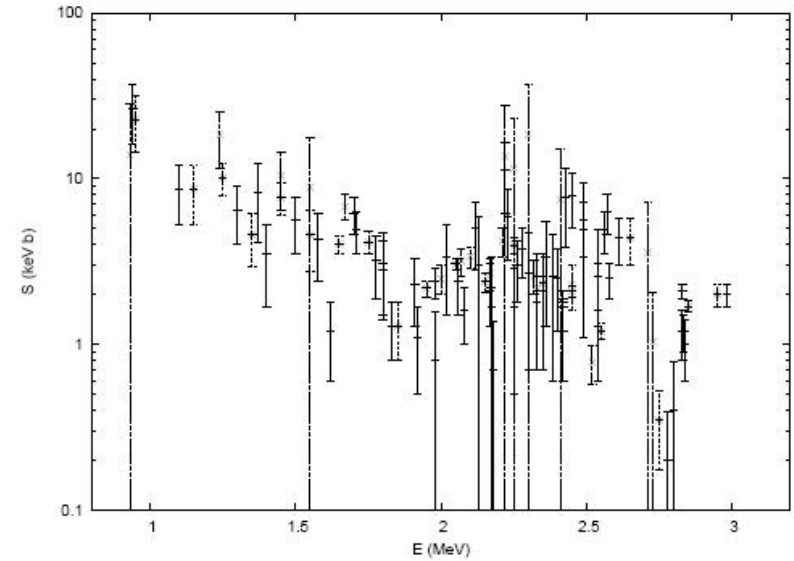
stops at ^{20}Ne

But some C is converted into O ...

E1 and E2 $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ Data



E1 data and S-factor fit



E2 data

Total $S(300 \text{ keV})$ is about $170 \pm 40 \text{ keV b}$.

Therefore:

Uncertainty in the $^{12}\text{C}(\alpha,\gamma)$ rate is the single most important nuclear physics uncertainty in astrophysics

- Affects:
- C/O ration \rightarrow further stellar evolution (C-burning or O-burning ?)
 - iron (and other) core sizes (outcome of SN explosion)
 - Nucleosynthesis (see next slide)

Some current results for $S(300 \text{ keV})$:

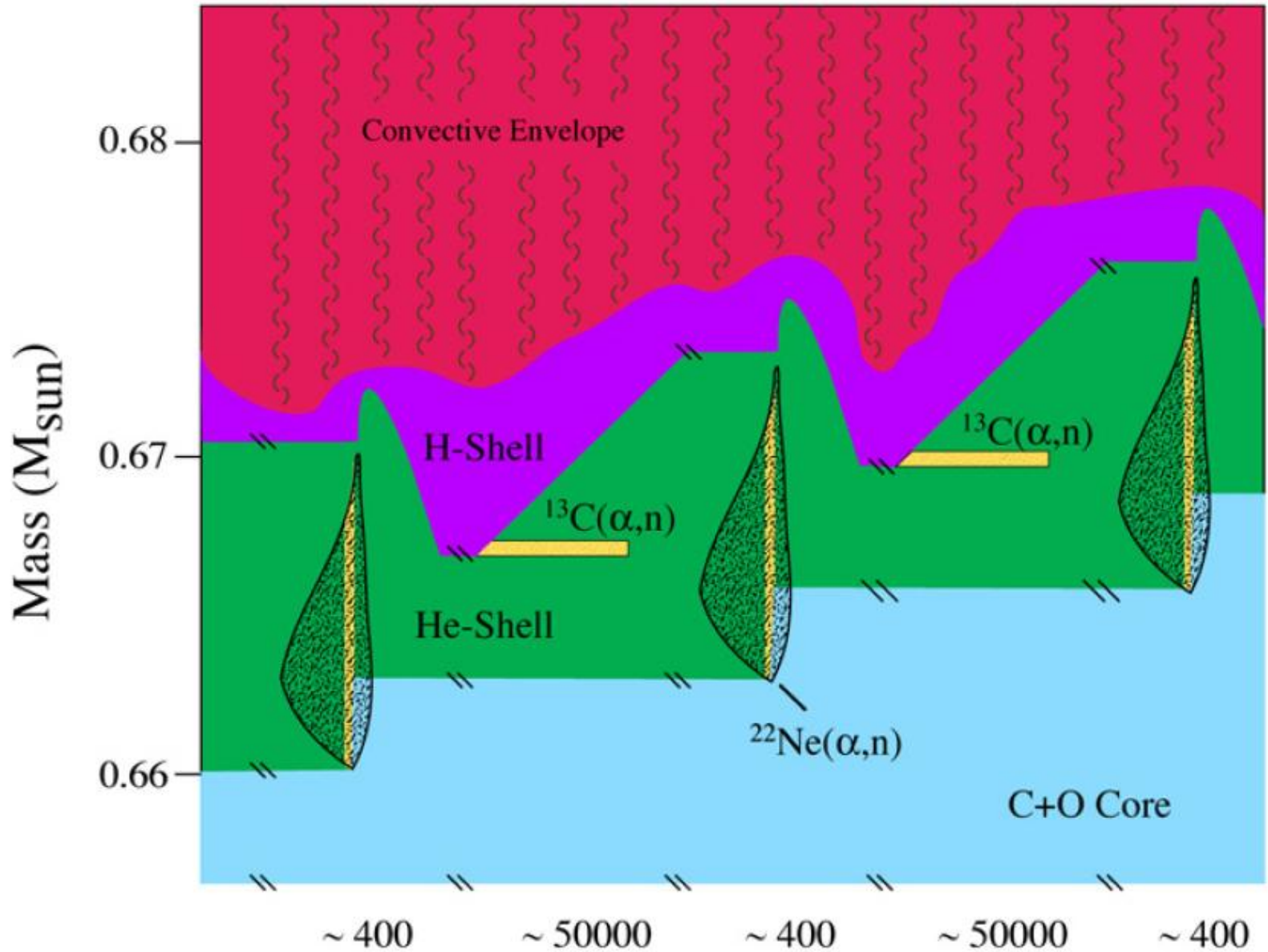
$S_{12} = 53 + 13 - 18 \text{ keV b}$ (Tischhauser et al. PRL88(2002)2501)

$S_{12} = 79 + 21 - 21 \text{ keV b}$ (Azuma et al. PRC50 (1994) 1194)

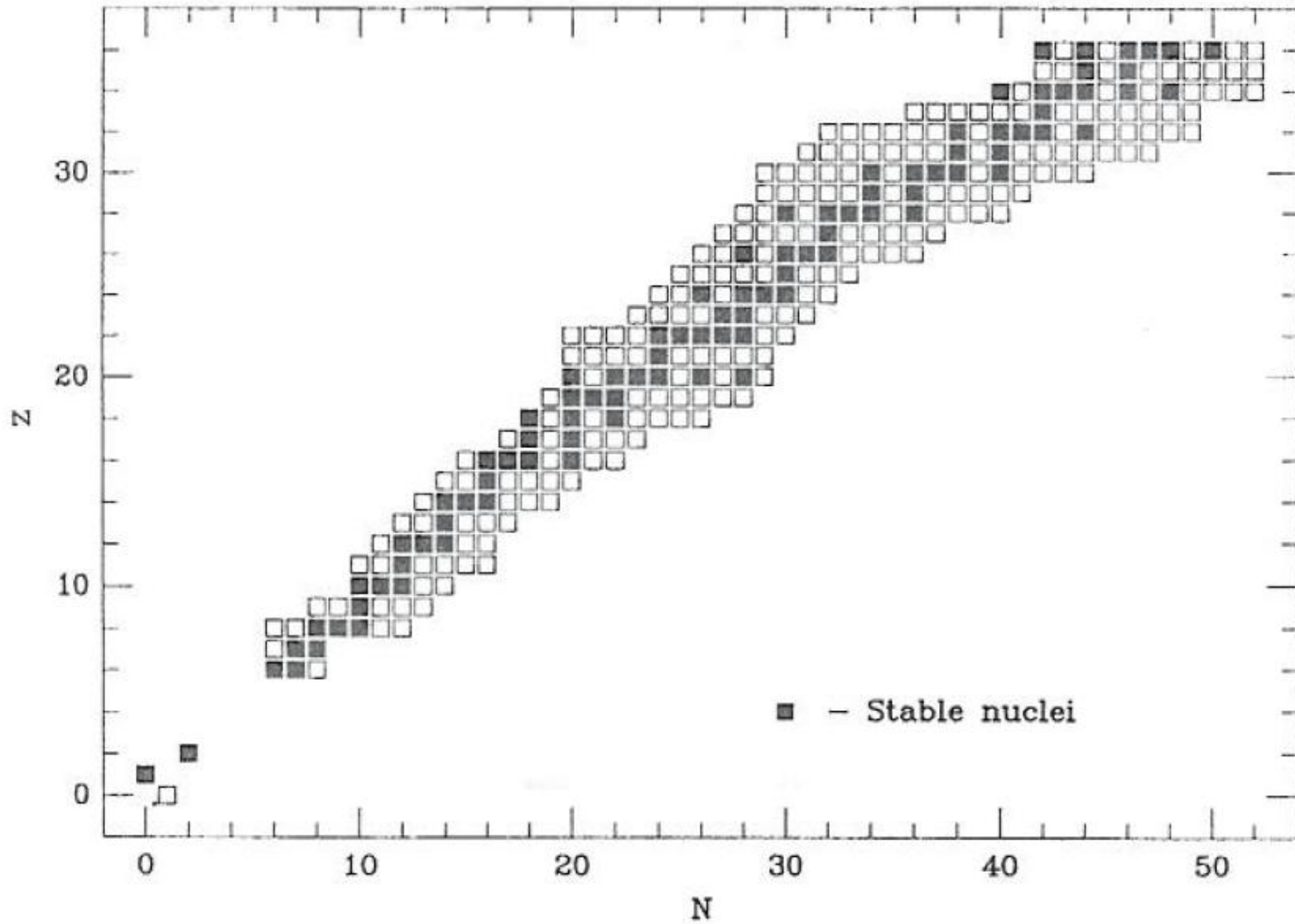
But others range among groups larger !

The 3rd Dredge Up in Massive Stars

25 Mo

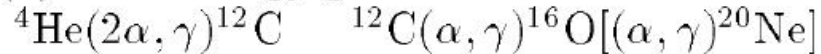


Extended alpha network for massive stars

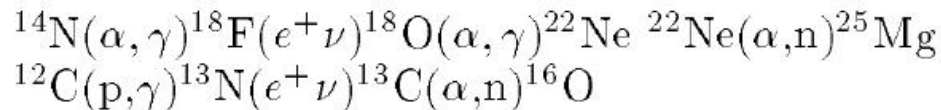


Major Reactions during He Burning

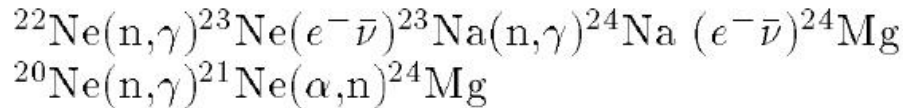
(a) basic energy generation



(b) neutron sources



(c) high temperature burning with neutron sources

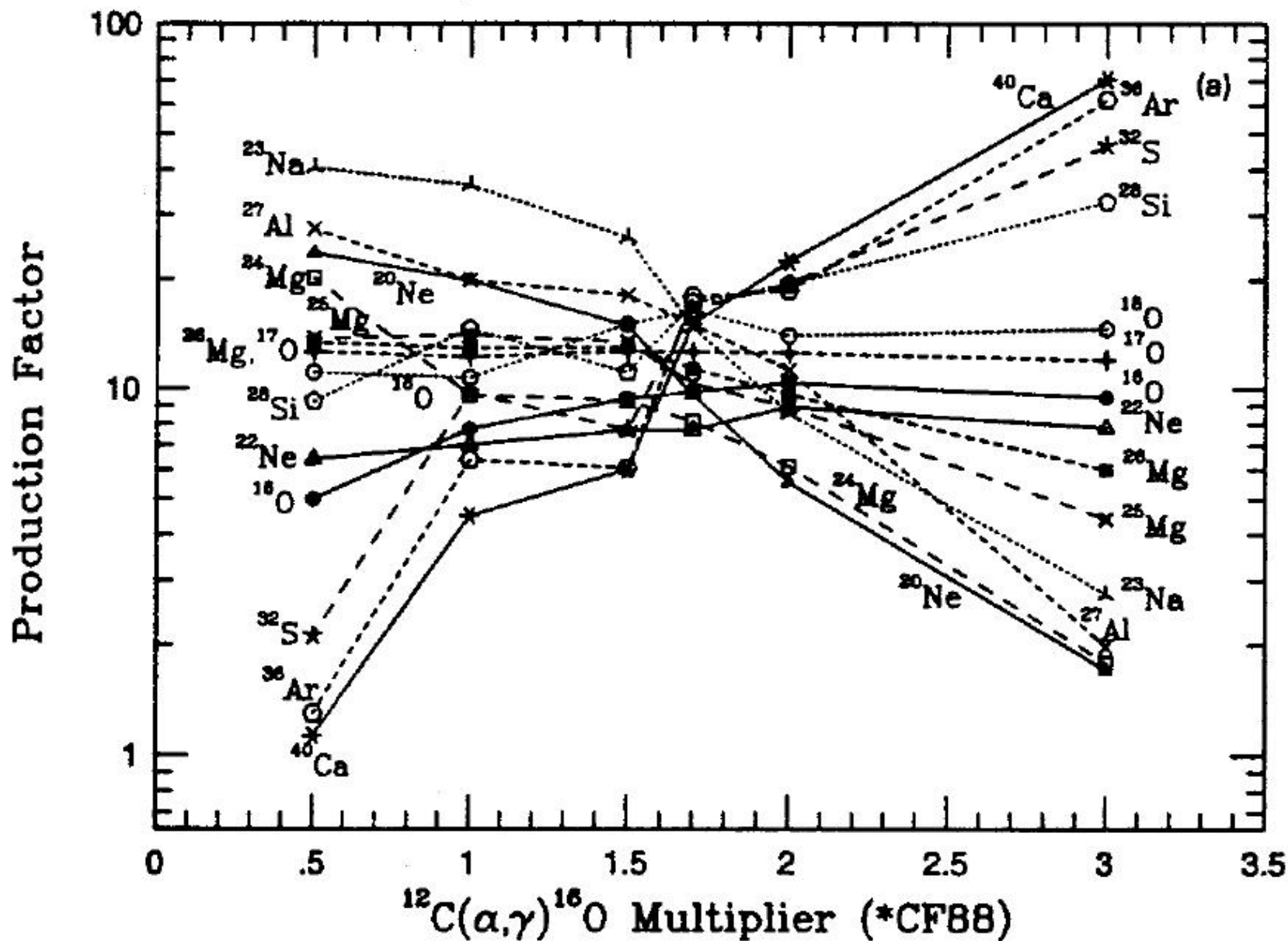


further s-processing via neutron captures and β -decays ${}^{24}\text{Mg}(n, \gamma){}^{25}\text{Mg}$ etc.
production of heavy elements ${}^{56}\text{Fe}(n, \gamma){}^{57}\text{Fe}(n, \gamma){}^{58}\text{Fe}$ etc.

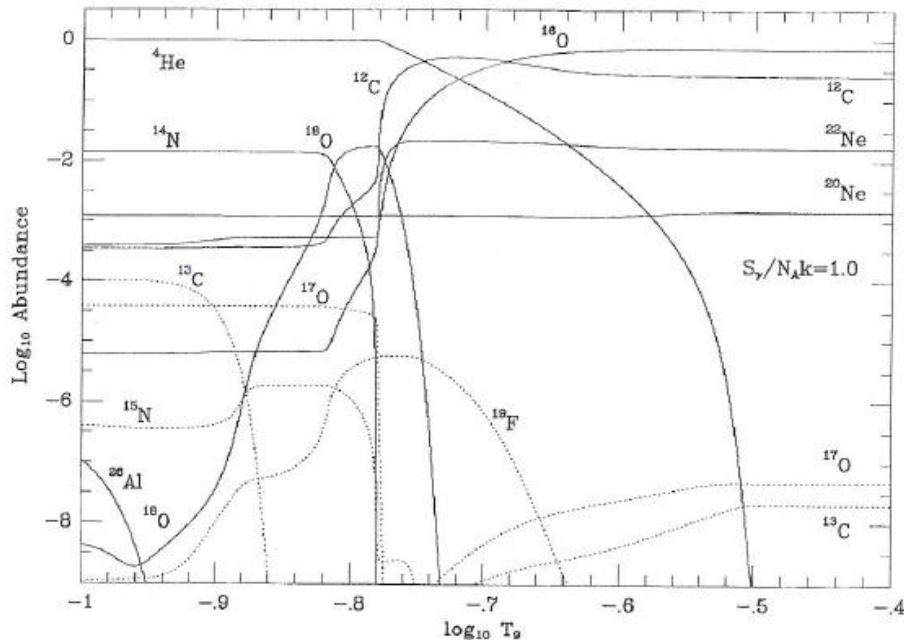
s-process: Slow neutron Capture

- as soon as you produce neutrons, they react because the lack

Influence of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ on Nucleosynthesis



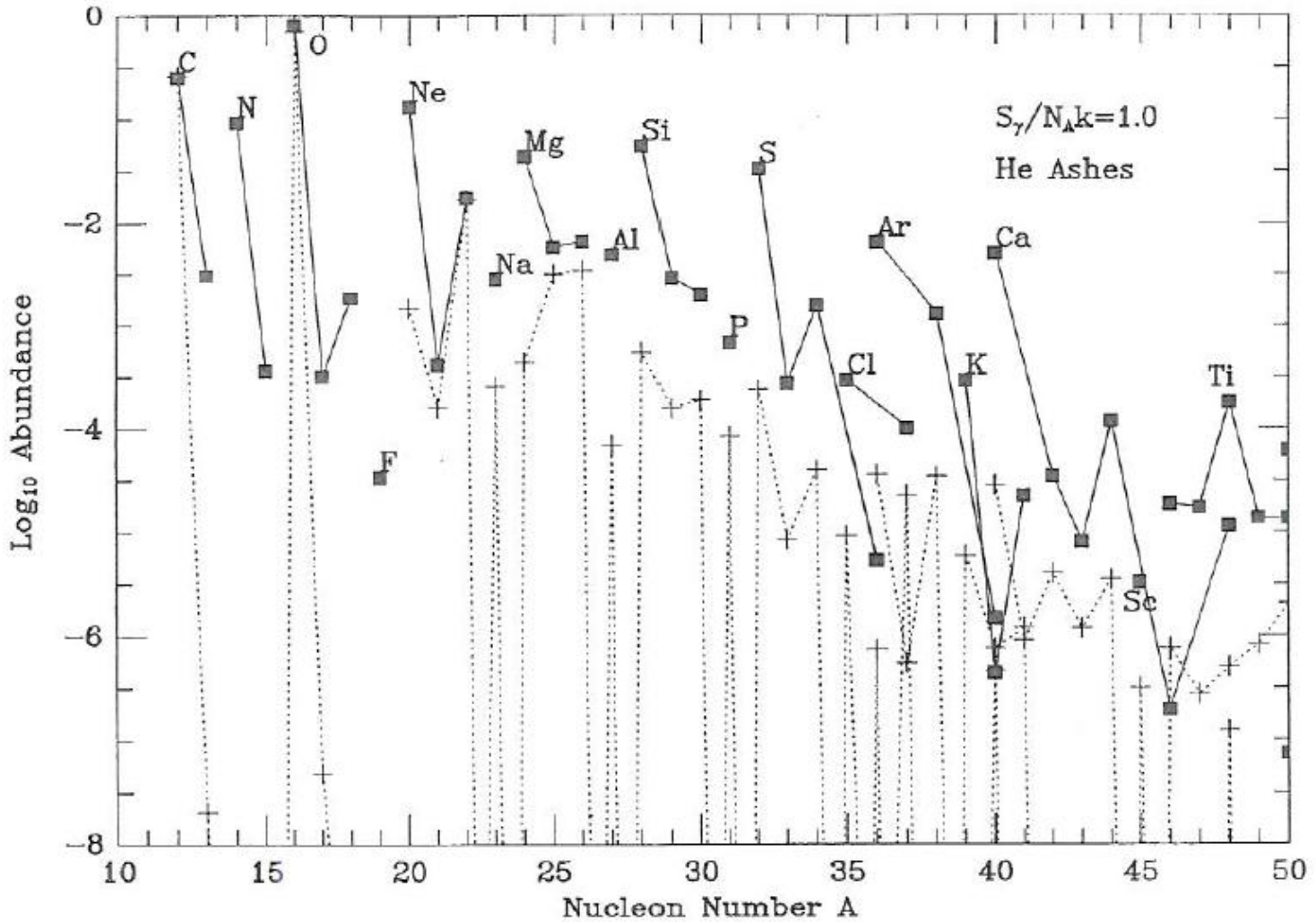
Abundance Structure at the end of the He-core Burning (25Mo)



$T_8 = 1.25 - 1.5$: ^{14}N is depleted, replaced by ^{18}O via $^{14}\text{N}(\alpha, \gamma)^{18}\text{F}(\beta\nu)^{18}\text{O}$;
first nucleus with neutron excess

$T_8 \approx 1.6$: ^4He depletion and main phase of helium burning begins; first ^{12}C , then ^{16}O are produced; ^{22}Ne is produced by competition of $^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}$ and $^{22}\text{Ne}(\alpha, \gamma)^{26}\text{Mg}$ and $^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$; **free neutrons appear**

Abundances compared to solar values



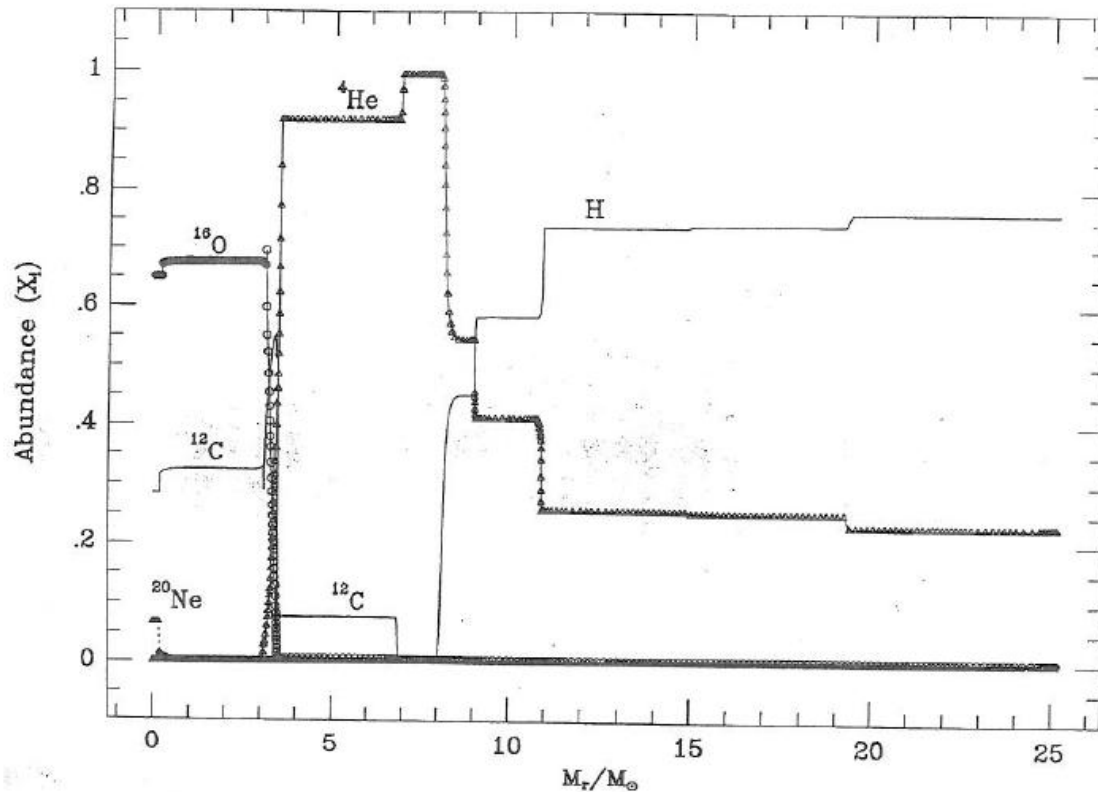
Squares: solar

Post-Main-Sequence Evolution beyond He

- Post He-burning Evolution of Massive Stars
- Alpha capture, NSE and advanced stages of stellar evolution

Literature: Iliadis, Chapt. 5.3

Stellar Structure after central He Burning

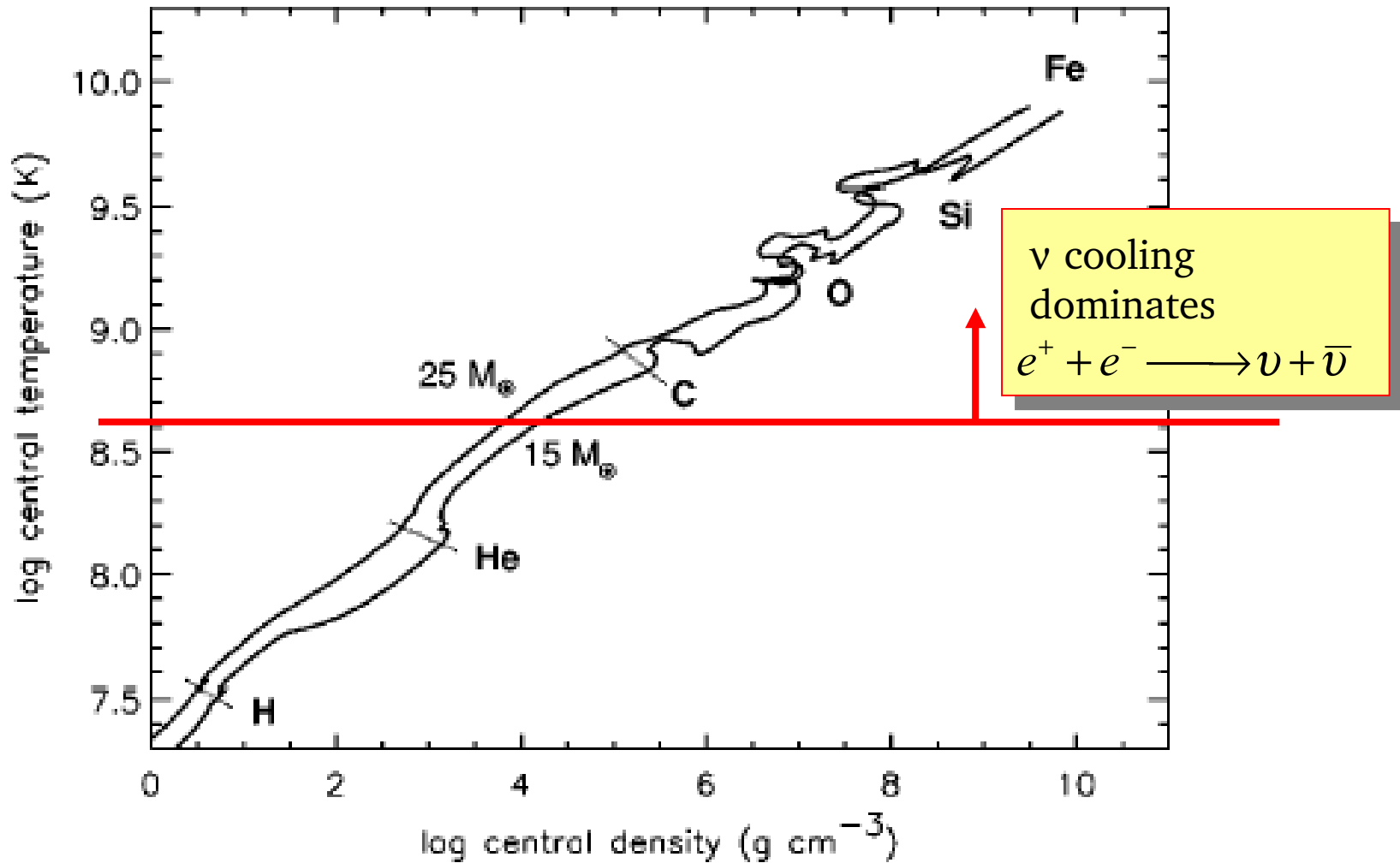


mass fractions of $25 M_{\odot}$ star

- $M(r) > 11 M_{\odot}$: matter not processed
- $8 M_{\odot} < M(r) < 11 M_{\odot}$: partial hydrogen burning
- $6 M_{\odot} < M(r) < 8 M_{\odot}$: complete H, but no He burning
- $3.3 M_{\odot} < M(r) < 6 M_{\odot}$: incomplete He burning, ^{12}C enhanced, no ^{16}O
- $M(r) < 3.3$: complete He burning, proceeds to advanced burning

(Weaver & Woosley, 1998)

Further evolution of burning conditions



Carbon burning

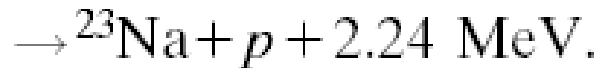
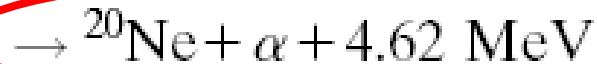
Burning conditions:

for stars $> 8 M_{\odot}$ (solar masses) (ZAMS)

$T \sim 600\text{-}700 \text{ Mio}$

$\rho \sim 10^5\text{-}10^6 \text{ g/cm}^3$

Major reaction sequences:



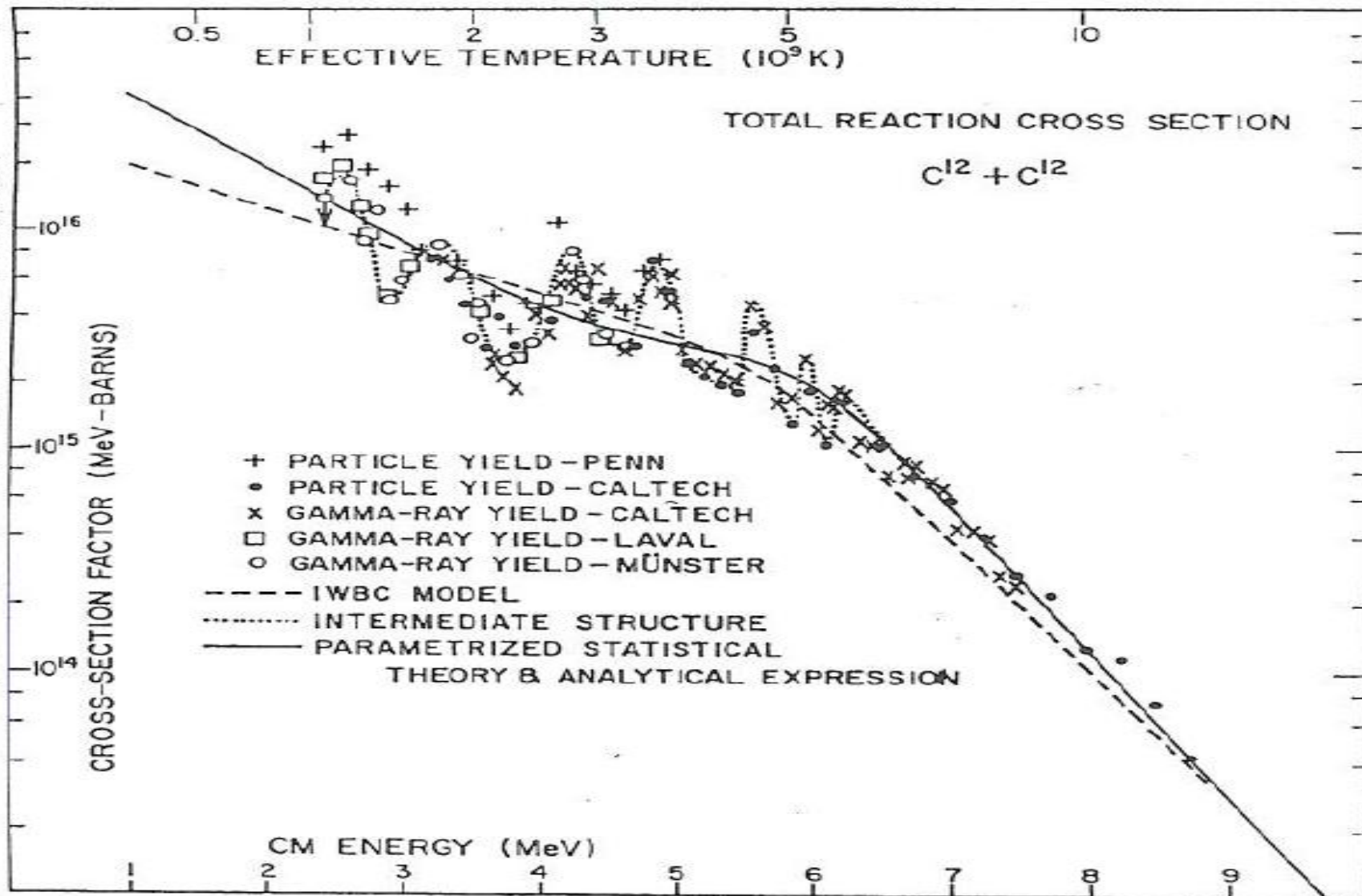
dominates
by far

of course p's, n's, and a's are recaptured ... ^{23}Mg can b-decay into ^{23}Na

Composition at the end of burning:

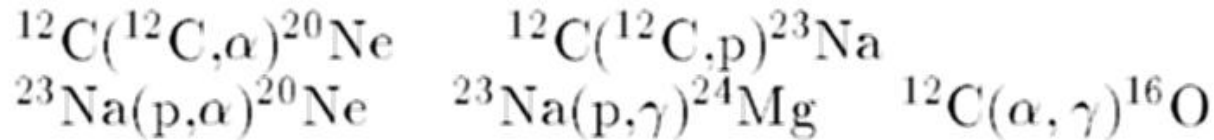
mainly ^{20}Ne , ^{24}Mg , with some $^{21,22}\text{Ne}$, ^{23}Na , $^{24,25,26}\text{Mg}$, $^{26,27}\text{Al}$

$^{12}\text{C}(^{12}\text{C},\gamma)^{24}\text{Mg}$

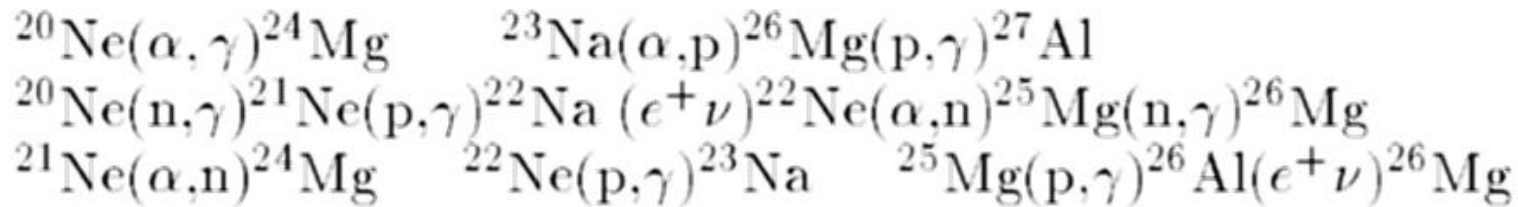


Major Reactions during Carbon Burning

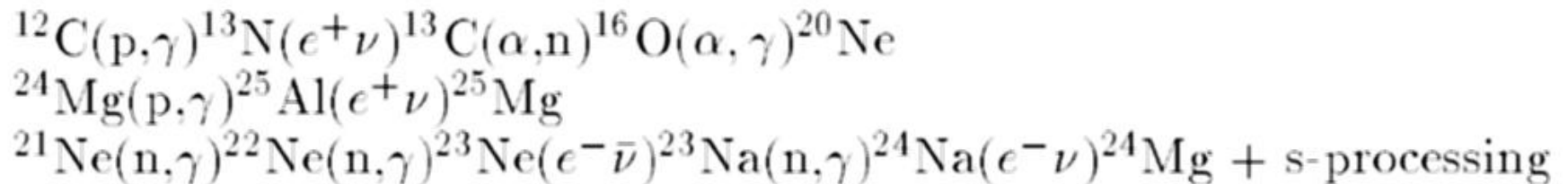
(a) basic energy generation



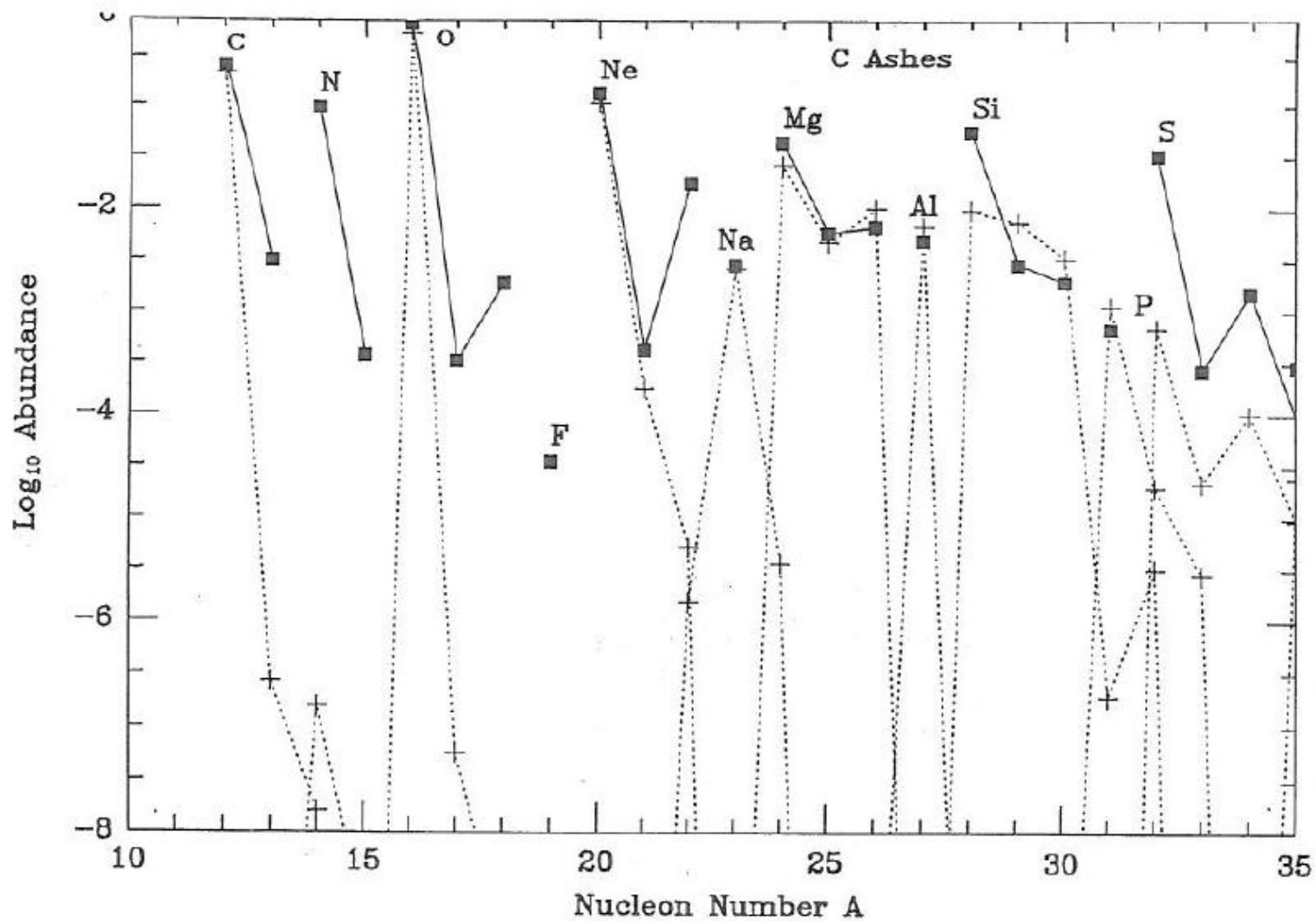
(b) fluxes $> 10^2 \times$ (a)



(c) low temperature, high density burning



Final composition of Carbon Dummies



Neon burning

Burning conditions:

for stars $> 12 M_{\odot}$ (solar masses) (ZAMS)

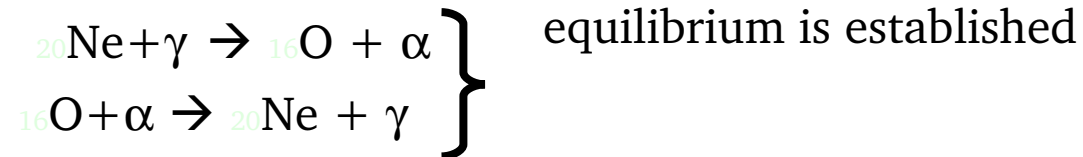
$T \sim 1.3\text{-}1.7 \text{ Bio K}$

$\rho \sim 10^6 \text{ g/cm}^3$

Why would neon burn before oxygen ???

Answer:

Temperatures are sufficiently high to initiate **photodisintegration** of ${}^{20}\text{Ne}$



this is followed by (using the liberated helium)

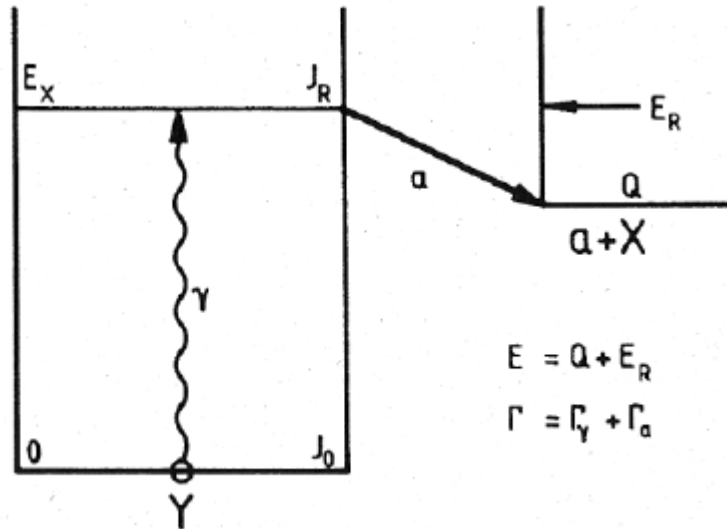
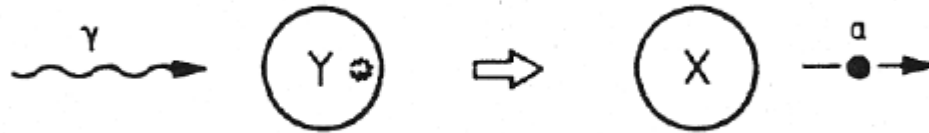


so net effect:



Photodisintegration

PHOTODISINTEGRATION $Y(\gamma, \alpha)X$

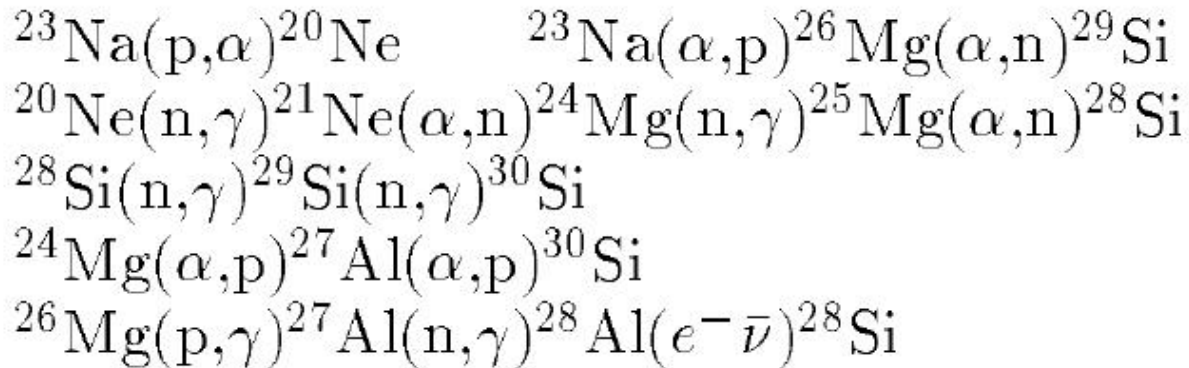


Major Production During Ne-Burning

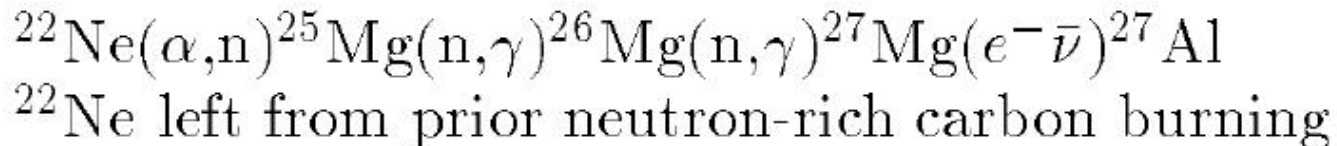
(a) basic energy generation



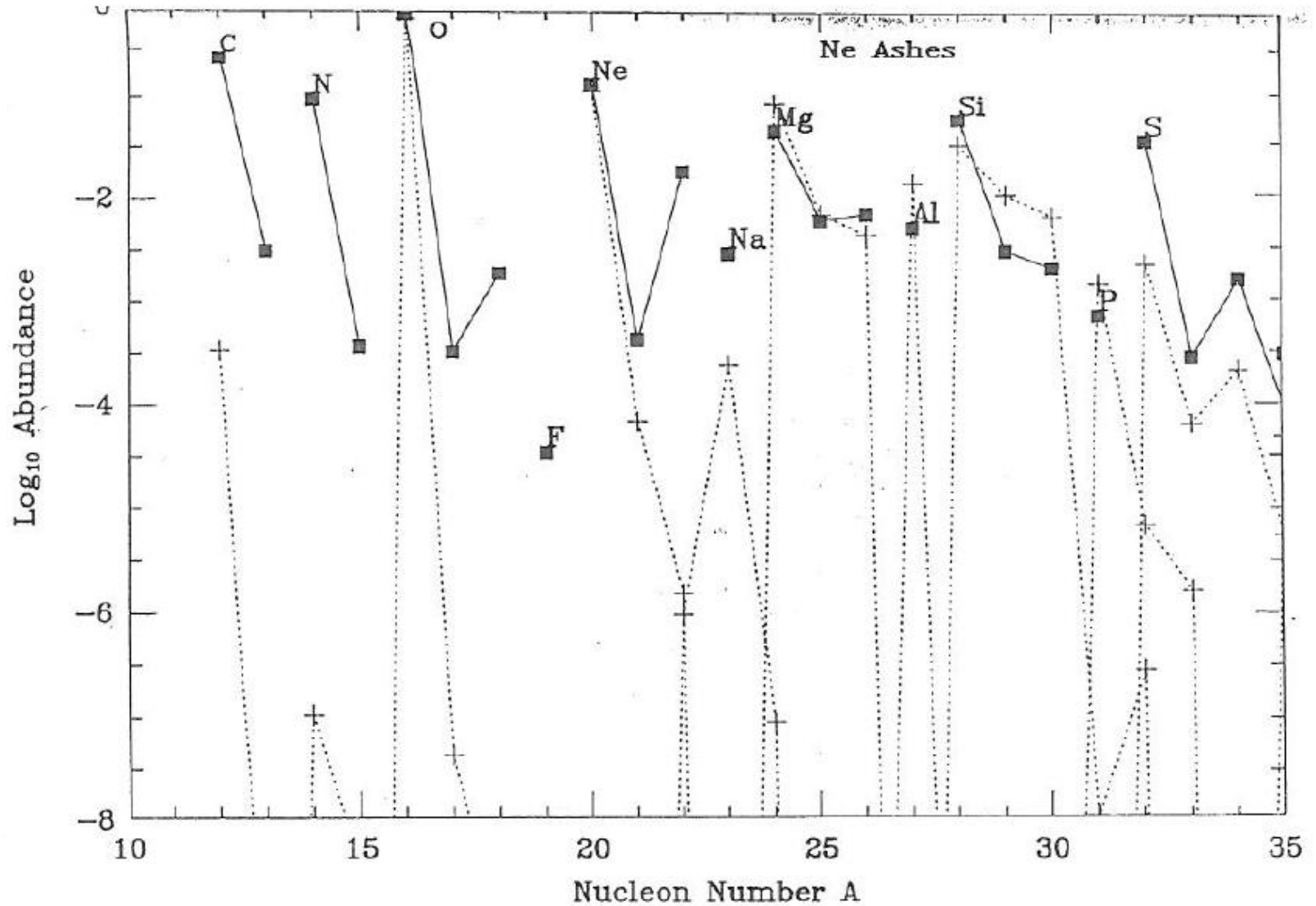
(b) fluxes $> 10^2 \times$ (a)



(c) low temperature, high density burning



Outcome of Ne Burning



Calculations of inverse reaction rates

A reaction rate for a process like ${}^{20}\text{Ne} + \gamma \rightarrow {}^{16}\text{O} + \alpha$ can be easily calculated from the inverse reaction rate ${}^{16}\text{O} + \alpha \rightarrow {}^{20}\text{Ne} + \gamma$ using the formalism developed so far.

In general there is a simple relationship between the rates of a reaction rate and its inverse process (if all particles are thermalized)

Derivation of “detailed balance principle”:

Consider the reaction $A + B \rightarrow C$ with Q-value Q in thermal equilibrium. Then the abundance ratios are given by the Saha equation:

$$\frac{n_A n_B}{n_C} = \frac{g_A g_B}{g_C} \left(\frac{m_A m_B}{m_C} \right)^{3/2} \left(\frac{kT}{2\pi^2} \right)^{3/2} e^{-Q/kT}$$

In equilibrium the abundances are constant per definition. Therefore in addition

$$\frac{dn_C}{dt} = n_A n_B \langle \sigma v \rangle - \lambda_C n_C = 0$$

or
$$\frac{\lambda_C}{\langle \sigma v \rangle} = \frac{n_A n_B}{n_C}$$

If $\langle \sigma v \rangle$ is the $A+B \rightarrow C$ reaction rate, and λ_C is the $C \rightarrow A+B$ decay rate

Therefore the rate ratio is defined by the Saha equation as well !

Using both results one finds

$$\frac{\lambda_C}{\langle \sigma v \rangle} = \frac{g_A g_B}{g_C} \left(\frac{m_A m_B}{m_C} \right)^{3/2} \left(\frac{kT}{2\pi \hbar^2} \right)^{3/2} e^{-Q/kT}$$

or using $m_C \sim m_A + m_B$ and introducing the reduced mass μ

Reminder: Nuclear Abundances in NSE

The ratio of the nuclear abundances in NSE to the abundance of free protons and neutrons is entirely determined by

$$Z \cdot \mu_p + N \cdot \mu_n = \mu_{(Z,N)}$$

which only depends on the chemical potentials

$$\mu = mc^2 + kT \ln \left[\frac{n}{g} \left(\frac{h^2}{2\pi mkT} \right)^{3/2} \right]$$

So all one needs are **density**, **temperature**, and for each nucleus **mass** and **partition function** (**one does not need reaction rates** !! - except for determining whether equilibrium is indeed established)

Solving the two equations on the previous page yields for the abundance ratio:

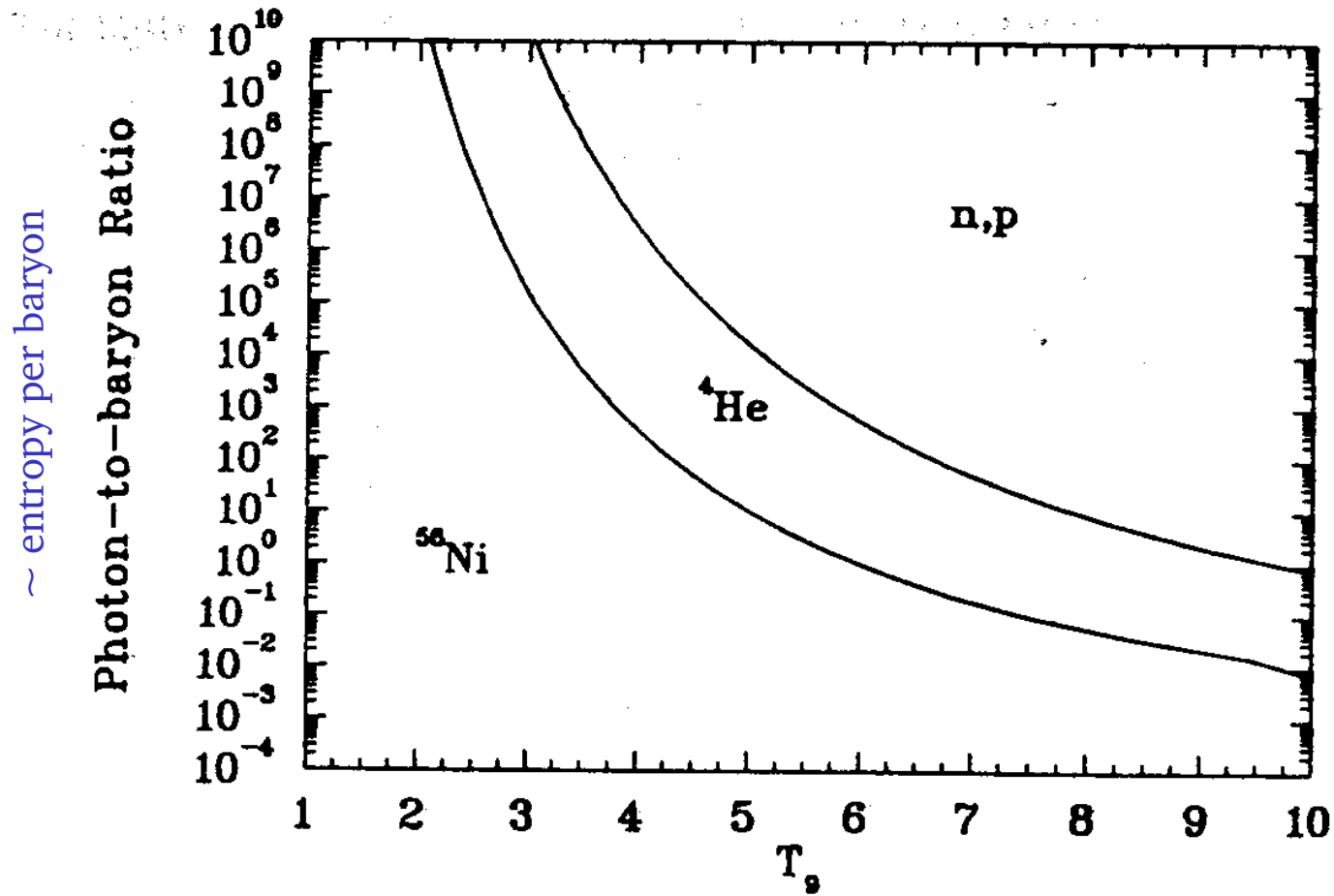
$$Y(Z, N) = Y_p^Z Y_n^N G(Z, N) (\rho N_A)^{A-1} \frac{A^{3/2}}{2^A} \left(\frac{2\pi^2}{m_u kT} \right)^{\frac{3}{2}(A-1)} e^{B(Z, N)/kT}$$

$$\sum_i A_i Y_i = 1$$

$$\sum_i Z_i Y_i = Y_e$$

- higher density favors (heavier) nuclei
- higher temperature favors free nucleons (or lighter nuclei)
- nuclei with high binding energy are strongly favored

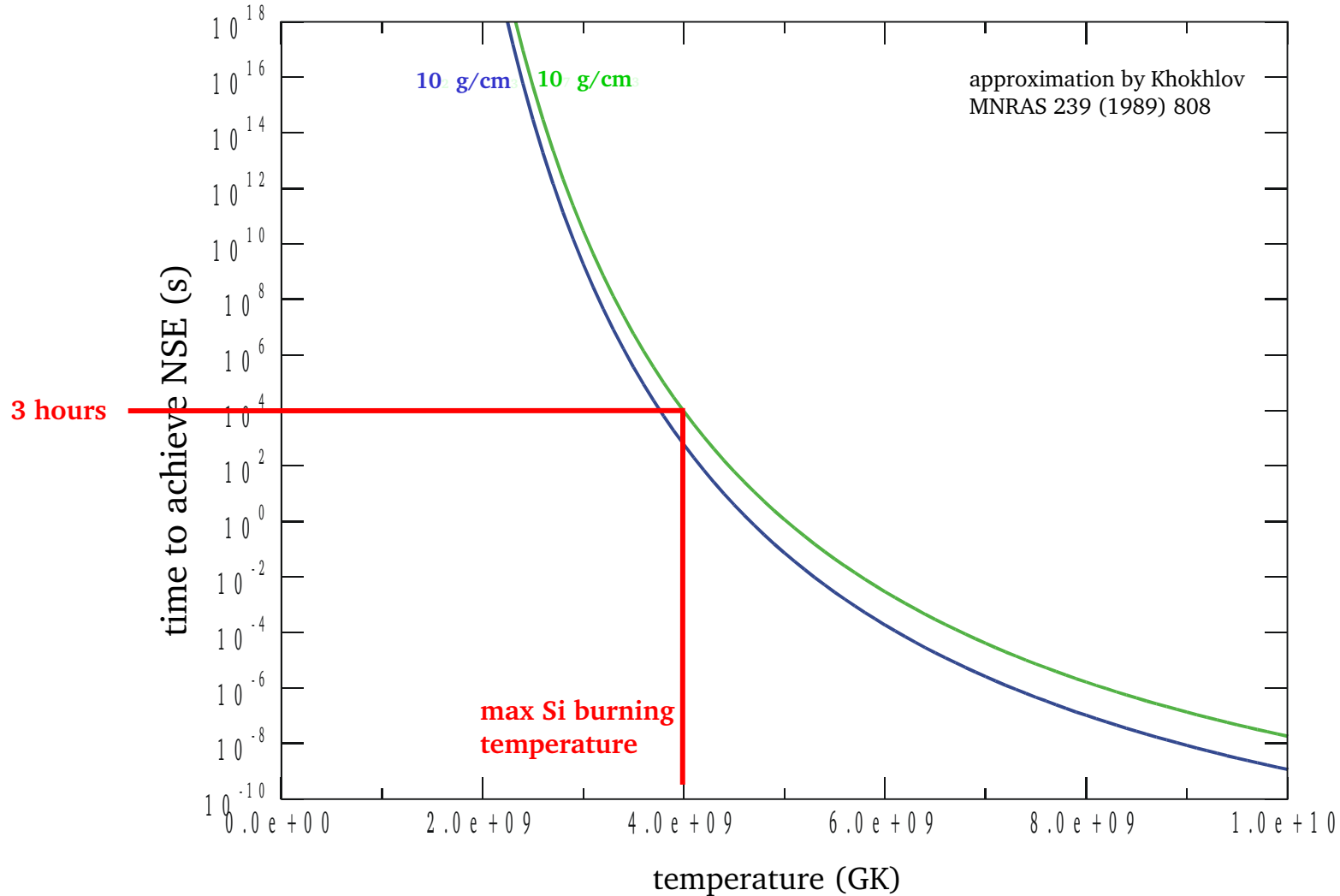
NSE composition ($Y_e=0.5$)



after Meyer, Phys Rep. 227 (1993) 257 “Entropy and nucleosynthesis”

NSE is established on the timescale of these reaction rates (the slowest reaction)

A system will be in NSE if this timescale is shorter than the timescale for the temperature and density being sufficiently high.



for temperatures above ~5 GK even explosive events achieve full NSE

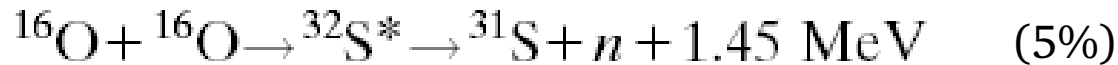
Oxygen burning

Burning conditions:

$$T \sim 2 \text{ Bio}$$

$$\rho \sim 10^7 \text{ g/cm}^3$$

Major reaction sequences:



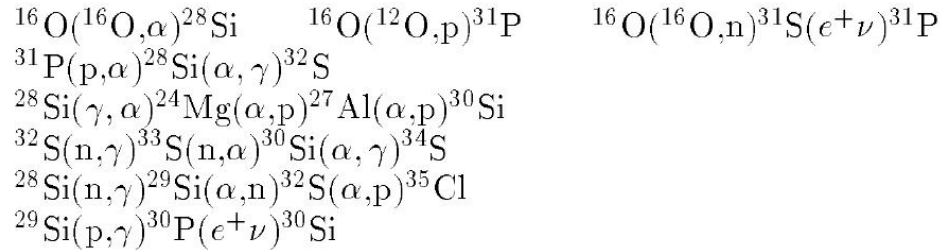
plus recapture of n,p,d, α

Main products: QSE

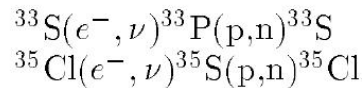
Ca

Major Processes during Oxygen Burning

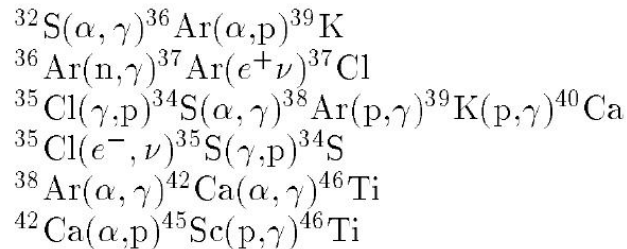
(a) basic energy generation



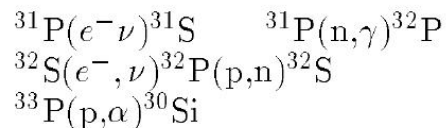
electron captures



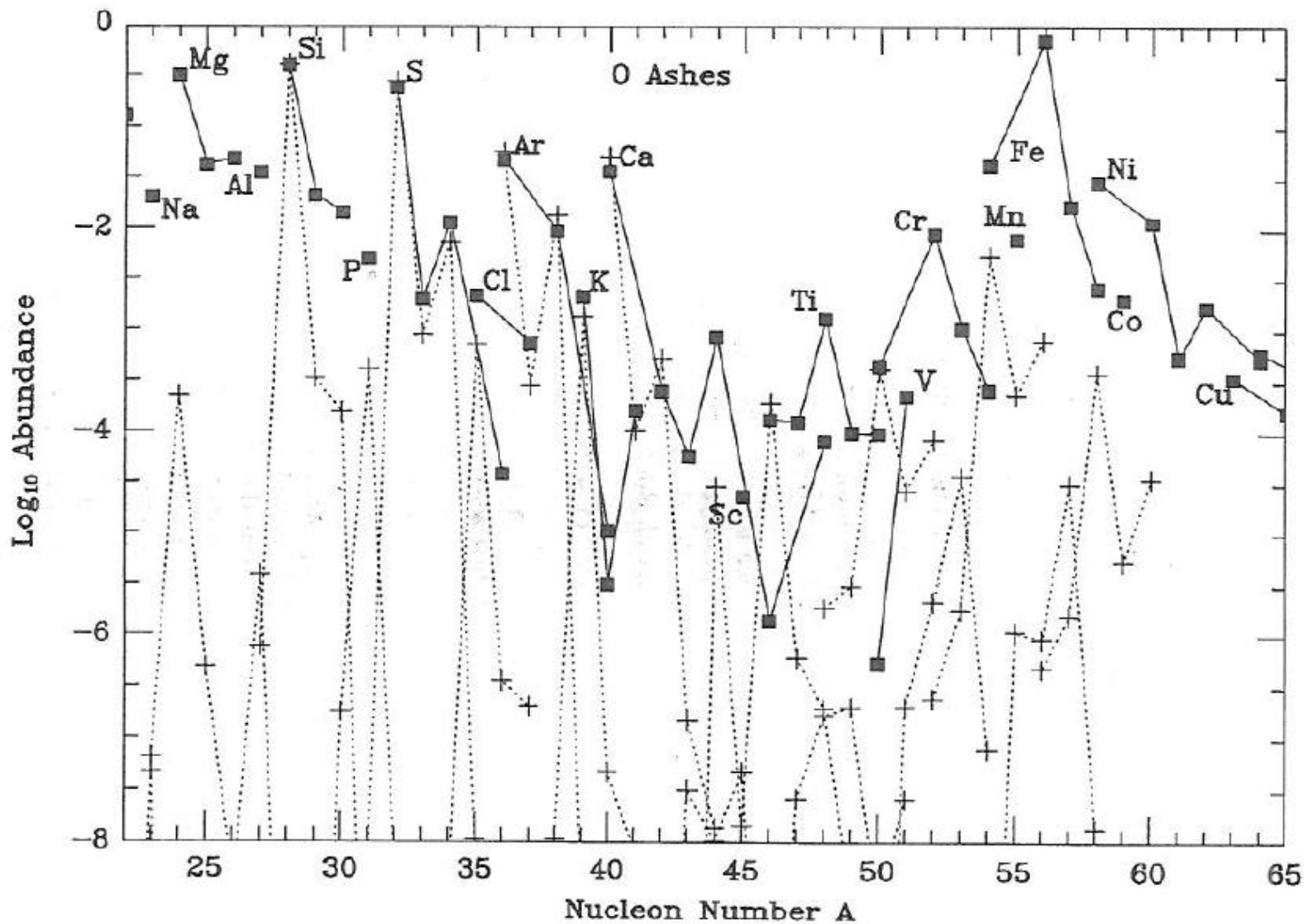
(b) high temperature burning



(c) low temperature, high density burning



Products of hydrostatic Ω burning



Silicon burning

Burning conditions:

$T \sim 3-4 \text{ Bio}$

$\rho \sim 10^9 \text{ g/cm}^3$

Reaction sequences:

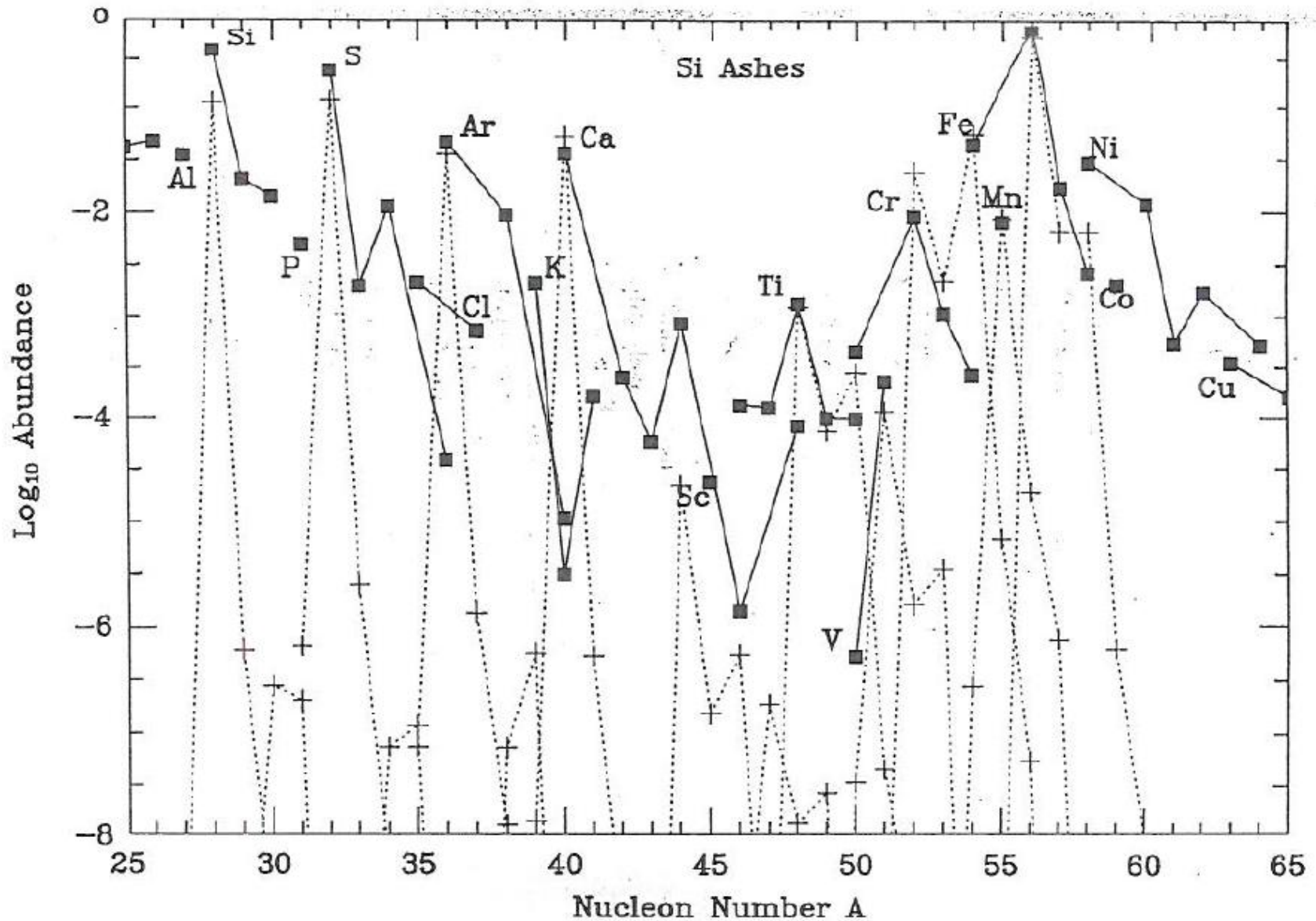
- Silicon burning is fundamentally different to all other burning stages.
- **Complex network of fast (γ, n) , (γ, p) , (γ, α) , (n, γ) , (p, γ) , and (α, γ) reactions**
 - The net effect of Si burning is: $2 \text{ } ^{28}\text{Si} \rightarrow \text{ } ^{60}\text{Ni}$,

need new concept to describe burning:

Nuclear Statistical Equilibrium (NSE)

Quasi Statistical Equilibrium (QSE)

Products of hydrostatic Si Burning



Summary stellar burning

TABLE 8.1 Evolutionary Stages of a $25 M_{\odot}$ Star^a

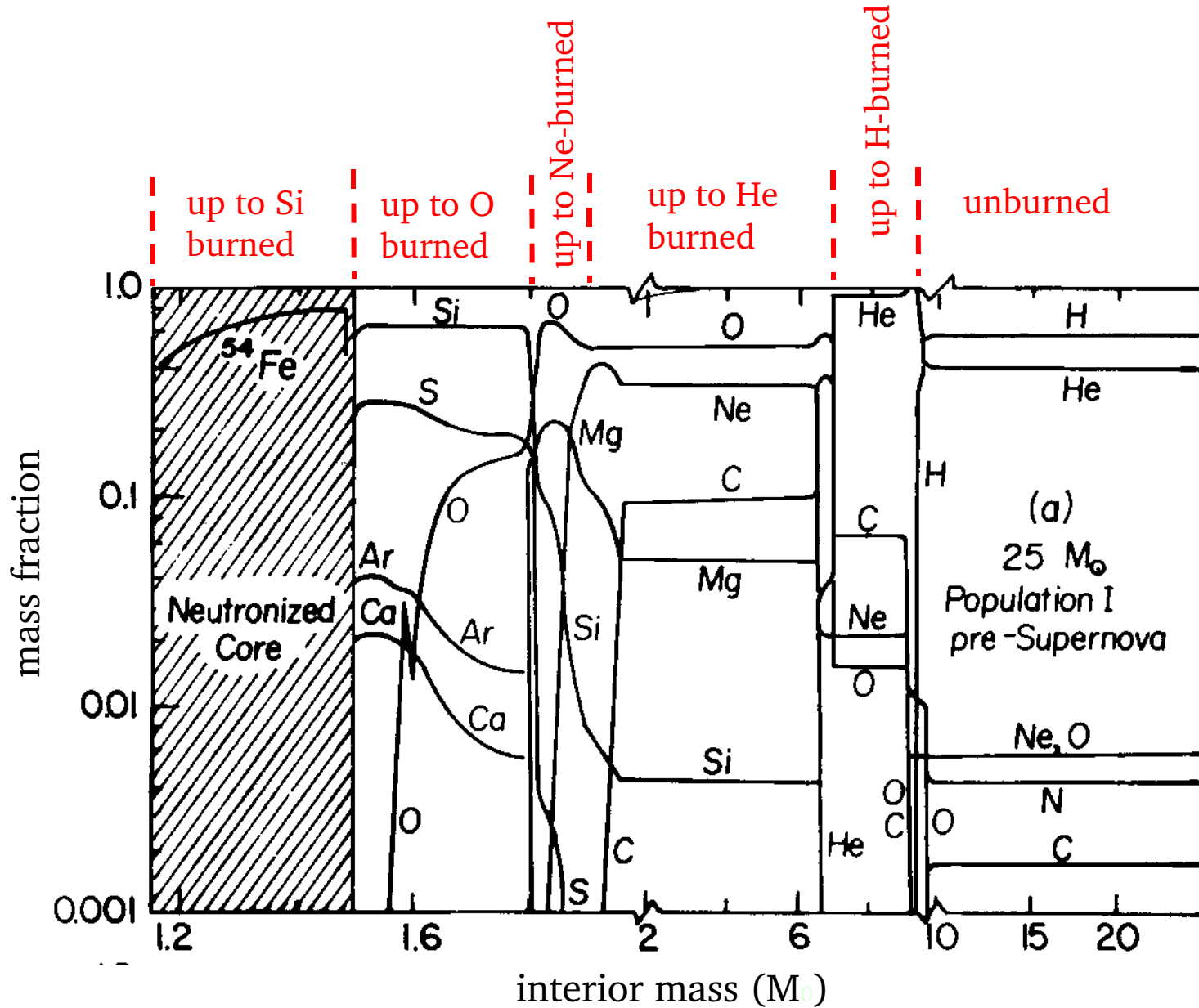
Stage	Time Scale	Temperature (T_9)	Density (g cm^{-3})
Hydrogen burning	7×10^6 y	0.06	5
Helium burning	5×10^5 y	0.23	7×10^2
Carbon burning	600 y	0.93	2×10^5
Neon burning	1 y	1.7	4×10^6
Oxygen burning	6 months	2.3	1×10^7
Silicon burning	1 d	4.1	3×10^7
Core collapse	seconds	8.1	3×10^9
Core bounce	milliseconds	34.8	$\simeq 3 \times 10^{14}$
Explosive burning	0.1–10 s	1.2–7.0	Varies

>0.8M_⊙ ↓
>8M_⊙ ↓
>12M_⊙ ↓

Why do timescales get smaller ?

Note: Kelvin-Helmholtz timescale for red supergiant $\sim 10,000$ years, so for massive stars, no surface temperature - luminosity change for C-burning and beyond

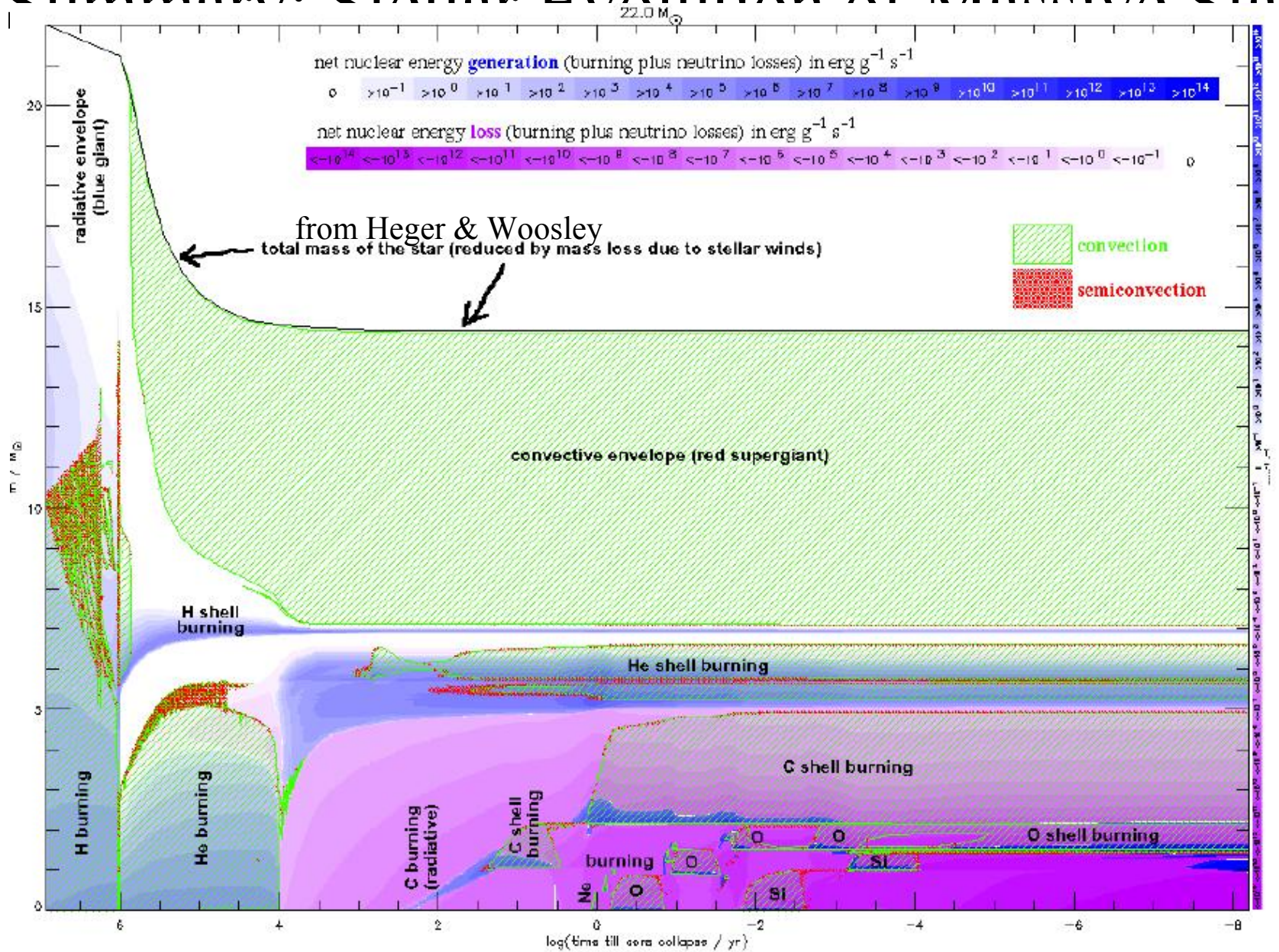
Final composition of a 25 M_⊙ star:



Summary: Main Products of Hydrostatic Burning

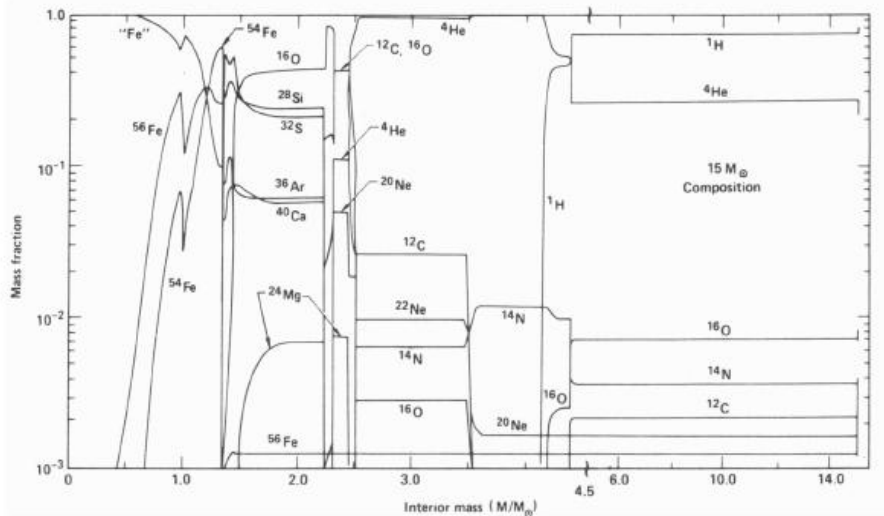
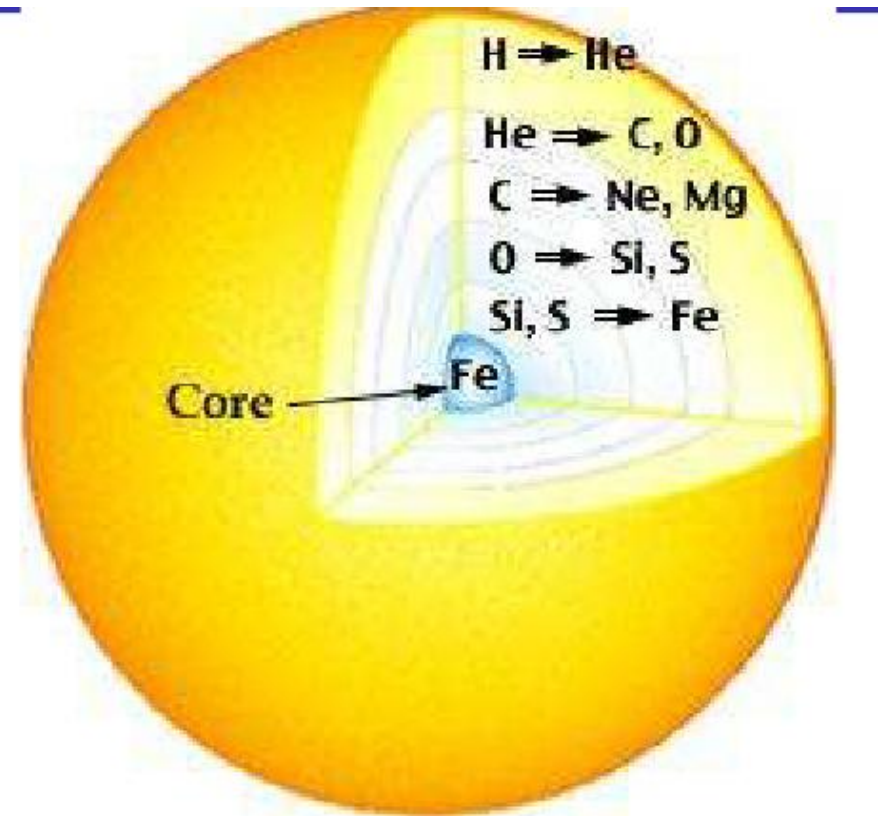
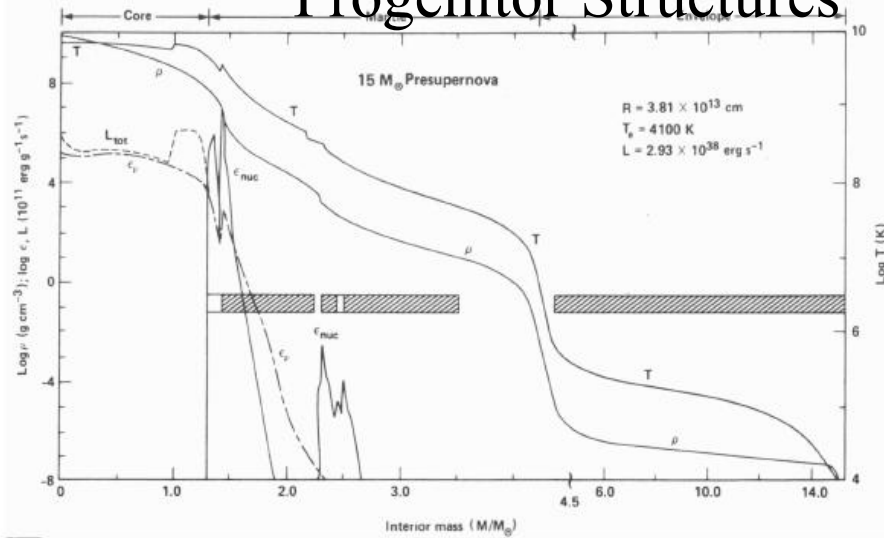
Fuel	Main Product	Secondary Product	T (10 ⁹ K)	Time (yr)	Main Reaction
H	He	¹⁴ N	0.02	10 ⁷	^{CNO} 4 H → ⁴ He
He	O, C	¹⁸ O, ²² Ne s-process	0.2	10 ⁶	3 He ⁴ → ¹² C ¹² C(α,γ) ¹⁶ O
C	Ne, Mg	Na	0.8	10 ³	¹² C + ¹² C
Ne	O, Mg	Al, P	1.5	3	²⁰ Ne(γ,α) ¹⁶ O ²⁰ Ne(α,γ) ²⁴ Mg
O	Si, S	Cl, Ar, K, Ca	2.0	0.8	¹⁶ O + ¹⁶ O
Si	Fe	Ti, V, Cr, Mn, Co, Ni	3.5	0.02	²⁸ Si(γ,α)...

Summary: Stellar Evolution of Massive Stars



Next: Exploding Stars /Supernovae

Progenitor Structures



Summary: Stellar Evolution Stages

hydrogen burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^7K	g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	Myr
1	1.57	153	1.00	0.001	1.00	$\sim 1,100$
15	3.53	5.81	14.9	28.0	6.75	11.1
20	3.69	4.53	19.7	62.6	8.03	8.13
25	3.81	3.81	24.5	110	9.17	6.70
75	4.26	1.99	67.3	916	21.3	3.16

helium burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^8K	10^3g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	Myr
1	1.25	20	0.71	0.044	~ 100	110
15	1.78	1.39	14.3	41.3	461	1.97
20	1.88	0.968	18.6	102	649	1.17
25	1.96	0.762	19.6	182	1,030	0.839
75	2.10	0.490	16.1	384	1.17	0.478

carbon burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^8K	10^5g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	kyr
15	8.34	2.39	12.6	83.3	803	2.03
20	8.70	1.70	14.7	143	1,070	0.976
25	8.41	1.29	12.5	245	1,390	0.522
75	8.68	1.39	6.37	164	0.644	1.07

neon burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^9K	10^6g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	yr
15	1.63	7.24	12.6	86.5	821	0.732
20	1.57	3.10	14.7	147	1,090	0.599
25	1.57	3.95	12.5	246	1,400	0.891
75	1.62	5.21	6.36	167	0.715	0.569

oxygen burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^9K	10^6g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	yr
15	1.94	6.66	12.6	86.6	821	2.58
20	1.98	5.55	14.7	147	1,090	1.25
25	2.09	3.60	12.5	246	1,400	0.402
75	2.04	4.70	6.36	172	0.756	0.908

silicon burning						
M_{initial}	T	ρ	M	L	R	τ
M_{\odot}	10^9K	10^7g cm^{-3}	M_{\odot}	$10^3 L_{\odot}$	R_{\odot}	d
15	3.34	4.26	12.6	86.5	821	18.3
20	3.34	4.26	14.7	147	1,090	11.5
25	3.65	3.01	12.5	246	1,400	0.733
75	3.55	3.73	6.36	173	0.755	2.09

