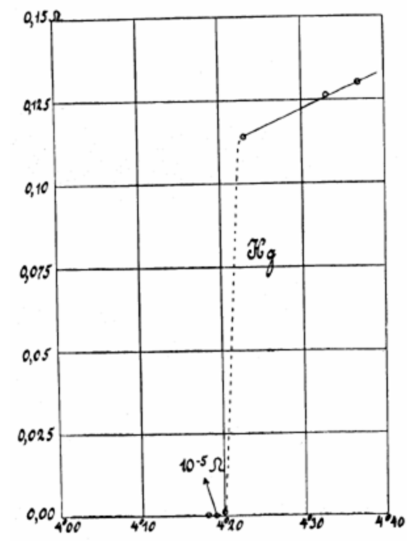
8. Superconductivity

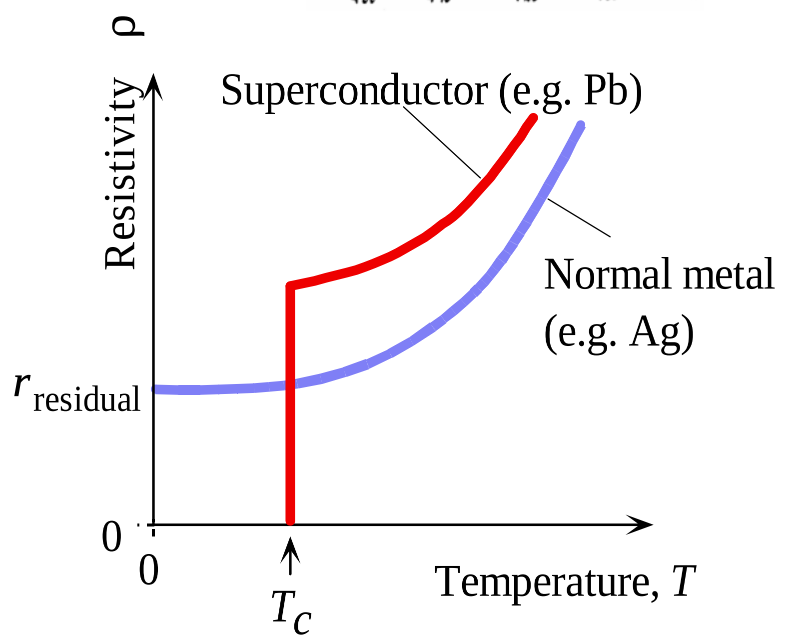
8.1 Basic phenomenon

- Discovery of superconductivity by H.K. Onnes (1911):

Resistance of Hg abruptly drops to zero below ~4.2K.

Critical temperature (T_c).



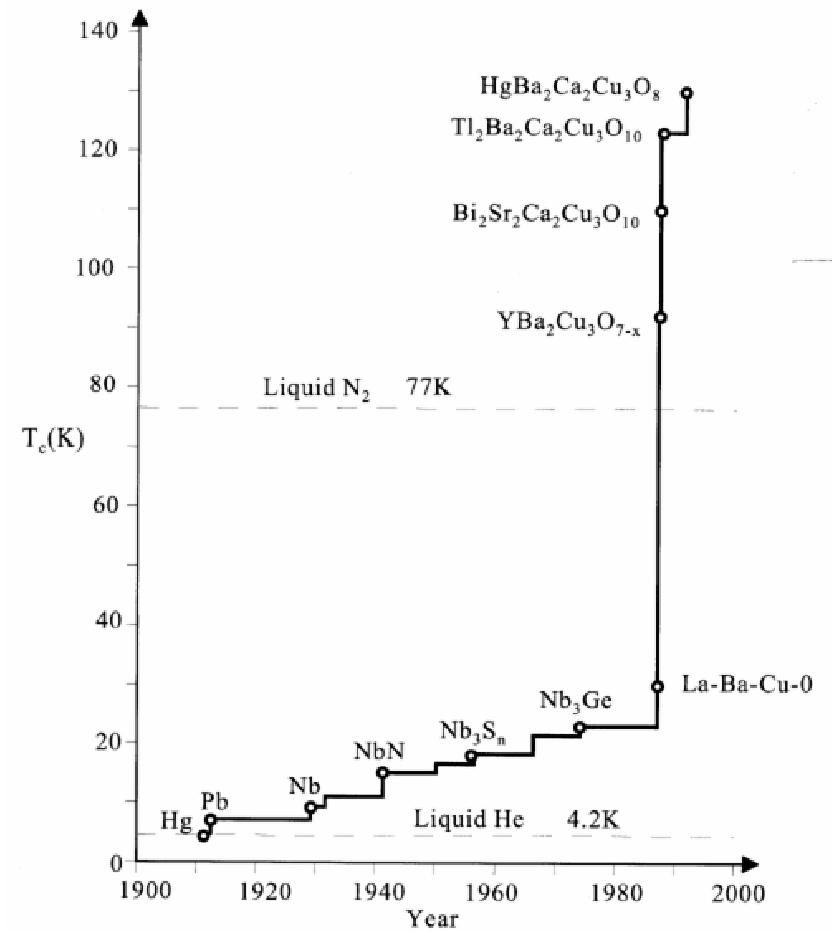


- Persistent current in a superconducting loop:
An induced current in a superconducting loop circulates forever.

- Occurrence of superconductivity

a) Conventional superconductivity: metals ($T_c < 10$K), alloys and compounds ($T_c < 40$K) , organic materials ($T_c < 40$K) ;

b) High-T_c superconductivity: Copper perovskites (cuprates) ($T_c < 140$K)



8.2 Effect of Magnetic Field

- Critical Field

A strong enough magnetic field (H > Hc) destroys superconductivity even below Tc.

$$H_c(T) = H_c(0)[1 - (\frac{T}{T_c})^2]$$

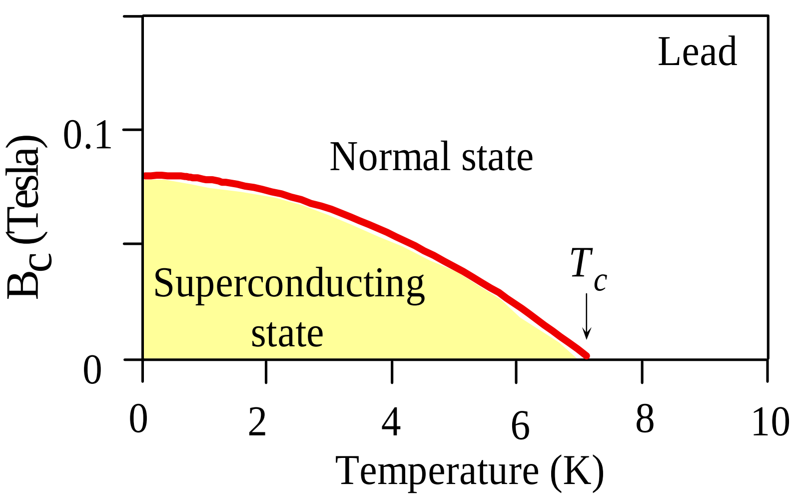


Fig. 8.46: The critical field vs temperature in Type I superconductors.

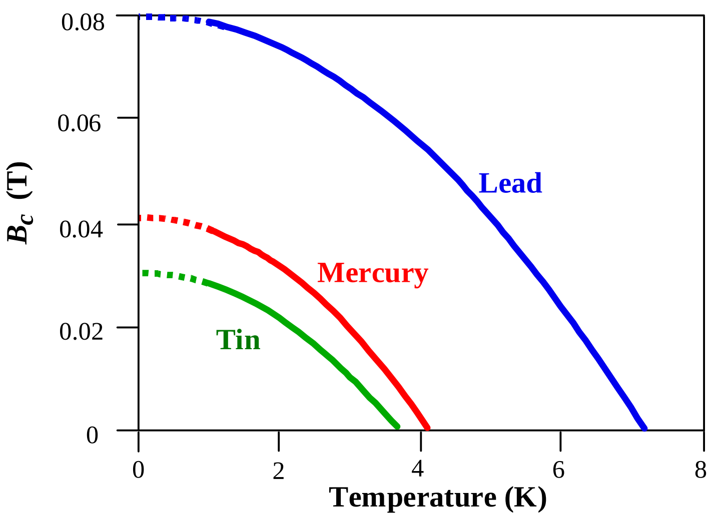


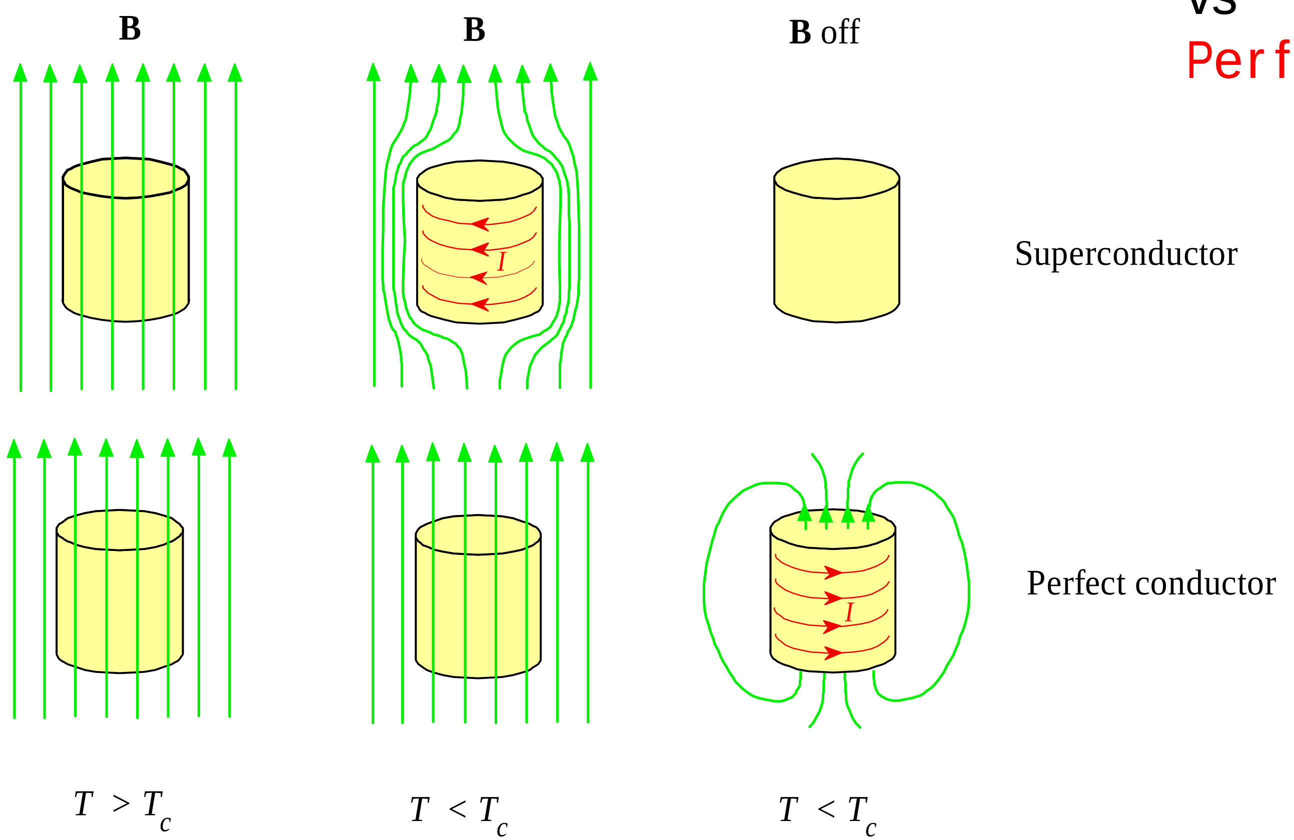
Fig. 8.47: The critical field vs temperature in three examples of Type I superconductors.

- Meissner Effect

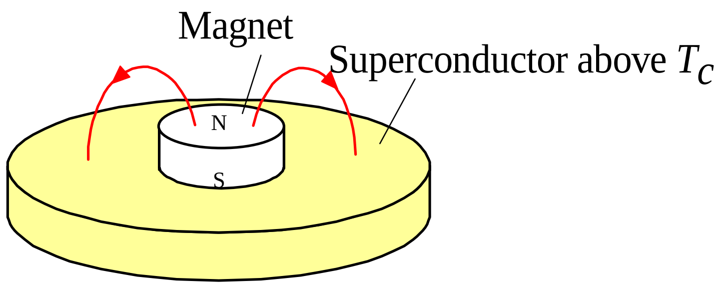
A superconductor expels magnetic flux completely – perfect diamagnetism.

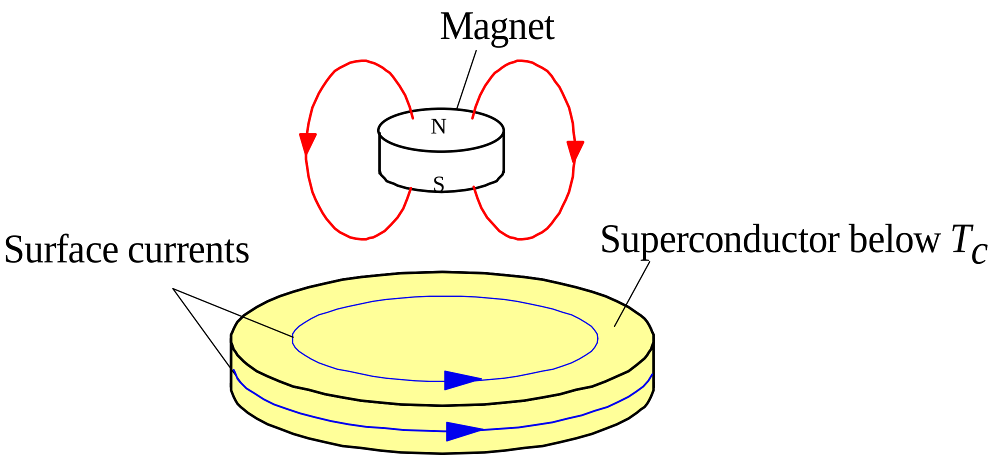
$B = 0 \rightarrow M = -H$

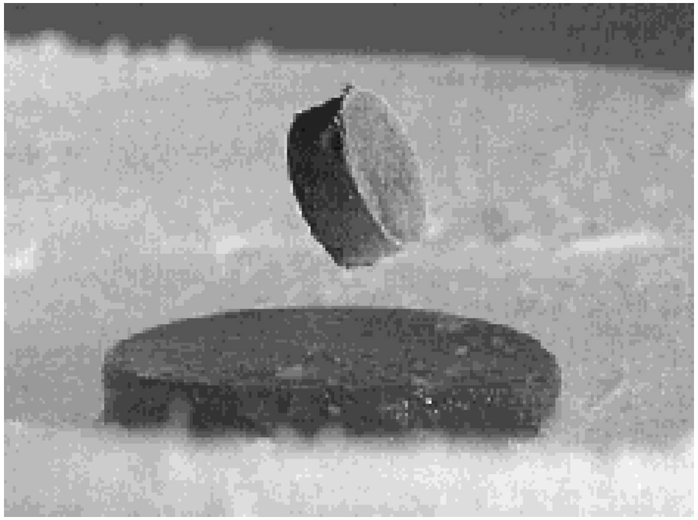
Superconductor
vs
Perfect conductor



Magnetic Levitation:



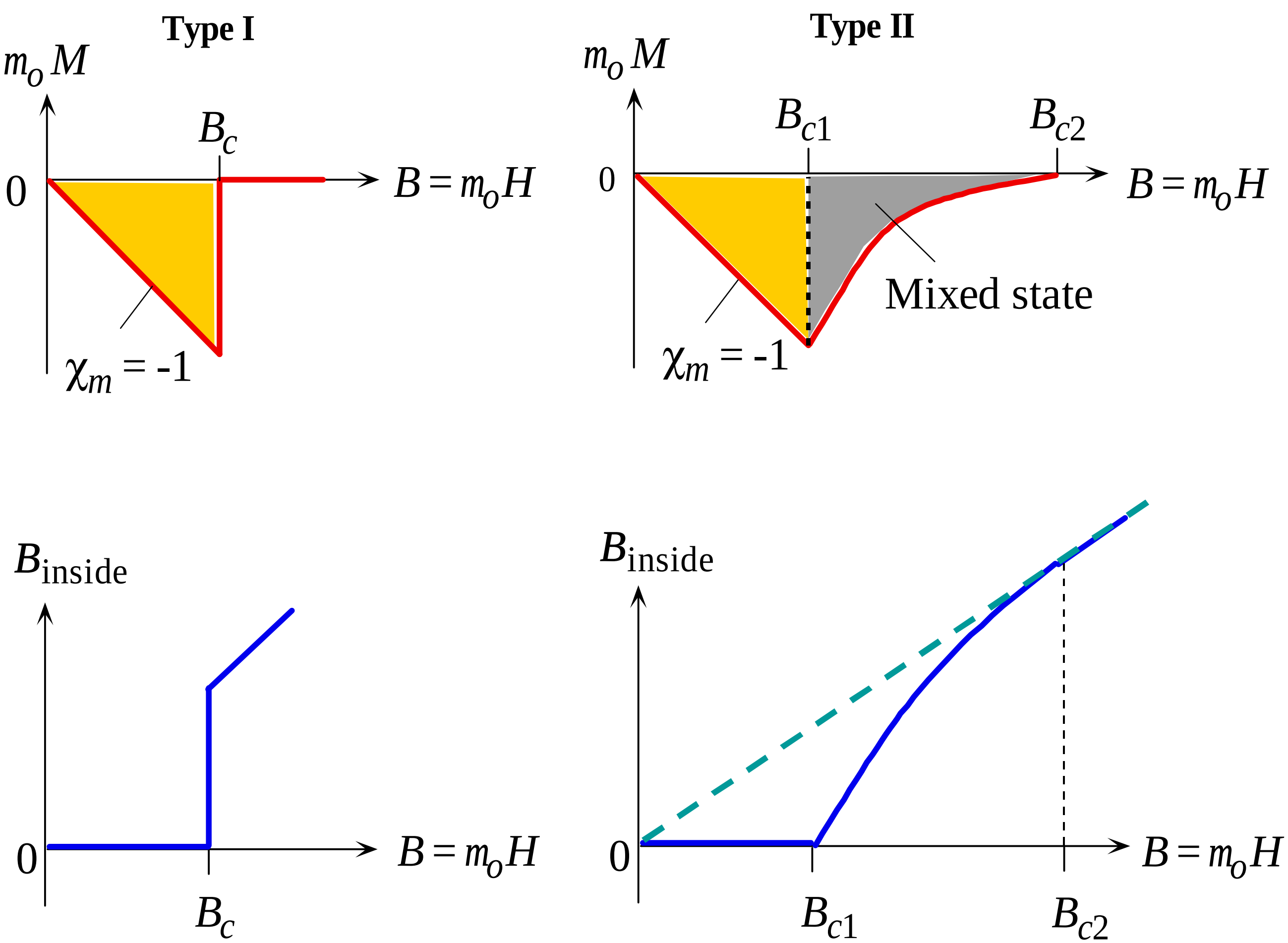




Future of transportation?

- Type II Superconductors

In a class of superconductors, the transition from the Meissner state to normal state is not abrupt. The transition goes through an intermediate (mixed) state where superconducting regions and normal regions coexist.



B_{c1}:
lower critical field

B_{c2}:
upper critical field

- For $\mu_0 H < B_{c1}$, Meissner state;
$B_{c2} > \mu_0 H > B_{c1}$, mixed state;
$\mu_0 H > B_{c2}$, normal state.

In the mixed state, the normal regions are in the form of small cylinders (filaments) that penetrate the sample. Each filament is a vortex (fluxoid) of flux lines.

- Magnetic field penetrate the superconductor in the form of a regular array of flux lines.

- A supercurrent circulates around the wall of each vortex.

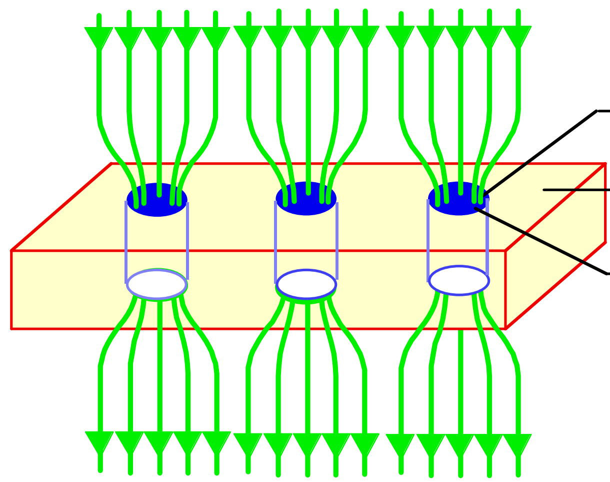
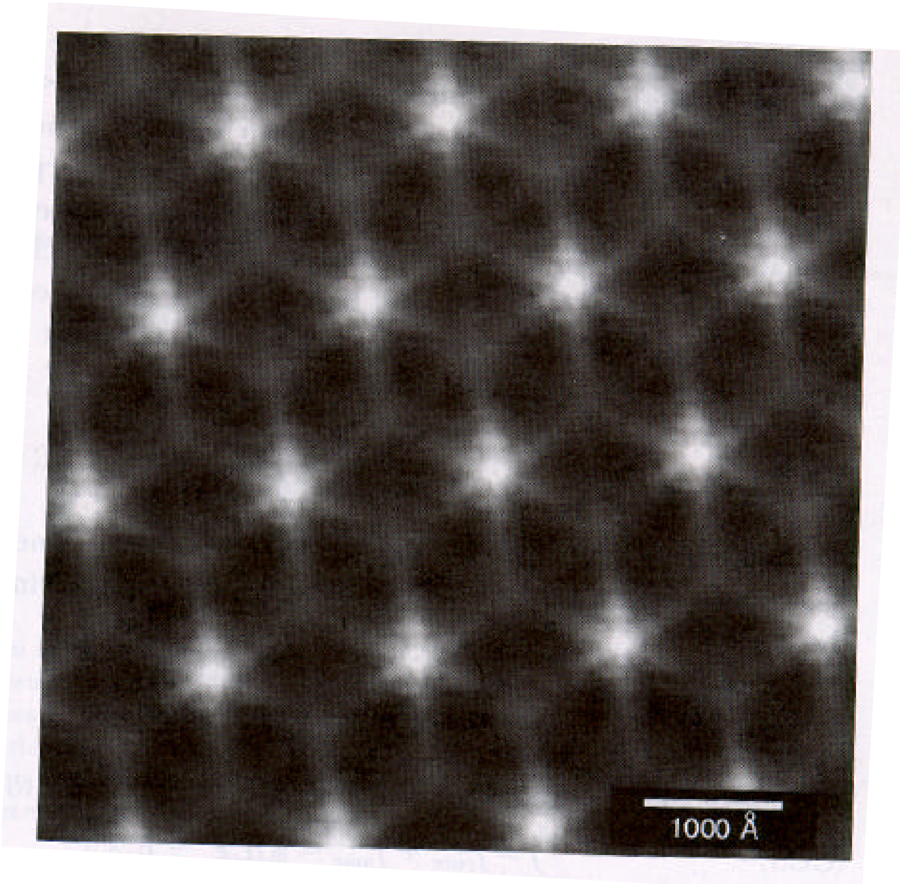
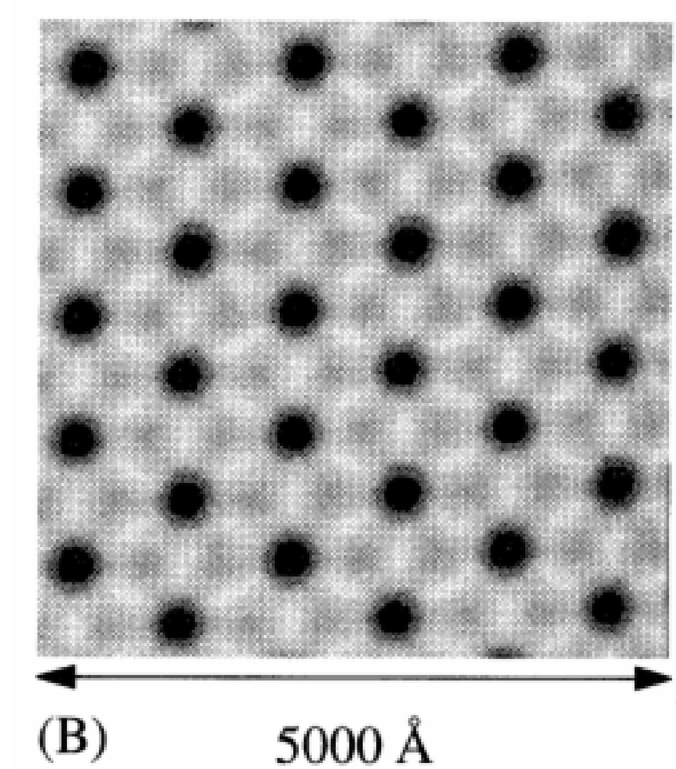


Fig. 8.49: The mixed or vortex state in a Type II superconductor.

Images of vortex lattice





Phase diagram of a type II superconductor:

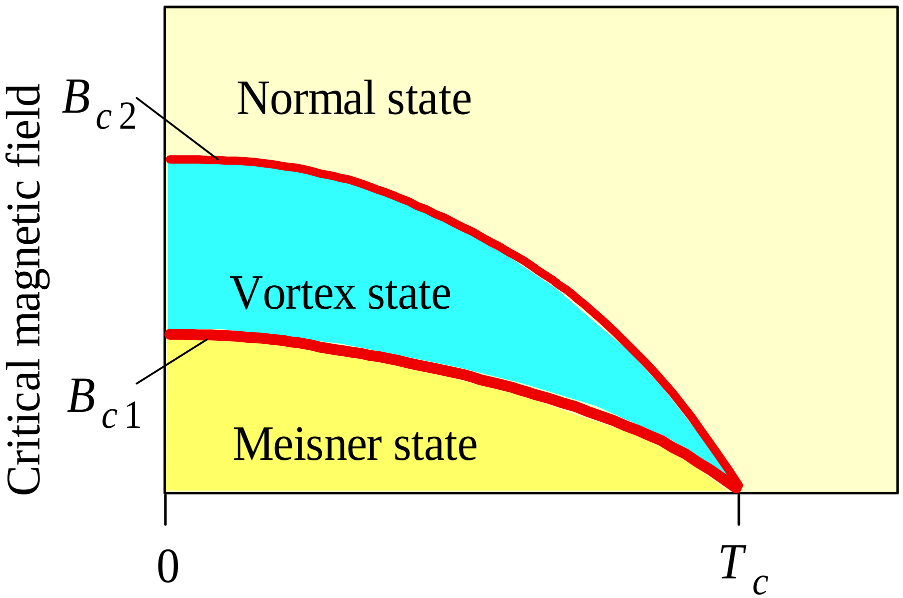


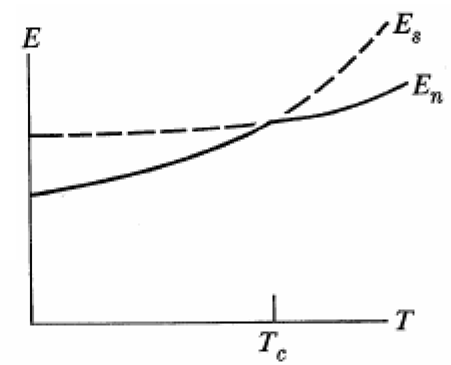
Fig. 8.50: Temperature dependence of B_{c1} and B_{c2}.

8.3 Thermodynamics of superconductors

- Condensation Energy

Below T_c the superconducting state should be a lower energy state than the normal state. Energy difference in zero field:

$$\Delta E = E_n - E_s = \frac{1}{2} m_0 H_c^2$$



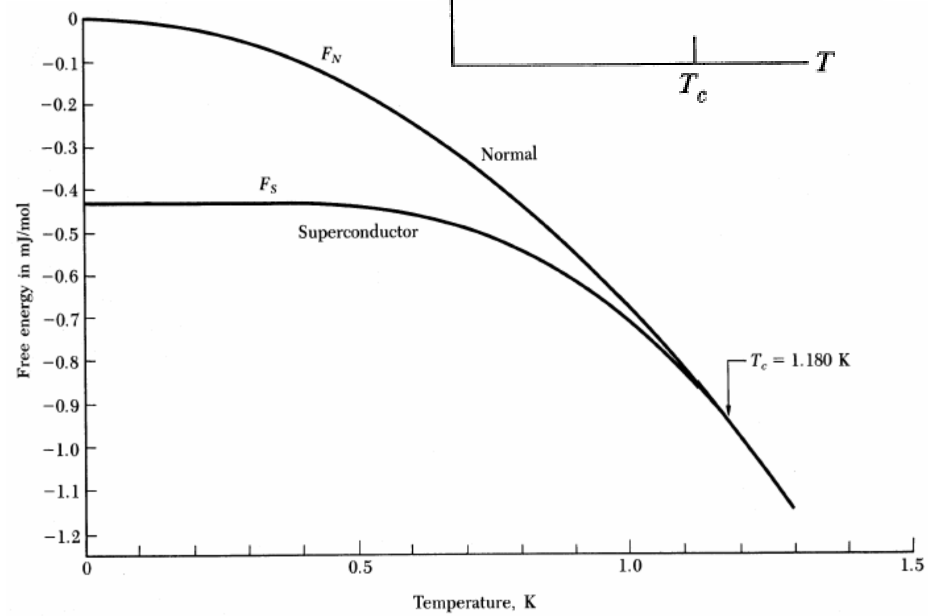
- Gibbs Energy

Superconducting state:

$$G_s(H) = G_s(0) + \frac{1}{2} m_0 H^2 V$$

Normal state:

$$G_n = G_s(0) + \frac{1}{2} m_0 H_c^2 V$$

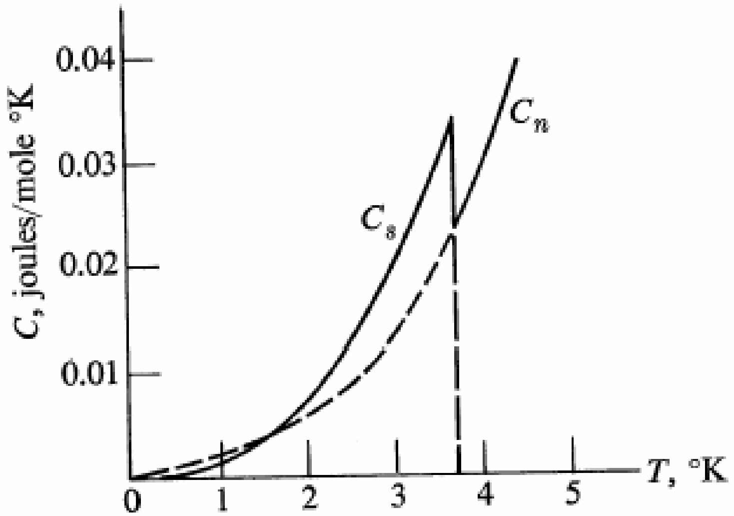


- Specific Heat

i) Discontinuity at T_c

$$c_n - c_s = -\{m_0 T V_m (\frac{dH_c}{dT})^2\}_{T=T_c}$$

➔ Second order phase transition

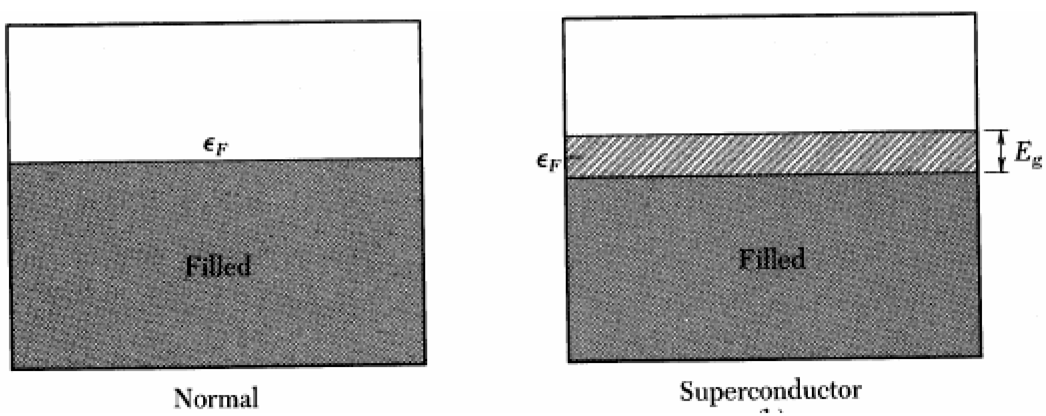


ii) Exponential T-dependence

$$c_s = ae^{-b(T_c/T)}$$

➔ Energy gap at E_F!

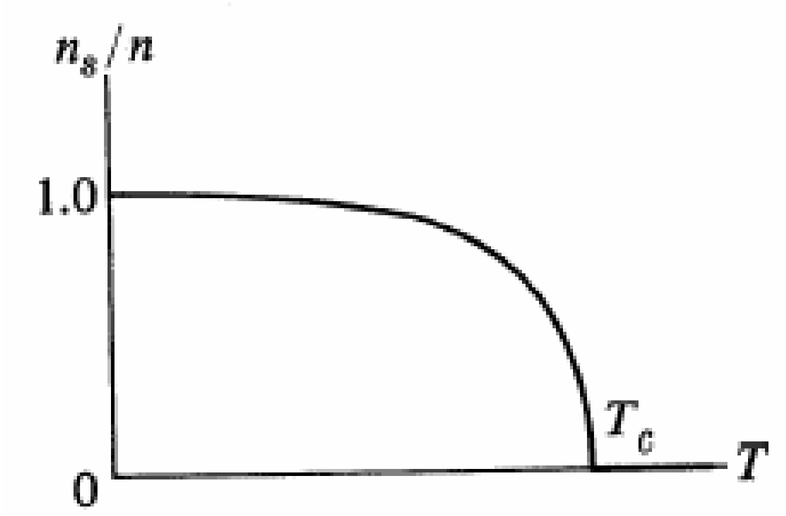
$$E_g = 2\Delta \quad (\sim kT_c)$$



$$H_c(0) \approx \sqrt{\frac{2nk^2}{m_0 E_F}} T_c$$

- Two fluid model:

Conduction electrons in a superconductor can be divided into two classes: superelectrons and normal electrons.



Concentration of superelectrons:

$$n_s = n[1-(\frac{T}{T_s})^4]$$

The superelectrons do not suffer any scatterings and have zero resistance, and short-circuit the normal electrons.

What's the difference between superelectrons and normal electrons???

8.4 Electrodynamics of Superconductors

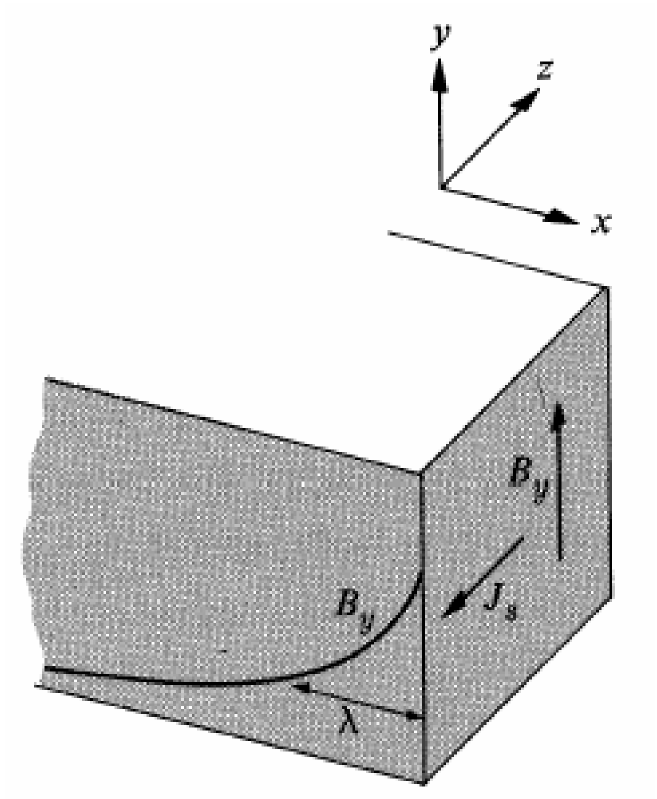
i) Electric Field inside a superconductor is zero
E = 0

ii) London Equation:

$$\vec{B} = -\frac{m}{n_s e^2} \nabla \times \vec{J}_s$$

➔ $$B_y(x) = B_y(0) e^{-x/l}$$

$$l = \sqrt{m / m_0 n_s e^2}$$



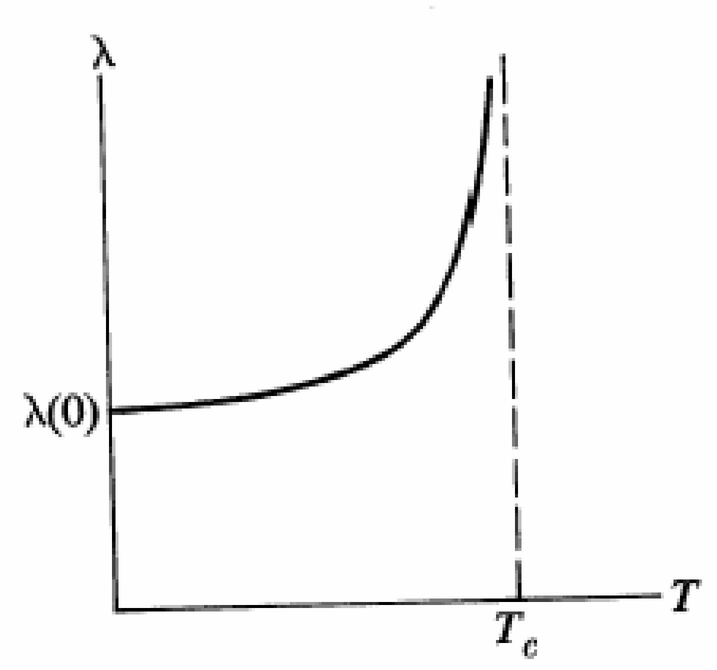
λ: London penetration depth

Magnetic field <u>does</u> penetrate into a superconductor, but only to a small depth near the surface!

iii) T-dependence of λ

$$l = l(0)(1-\frac{T^4}{T_c^4})^{-1/2}$$

The field penetrate the entire sample at T_c (of course!).

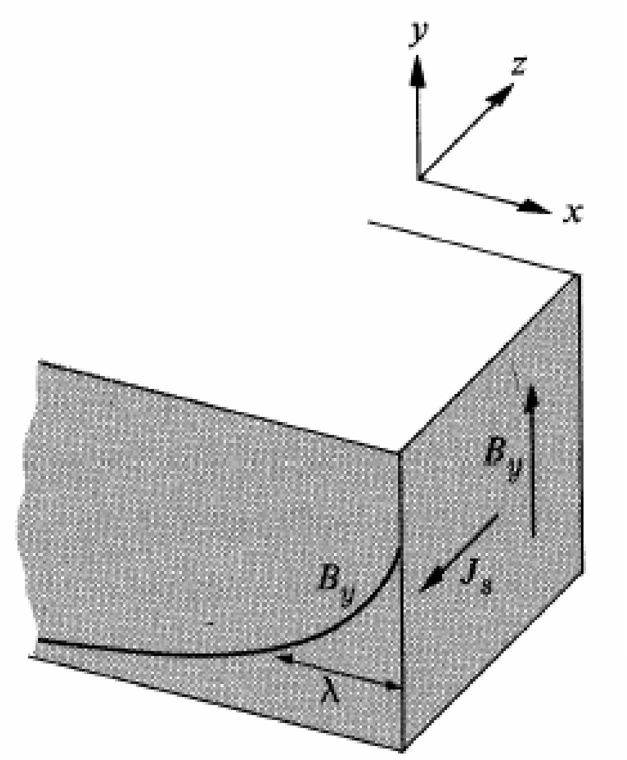


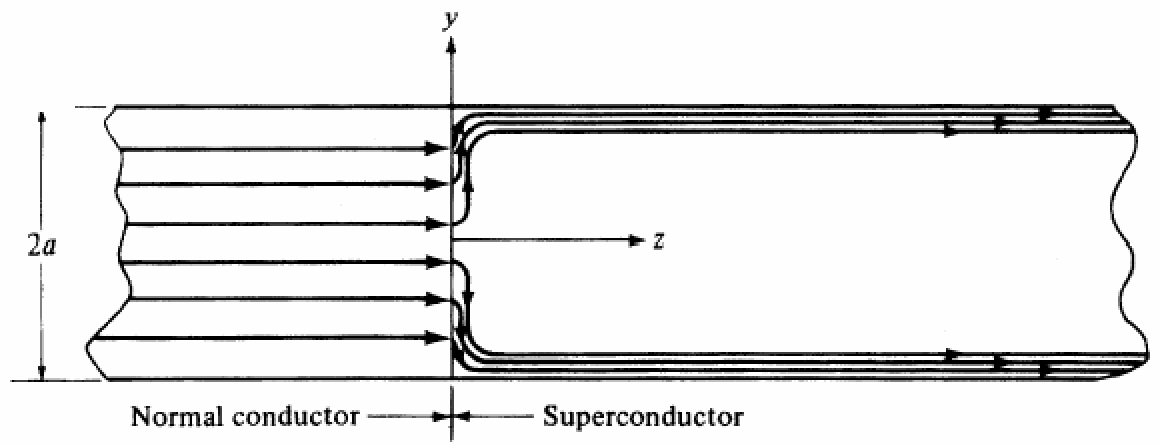
iv) Spatial distribution of supercurrent

$$J_z(x) = -(\frac{n_s e^2}{m_0 m})^{1/2} B_y(x) = -J_s(0) e^{-x/l}$$

The electric current flow in a superconductor is restricted to a surface layer of the depth of London penetration depth.

Field and current penetration:

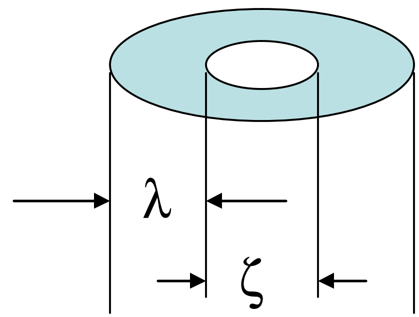




v) Concept of coherence length

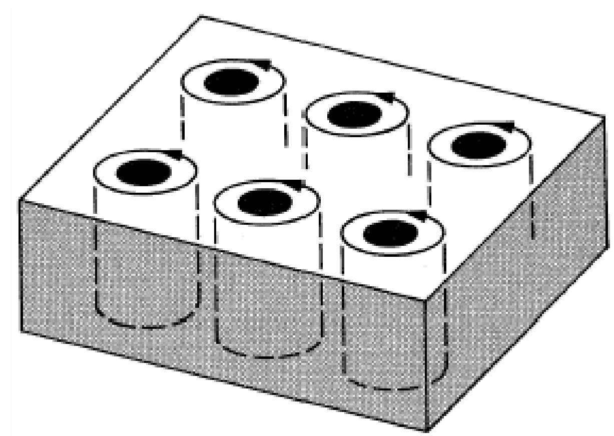
The superconducting coherence, ζ, represents the extent of the superelectron wave function. Superconductivity cannot vary greatly over this distance.

$$x \approx \frac{\hbar}{\Delta p} \approx \frac{\hbar v_F}{2\Delta} \propto \frac{1}{T_c}$$



vi) Detailed picture of flux lattice

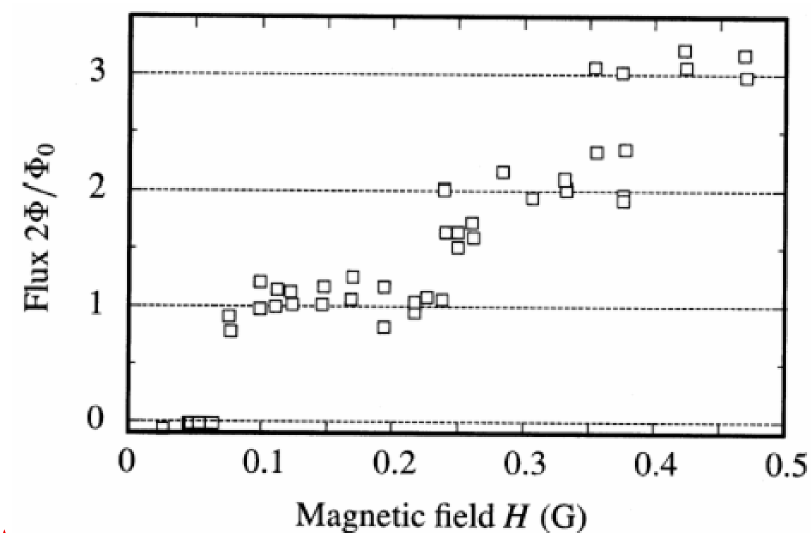
Each vortex has a normal core of diameter ζ and a circulating supercurrent around the normal core of depth λ.

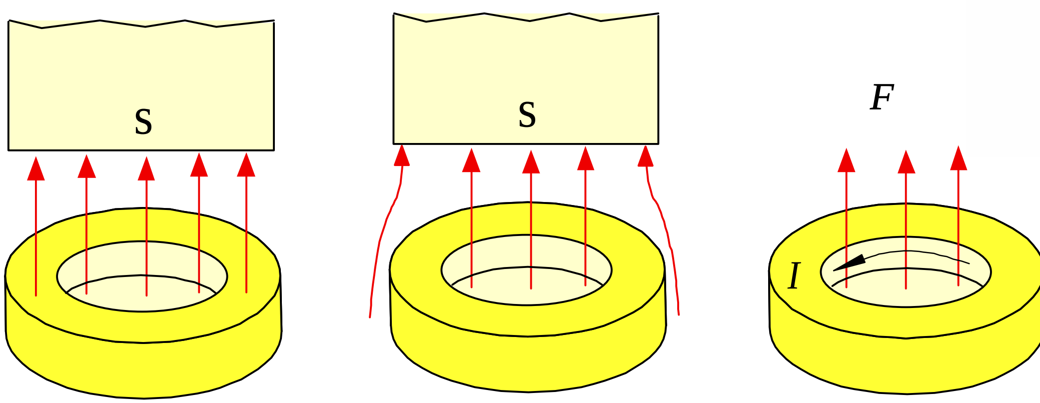


vii) Flux Quantization

The magnetic flux threading a superconducting ring is quantized:

$$\Phi = n\frac{h}{2e} = n\Phi_0$$





(a) $T > T_c$ (b) $T < T_c$ (c) $T < T_c$

Fig: 8.60: (a) Above T_C, the flux line enter the ring (b) The ring

8.5 Origin of Superconductivity: BCS Theory

i) Cooper pairs

- The superelectrons form pairs;
- Each pair consists of two electrons of opposite momentum and spin ($\vec{k}\uparrow, -\vec{k}\downarrow$);
- Each electron in a pair has a lower energy (by amount of the energy gap Δ) than a normal electron ➔ condensation energy;

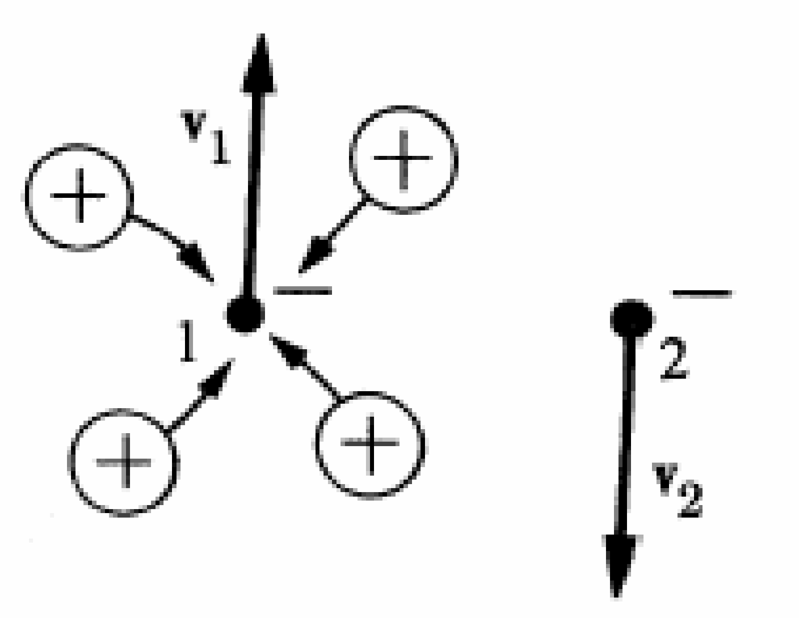
$$\Delta E \sim g(0)\Delta^2$$

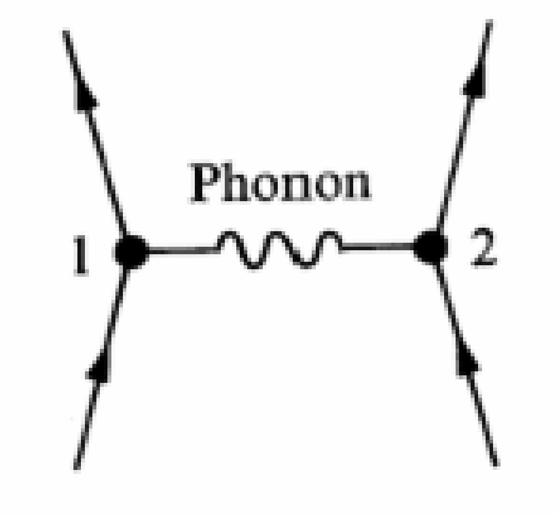
- An energy of 2Δ is required to break up a Cooper pair;
- The Cooper pairs do not suffer any scattering and have zero resistance.

Where does the attraction come from???

ii) Attraction through electron-phonon interaction

One electron interacts with the lattice, distorts the lattice and creates a local positive net charge, which attracts a second electron nearby.

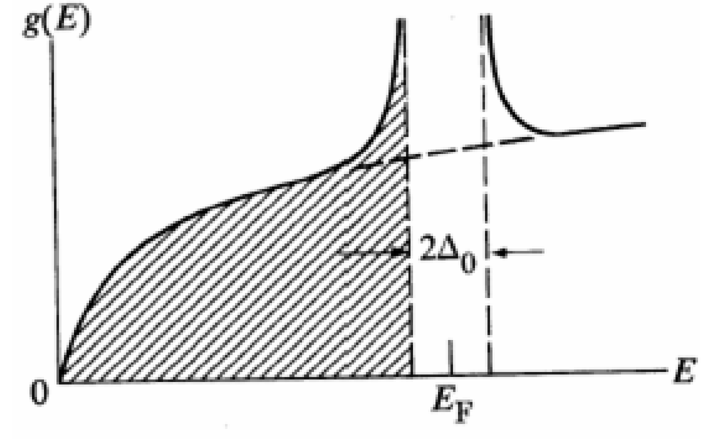




Consequences of BCS theory:

i) Electronic DOS of a superconductor:

$$g(E) = g_n(E)\frac{|E|}{\sqrt{E^2 - \Delta^2}}$$



ii) θ_D: Debye temperature

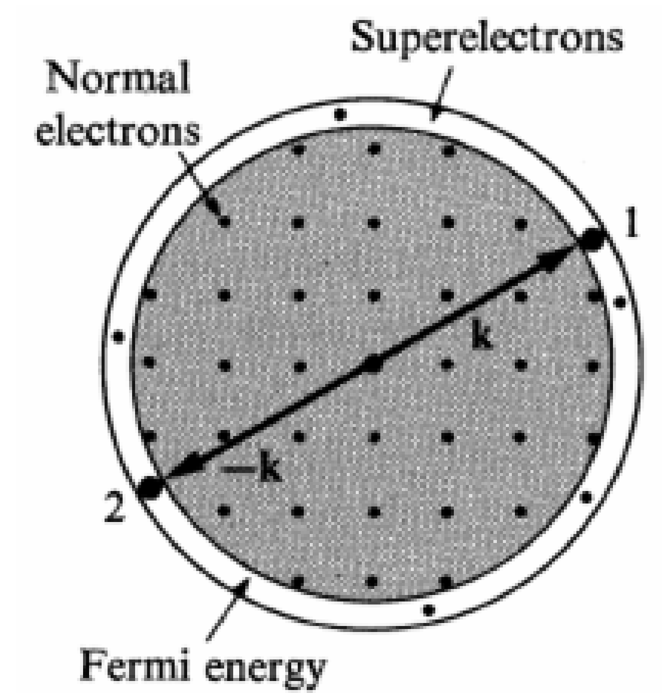
ω_D: Debye frequency

V': strength of electron-phonon interaction

$$T_c = 1.14 q_D \exp[-\frac{1}{g(E_F)V'}]$$

$$\Delta = 2\hbar w_D \exp[-\frac{1}{g(E_F)V'}]$$

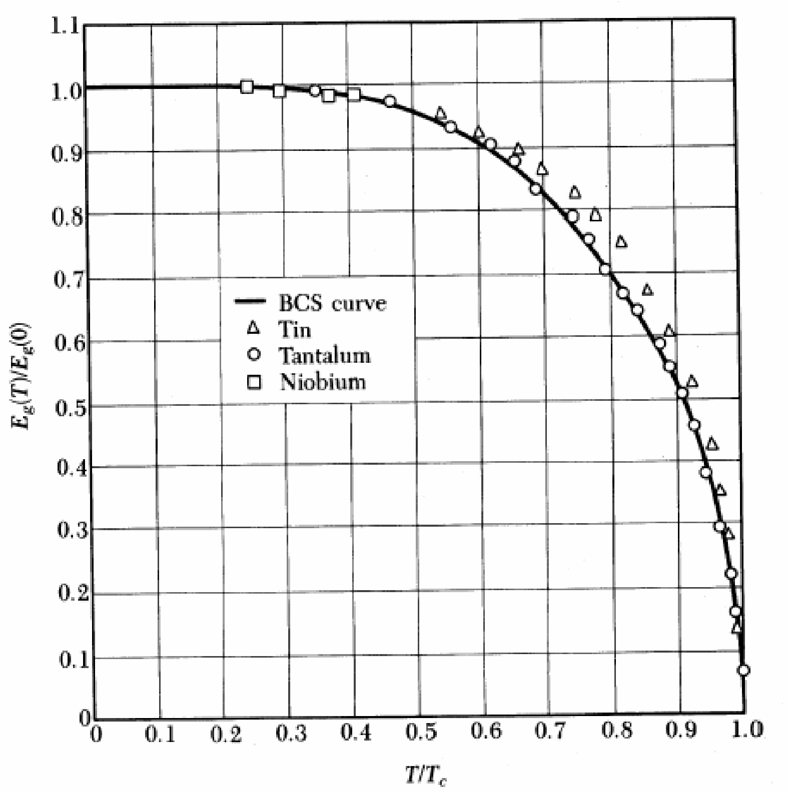
$$\frac{2\Delta}{kT_c} \approx 3.5$$



iii) Large V' means higher T_c, but also higher resistivity in the normal state. ➔ bad metals make good superconductors!

iv) Since $w_D \propto M^{-1/2}$ higher T_c for lighter masses → isotope effect.

v) Temperature dependence of the energy gap

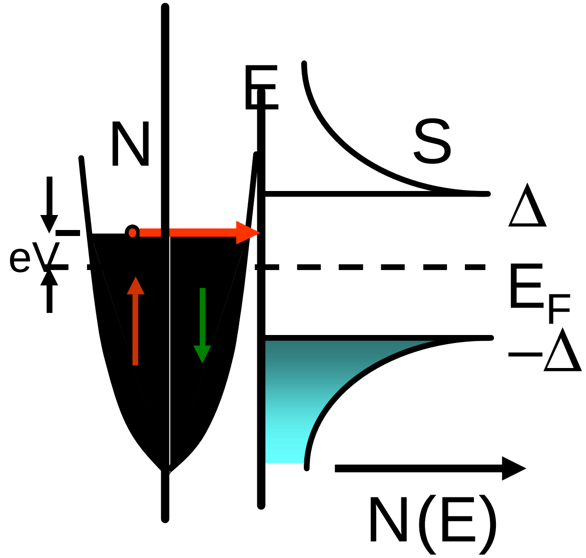


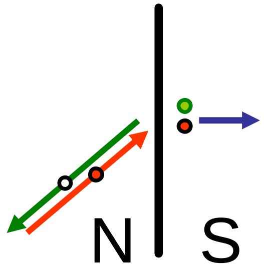
$$\frac{\Delta(T)}{\Delta_0} = \tanh[\frac{T_c \Delta(T)}{T \Delta_0}]$$

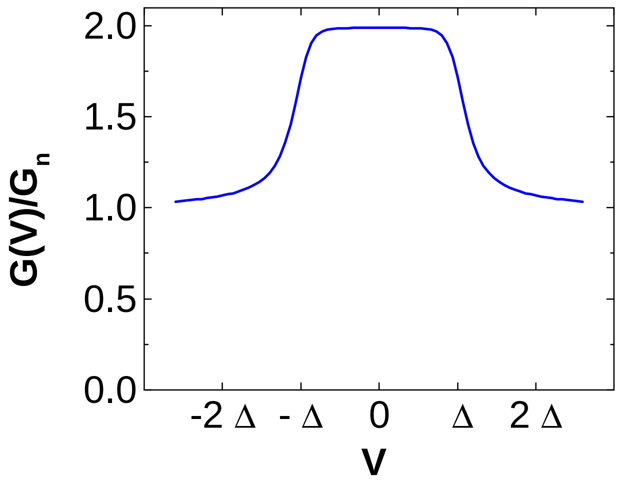
8.6 Superconducting Junctions

i) superconductor/normal metal (S/N): Andreev Reflection

How do electrons in N get into S?



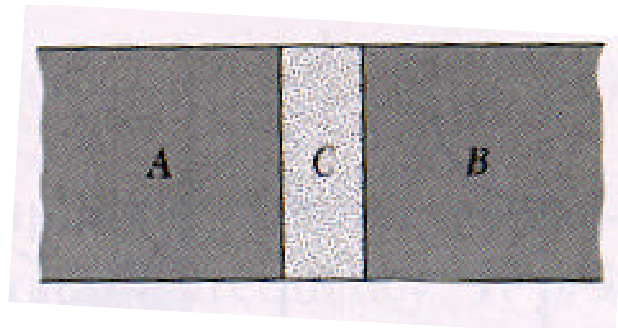


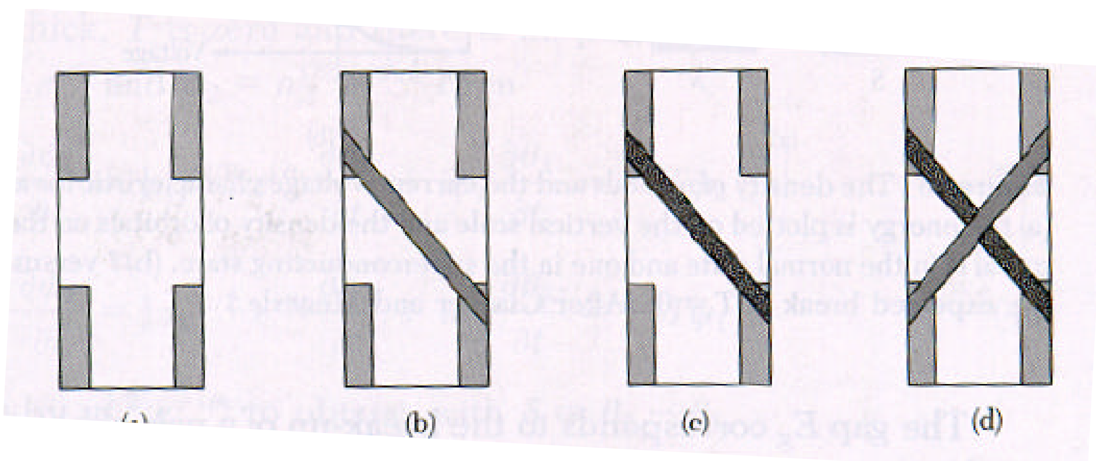


By pairing up with an electron of opposite momentum and spin, thus forming a Cooper pair and get into S. A hole has to be reflected back in N (why?). ➔ Andreev reflection

ii) S/Insulator/N (SIN): (single) electron tunneling

Fabrication of a tunnel junction: an example



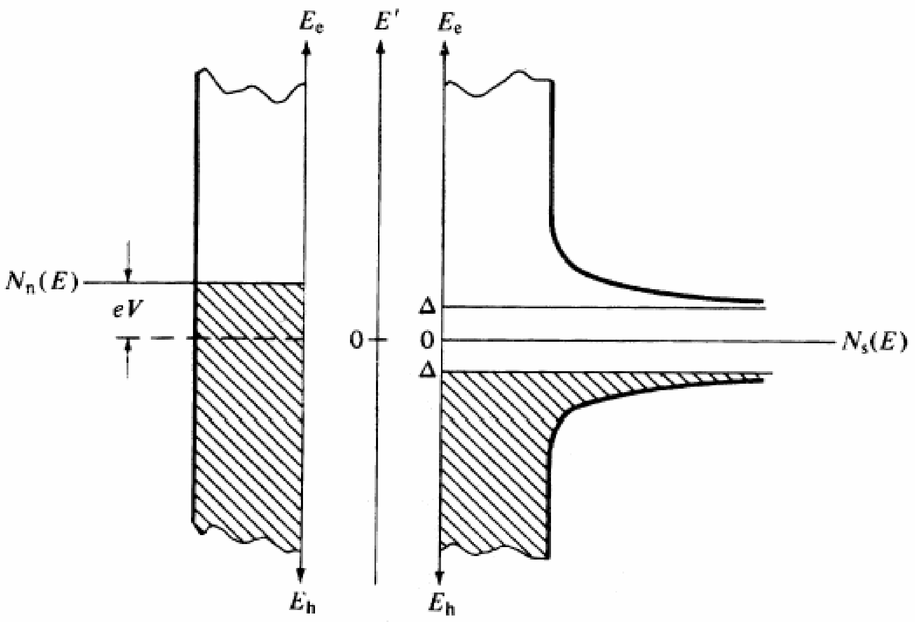


I-V and conductance spectrum:

- T = 0

$$I_{sn}(T=0)=\frac{2peA}{\hbar}\int_{\Delta}^{eV}|M|^2 g_n(E-eV)g_s(E)dE$$

$$\left(\frac{dI_{sn}}{dV}\right)_{T=0}=G_n\frac{|eV|}{\sqrt{(eV)^2-\Delta^2}}$$ for eV>Δ.

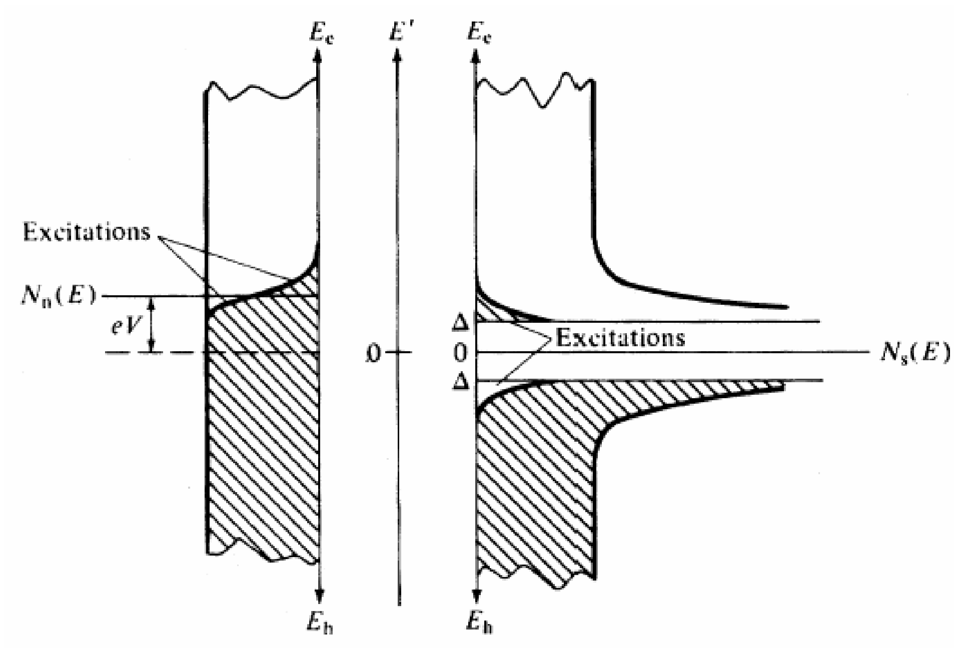


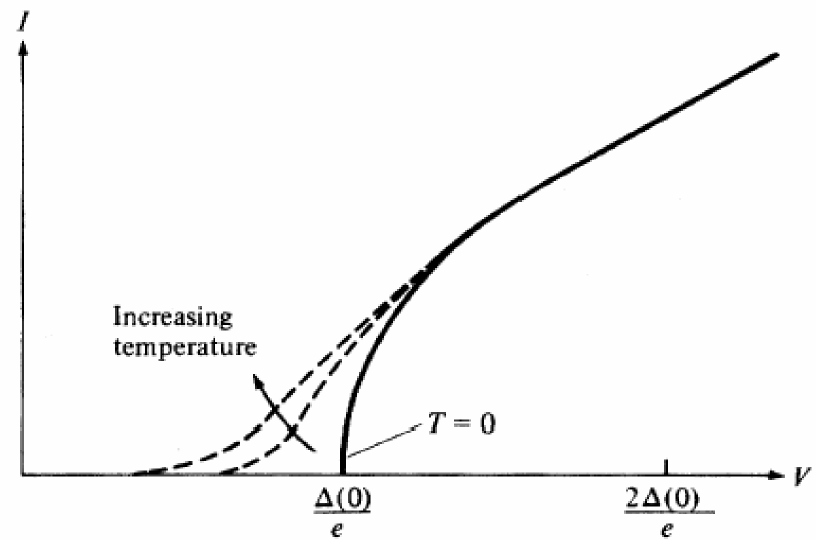
SIN electron tunneling is a very precise way of measuring the energy gap and DOS of a superconductor. Conversely, it is often used to determine the DOS of non-superconducting materials, in which the S is there to evidence tunneling.

- T > 0

$$I_{sn}(T) = \frac{G_n}{e}\int_{-\infty}^{+\infty}\frac{E}{\sqrt{E^2-\Delta^2}}[f(E-eV)-f(E)]dE$$

$$\left(\frac{dI_{sn}}{dV}\right)_T = G_n\int_{-\infty}^{+\infty}\frac{E}{\sqrt{E^2-\Delta^2}}\frac{df(E-eV)}{dV}dE$$

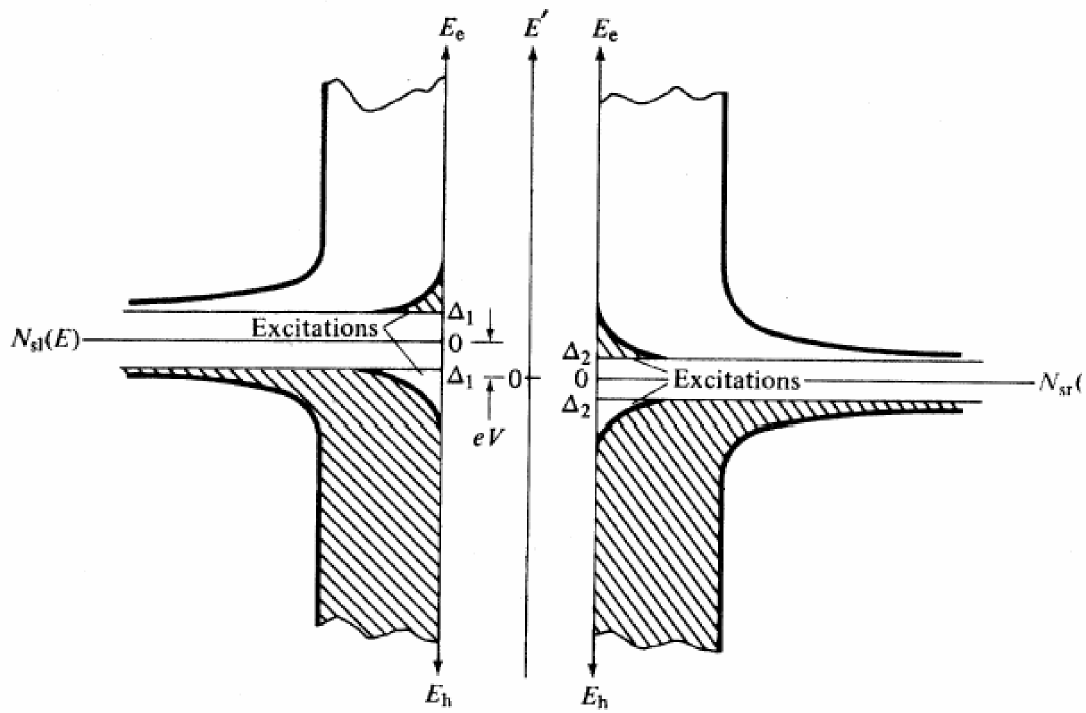


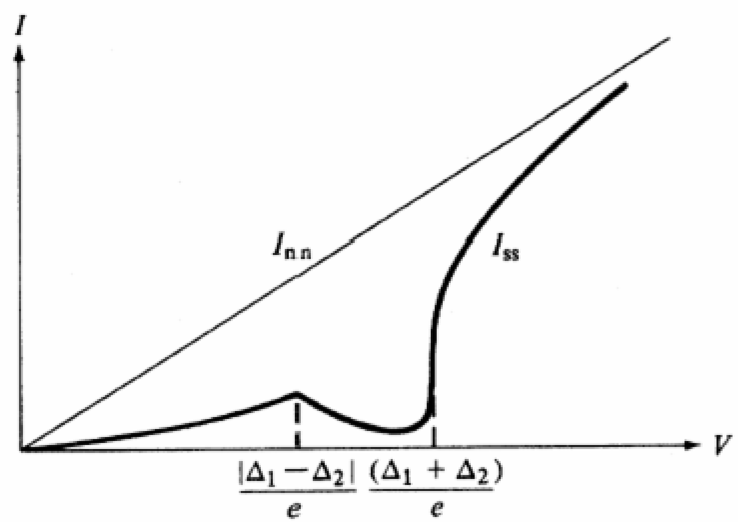


- small current and conductance below gap;
- sharp current rise at Δ (gap edge).

iii) SIS': (single) electron tunneling

$$I_{ss'}(T) = \frac{G_n}{e}\int_{-\infty}^{+\infty} \frac{|E-eV|}{\sqrt{(E-eV)^2-\Delta^2}} \frac{|E|}{\sqrt{E^2-\Delta^2}} [f(E-eV) - f(E)]dE$$





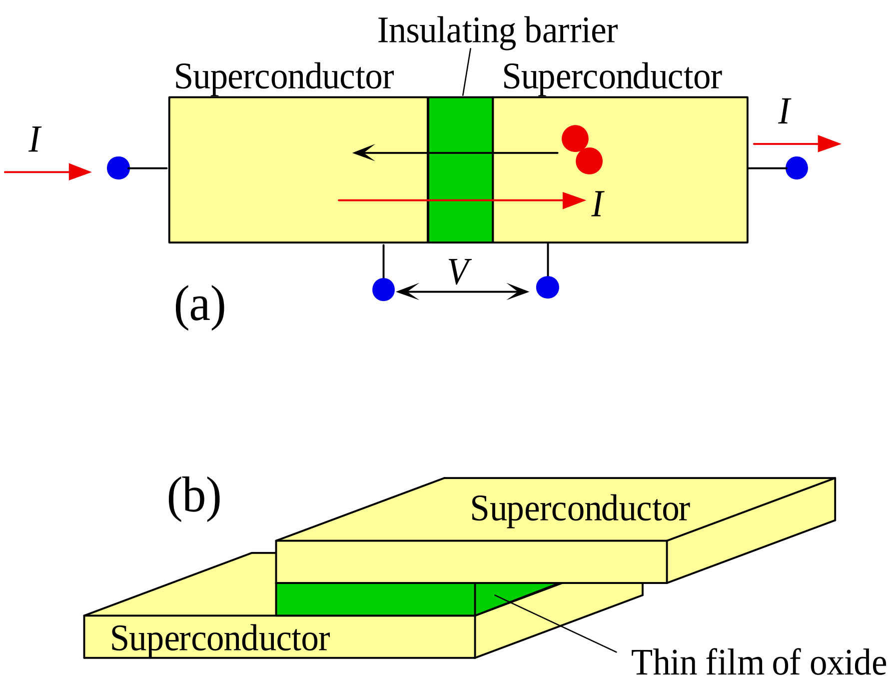
- Current maximum at $\Delta_2 - \Delta_1$ ➔ negative resistance;
- Sharp current rise (gap edge) at $\Delta_1 + \Delta_2$.

iv) S/(I, N)/S' junctions: Josephson Effect (Cooper pair tunneling)

A supercurrent (current without voltage) flows across the insulator without any dissipation because of tunneling of Cooper pairs.

Superconducting pair wave function:

$$\Psi = \Delta^{1/2} e^{if}$$



$$I = I_c \sin df$$

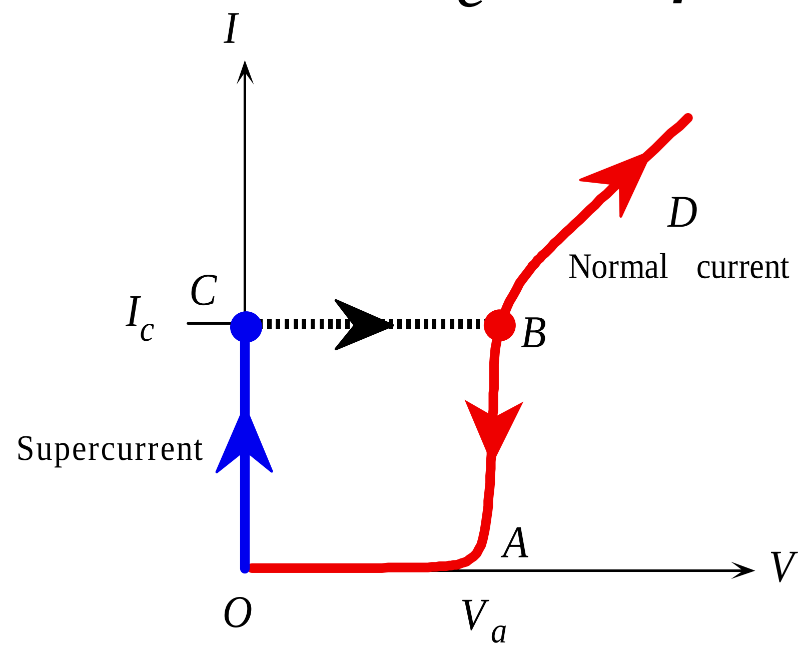
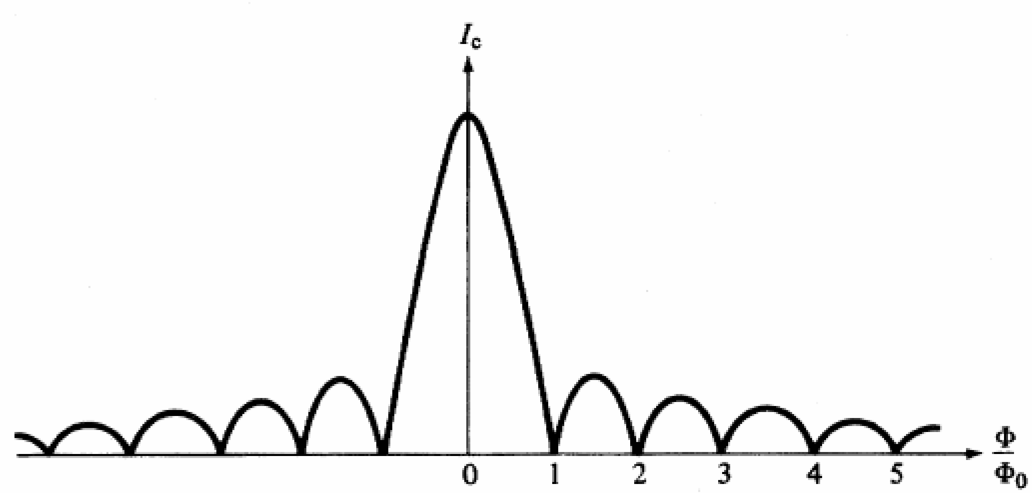


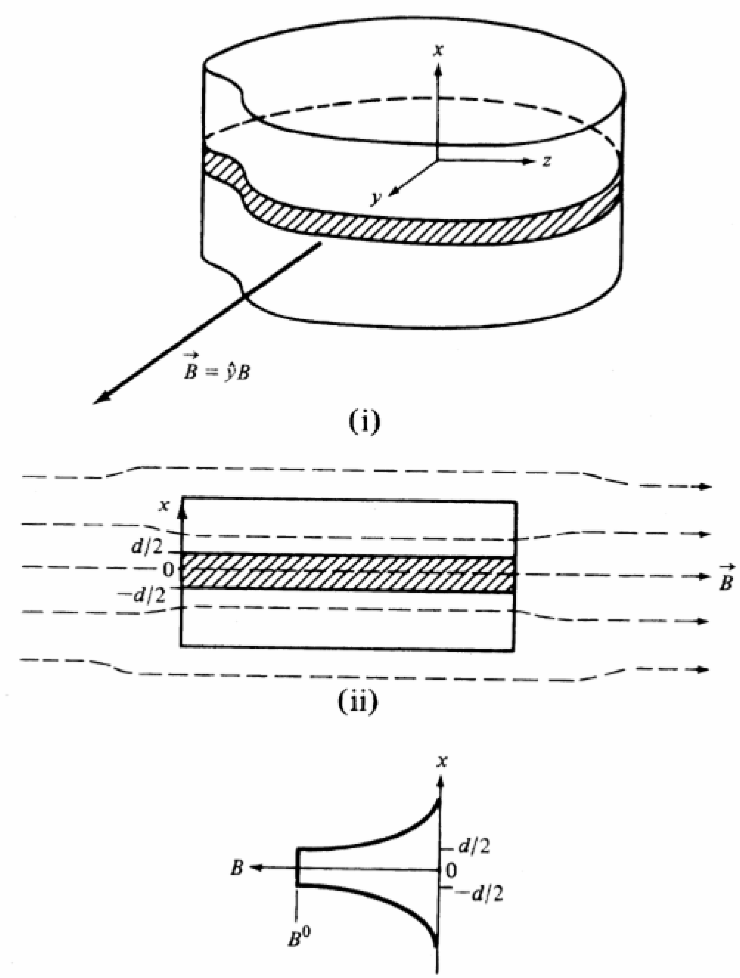
Fig: 8.58: (a) A Josephson junction is a junction between two Fig. 8.59: *I-V* characteristics of a Josephson junction for positive

Modulation of Josephson current by magnetic field:

The Josephson current depends on the phase difference between the two superconductors, which is modulation by a magnetic field passing through the junction area ➔ Fraunhofer pattern.

$$I_c(\Phi) = I_c(0)\left|\frac{\sin(p\Phi/\Phi_0)}{p\Phi/\Phi_0}\right|$$

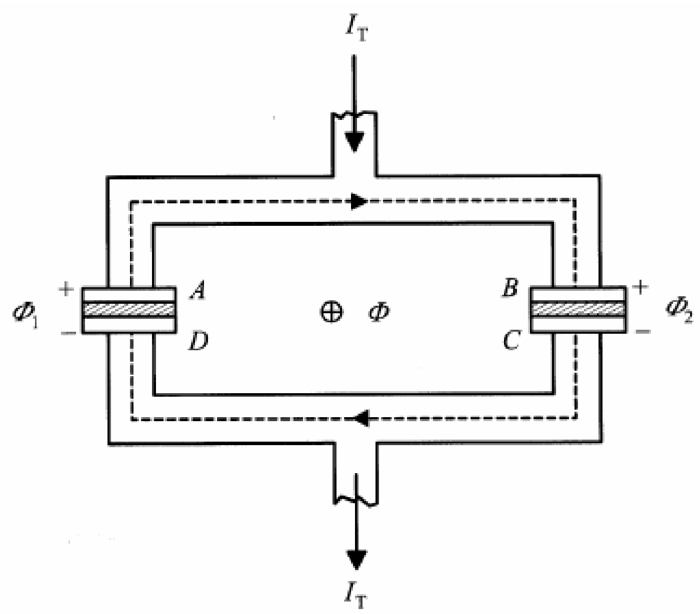


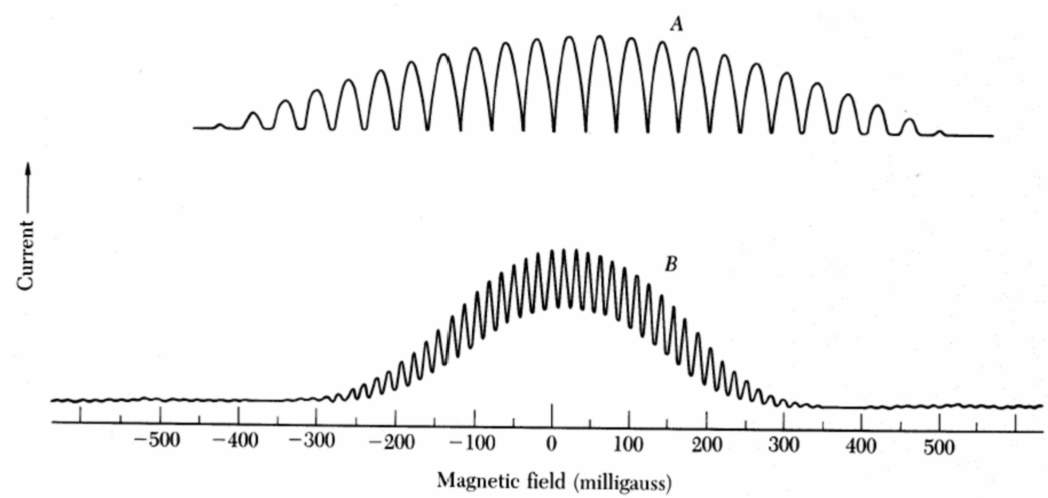


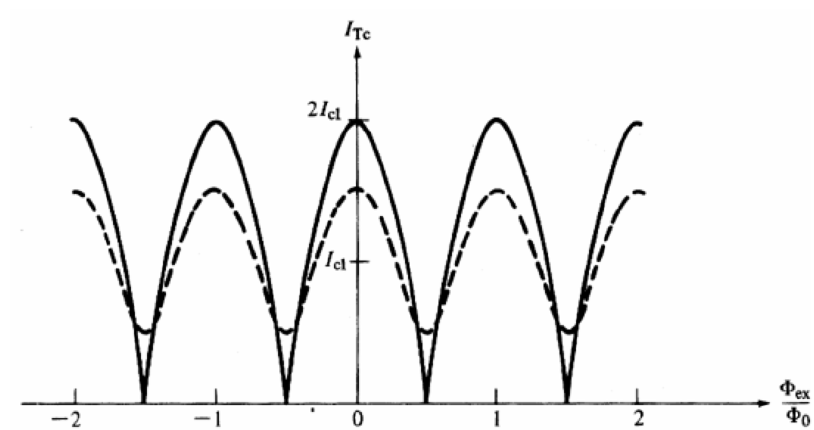
v) Macroscopic quantum interference: SQUID
(Superconducting QUantum Interference Device)

A loop containing two Josephson junctions, and the total critical current through the loop is modulated by the magnetic flux thread the loop.

$$I_T = I_{c1}\sin f_1 + I_{c2}\sin(f_1 - \frac{2p\Phi}{\Phi_0})$$







Most sensitive detector of magnetic flux and magnetic field.

8.7 Superconducting Magnets

High critical field and critical current required.