

Basic Nuclear Physics and Decays

- Basic Ingredients
- Models (Fermi, Droplet, Shell)
- Decays and Stability

Source: Chapter 1 & 2, and Cameron (1984)

Players and Properties of Nuclei

Nucleons:

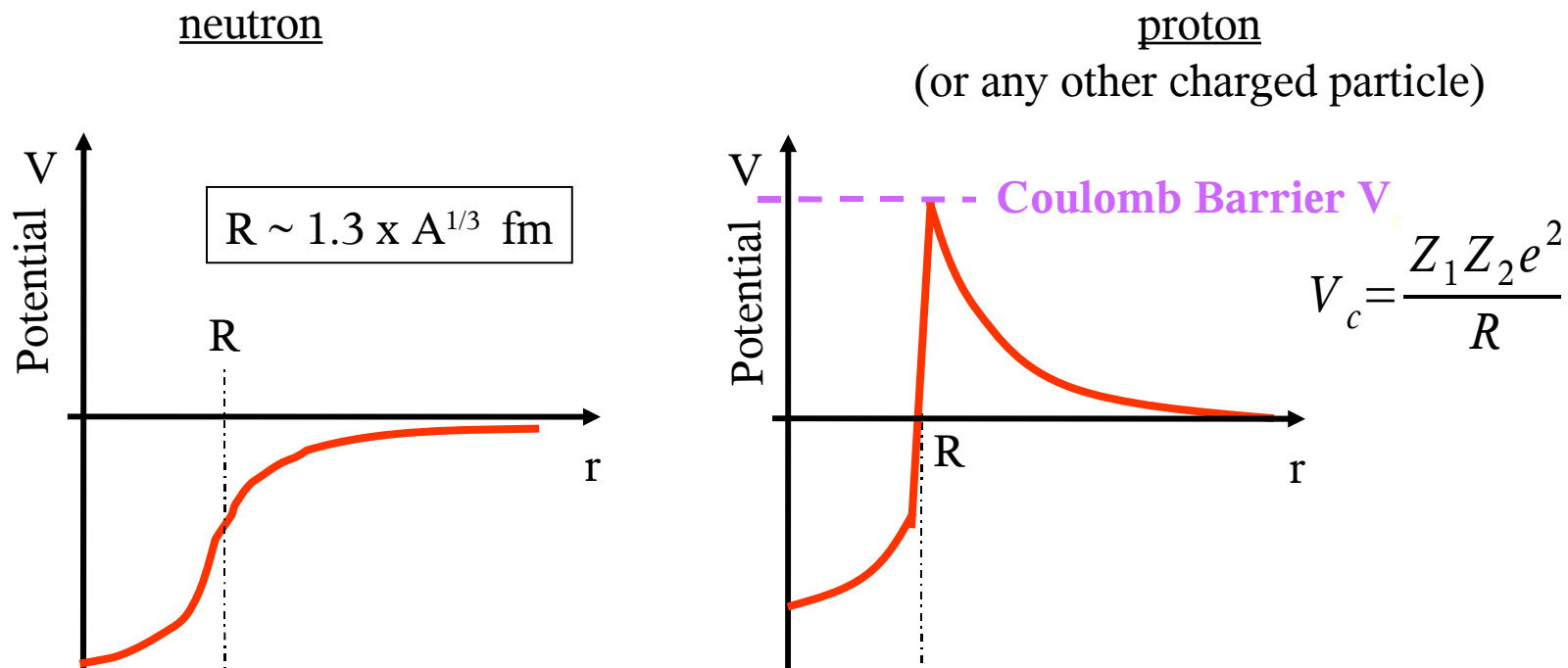
	Mass	Spin	Charge
Proton	938.272 MeV/c ²	1/2	+1e
Neutron	939.565 MeV/c ²	1/2	0

size: ~1 fm

Forces

Strong force (range ~ 1 fm) and electromagnetic force

Nuclear Potentials



Rem: Nuclear radii R are determined by electron scattering experiments²

Measured Core Radii from Electron Scattering

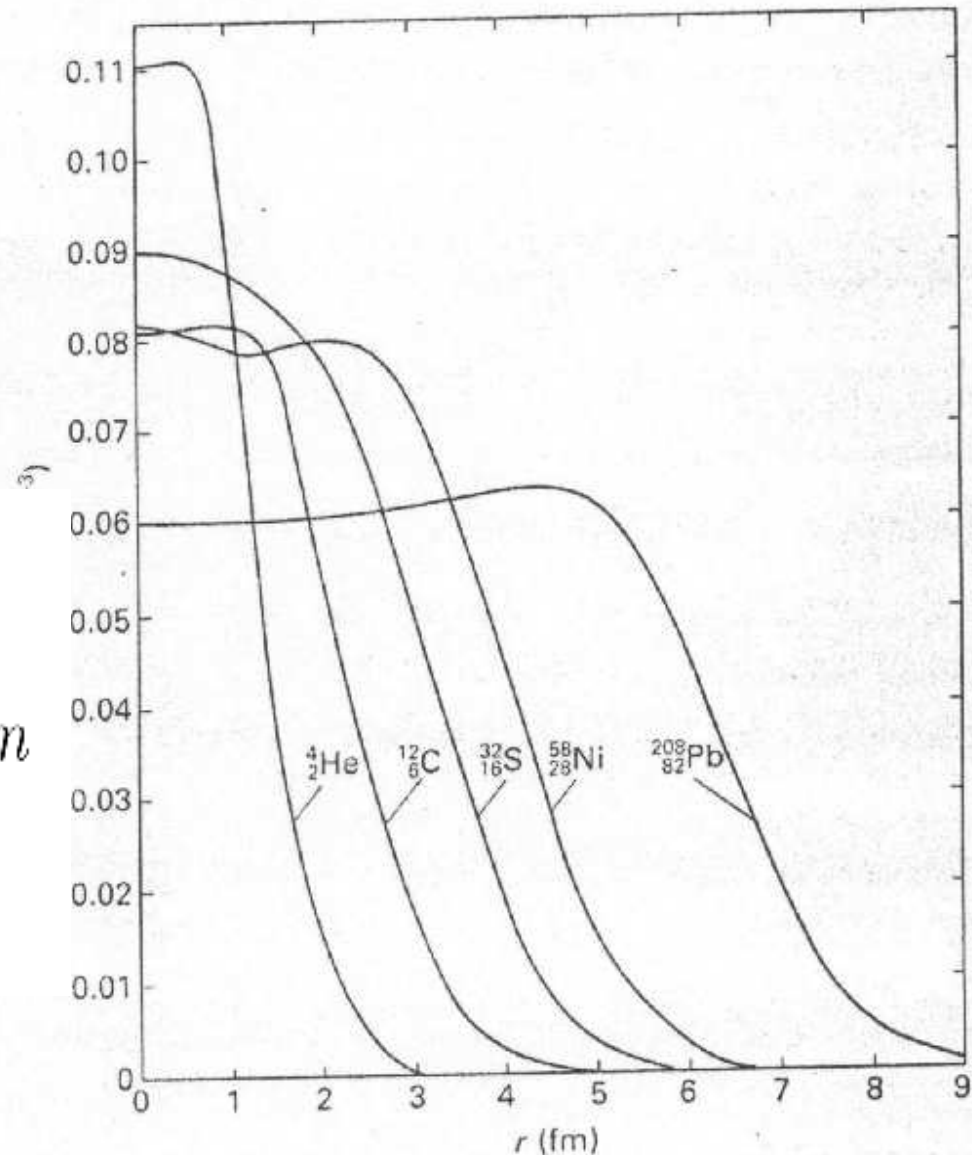
(see also Cross Sections)

Charge density can be approximated by a Saxon-Woods shape within about 10 %

$$\rho(r) = \frac{\rho_0}{1 + e^{r-R/a_V}}$$

$$\text{with } R = 1.18 \times A^{1/3} - 0.048 \text{ fm}$$

$$\text{and } a_V = 0.055 \pm 0.07 \text{ fm}$$



Binding Energy B of Nuclei with Z protons, N neutrons and Mass $M(Z,N)$

$$B(Z, N) = Zm_p + Nm_n - M(Z, N).$$

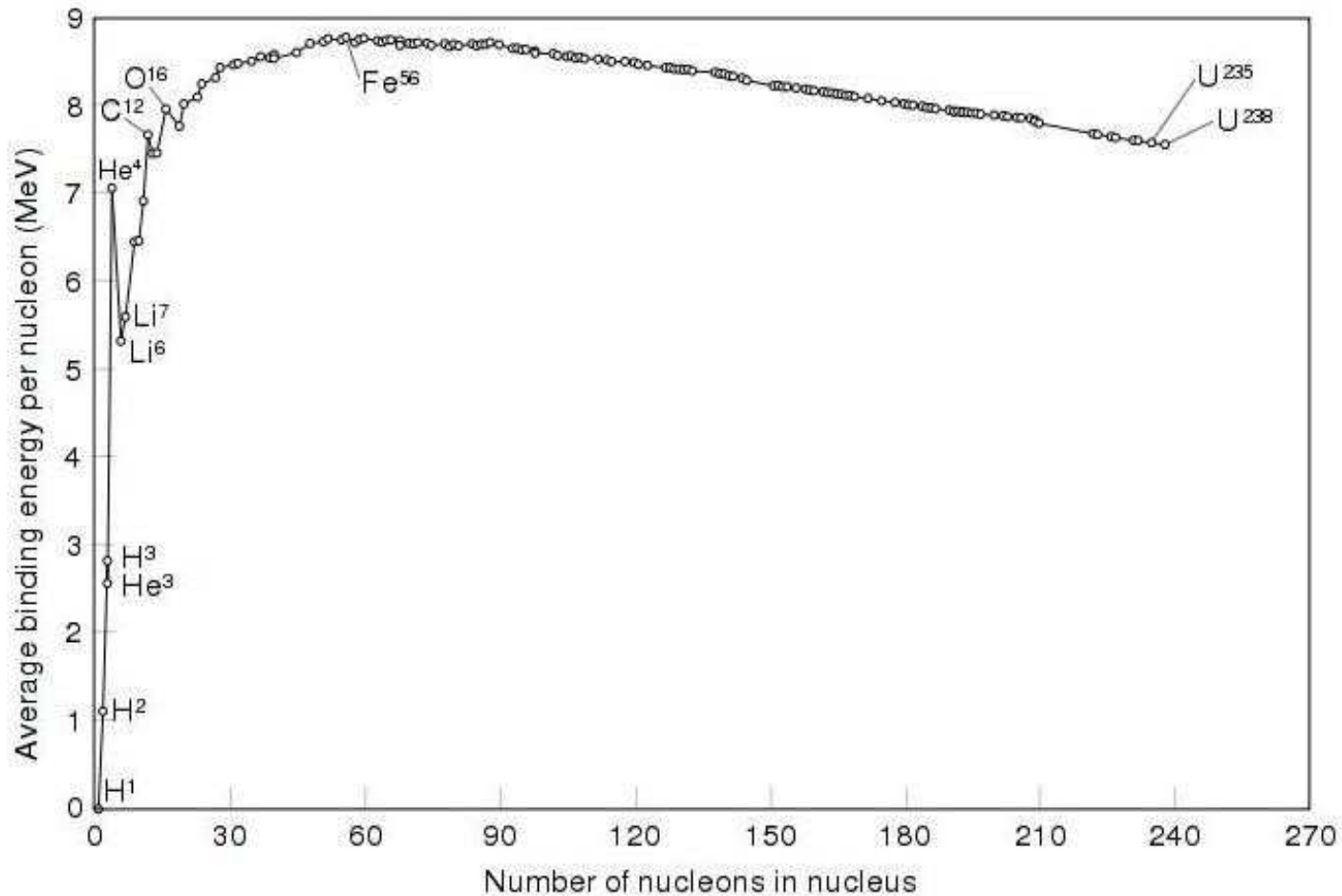
with $E=mc^2$, masses are measured in MeV.

Remark: Mass excess and atomic mass unit

$$M_{\text{ex}}(Z,N) = M(Z,N) - A m_u$$

u

Measured Nuclear Binding Energies of Stable Nuclei



Remark: Binding energy is released by nuclear burning

The Independent Particle Model

Assumption: Equilibrium distribution of cold Fermi-gas in a box like potential (last week, and see blackboard for details)

$$\begin{array}{l}
 \text{Phase} \\
 \text{Space}
 \end{array}
 \quad
 \begin{array}{l}
 \Phi(E) = \frac{4\pi}{3} \frac{g}{h^3} (2m)^{3/2} E^{3/2} \\
 \omega(E) = 2\pi \frac{g}{h^3} (2m)^{3/2} E^{1/2}.
 \end{array}
 \quad
 E_F = \frac{h^2}{2m} \left(\frac{3N}{8\pi V} \right)^{2/3}.$$

Mean binding energy per nucleon:

$$\begin{aligned}
 \bar{E} &= \frac{1}{N} \int_0^{E_F} E \leq (E) dE = \frac{1}{\Phi(E_F)} \int_0^{E_F} E \leq (E) dE \\
 &= \left(\frac{8\pi}{3} \frac{(2m)^{3/2}}{h^3} E_F^{3/2} V \right)^{-1} \cdot \frac{2}{5} \frac{4\pi}{h^3} (2m)^{3/2} E_F^{5/2} \cdot V \\
 \bar{E} &= \frac{3}{5} E_F. \quad \Rightarrow \text{Mean energy/nucleon 8-12MeV} \\
 &\quad \text{Potential } V = 40\text{MeV}
 \end{aligned}$$

Droplet Model (Nuclear Mass Formula)

(Weizsacker, 1935, Bethe and Backer 1936)

$$B(Z, A) = a_V A \quad \text{Volume Effect (B = A (Potential - Ebar))}$$

$$-a_S A^{2/3} \quad \text{Surface Effect (less neighbors for nuclear at surface)}$$

$$-a_C \frac{Z^2}{A^{1/3}} \quad \text{Coulomb repulsion of protons}$$

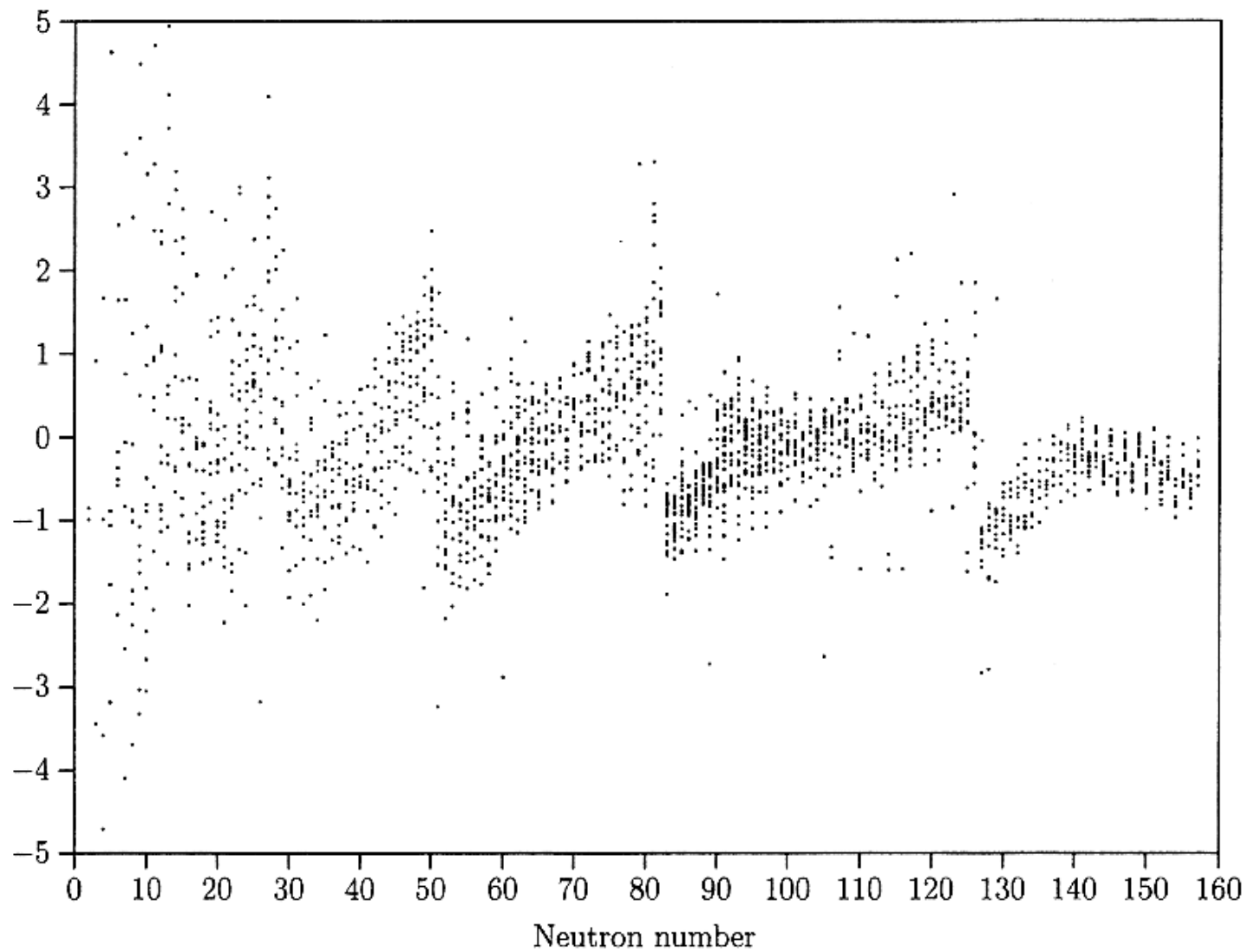
$$-a_A \frac{(Z - A/2)^2}{A} \quad \text{Asymmetry Effect from Pauli exclusion (see blackboard)}$$

$$\begin{array}{ll} +\frac{12}{\sqrt{A}} & \text{if even-even} \\ 0 & \text{if odd A} \\ -\frac{12}{\sqrt{A}} & \text{if odd-odd.} \end{array} \quad \text{Cooper Pairs (pairing gap of Nucleons)}$$

$$B(Z, N) = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - a_{sym} \frac{(N - Z)^2}{A} + B_{pair}(Z, N)$$

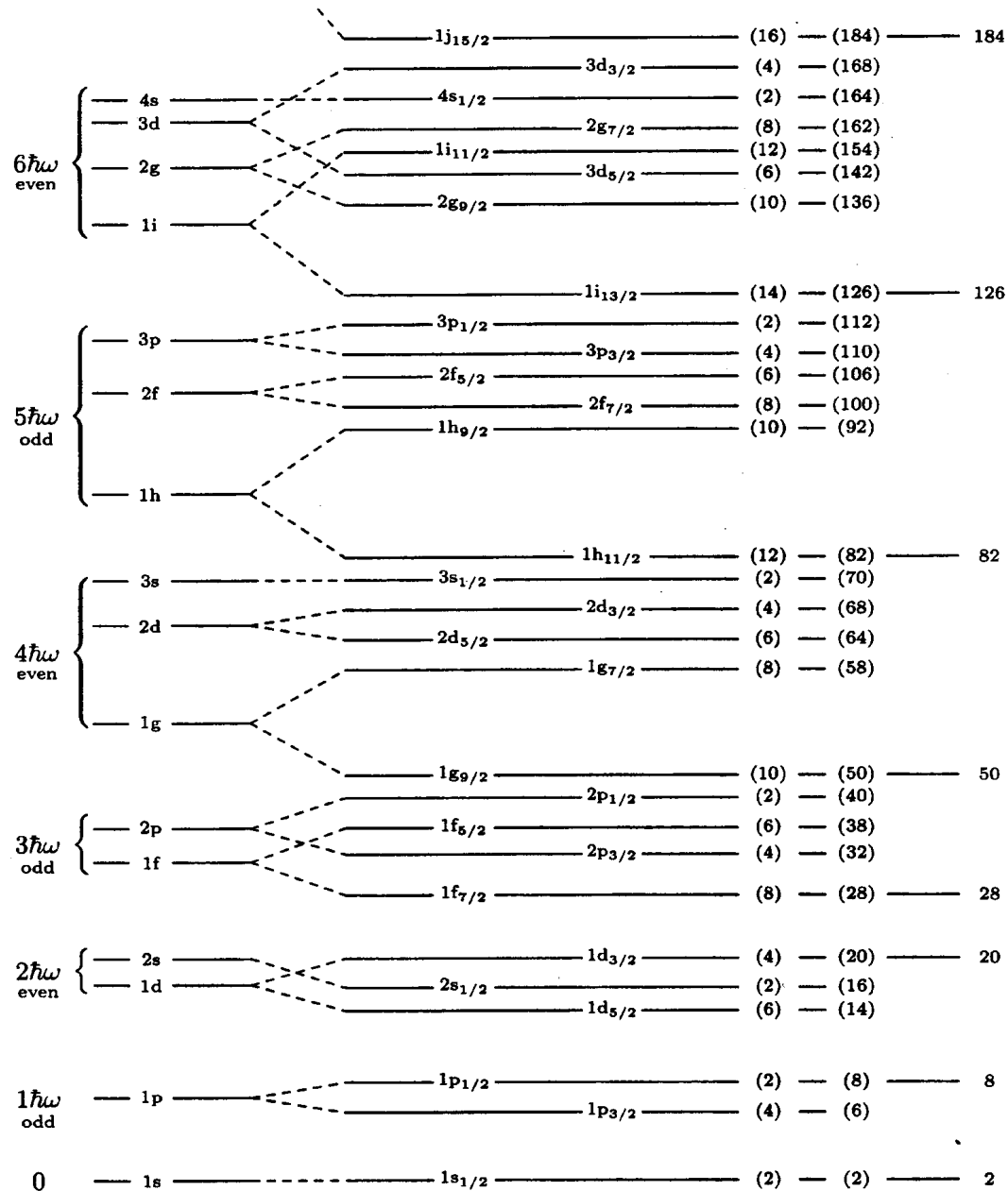
$$a_V = 16 \text{ MeV}, \quad a_S = 18.5 \text{ MeV}, \quad a_C = 0.72 \text{ MeV}, \quad \text{and} \quad a_{sym} = 23.4 \text{ MeV}.$$

Differences between observed and predicted $B(Z,N)$ [MeV]



(from Moeller et al. 1994)

Solution: Shell Model and Magic Numbers (e.g. Moeller et al. 94)



- Non-uniform distribution of level by splitting

- Gaps in E

- coupling

- Magic numbers