Term Paper

Class: Quantum Many-body Physics

Student: Thach Can

Double-layer Quantum Hall Ferromagnets

Introduction

The discovery of quantum Hall effect (QHE) was one of the most remarkable achievements in condensed matter physics in the second half of the last century. Together with superconductivity, QHE has extended our knowledge of quantum mechanics in many-body systems. In this effect, a 2D electron gas when subjected to a strong magnetic field exhibits a quantized Hall resistance and, at the same time, a nearly vanishing dissipative resistance in a range of magnetic field strength. The quantized Hall resistance can be characterized by integer quantum numbers in the integer quantum Hall effect (IQHE) or by some rational fractions in fractional quantum Hall effect (FQHE). In the IQHE regime, electrons occupy only the first few Landau levels because of the enormous degeneracy of these levels at high magnetic field. In the FQHE, the underlying physics is the Coulomb interaction and the correlation among electrons. It turns out that Coulomb interaction plays an important role not only in FQHE but also in QHE when we consider the spin dynamics of the system. In free space, the Zeeman splitting is exactly the same as the cyclotron splitting which is of the order of 100 K. But in real material, such as GaAs, the Zeeman splitting is reduced by about 2 orders of magnitude compared to the cyclotron splitting due to the reduction in effective mass and the gyromagnetic ratio of electrons. Therefore, at low temperature, the system lies in a special situation in which the orbital motion is fully quantized ($k_B T \ll \hbar\omega_c$) but the low-energy spin fluctuations are not forbidden ($k_B T \sim g^* \mu_B B$). Interestingly, in the ground state of quantum Hall system, all the spins are still aligned ferromagnetically not due to the Zeeman splitting but due to the Coulomb interaction. Coulomb interaction is independent of spin therefore we can expect the same effect in double-layer quantum Hall system with the layer indices play the role of spin.

In this report, we start out with an introduction to quantum Hall effect and quantum Hall ferromagnet. We then discuss two quantum many-body effects in double-layer quantum Hall ferromagnet by using pseudo spin treatment in which the layer indices play the role of spin. The first one is the interlayer phase coherent effect which associated with the interaction between electrons in different layers. The second one is the tunneling between the two layers and the effect of the in plane component of the magnetic field, which is related to the broken U(1) symmetry of the system.

Quantum Hall Effect

The advance in technology makes it possible to prepare 2D electron gas system with extremely low disorder and high mobility. The motion of an electron in such system in the presence of a uniform magnetic field perpendicular to the plane can be studied conveniently by choosing the Landau gauge for the vector potential $\vec{A}(\vec{r}) = xB\hat{y}$. Using this gauge, the physics of the system is invariant under the translation in y axis. The wave function for the motion in this axis is therefore simply plane wave characterized by the wave vector k. By separating variables one can see that the wave function for the motion in x axis is the same as that of a harmonic oscillator whose frequency equals to the cyclotron frequency and whose motion centers at $X_k = -kl^2$ where $l = \sqrt{\hbar c/eB}$ is the magnetic length. The energy levels corresponding to the motion in x axis are quantized as $\epsilon_{k,n} = (n + \frac{1}{2})\hbar\omega_c$ and called Landau levels. Landau levels are degenerate because they do not depend on the wave vector k. The numbers of states in each Landaus levels can be calculated by imposing the periodic boundary condition in y axis. Let's consider a rectangular sample with dimensions L_x, L_y and the left edge is at $x = -L_x$ and the right edge is at $x = 0$, then the condition for the centers of motion to be inside the sample $-L_x < X_c < 0$ gives $0 < k < L_x/l^2$. The total number of states in each Landau levels is then:

$$N = \frac{L_y}{2\pi}\int_0^{L_x/l^2} dk = \frac{L_x L_y}{2\pi l^2} = N_\phi$$

where $N_\phi = BL_x L_y/\phi_0$ is the number of flux quanta penetrating the sample; $\phi_0 = hc/e$ is the quantum of magnetic flux. So there is exactly one state per Landau level per flux quantum.

In general, applying an electric field along the x axis will result in a shift in the centers of the motions X_k and a k-dependence in Landau levels. If the electric field is non-uniform and there are disorders in the system, all the states in the bulk are localized due to Anderson's localization. In the other words, deep inside the sample, the Landau levels are still k-independent, the group velocity is therefore equal to zero. But at the edges $x = -L_x$ and $x = 0$ of the sample Landau levels does depend on k, furthermore, the group velocity

$$\vec{v}_k = \frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial k}\hat{y}$$

has the opposite sign on the two edges of the sample. Therefore, there are edge currents running in opposite directions along the y axis. The electric transport in the system can be analyzed in analogy with the Landauer formalism for the transport in narrow wide. The edge currents correspond to the left and the right moving states between two Fermi points. The net current can be calculated by adding up the group velocities of all the occupied states:

$$I = -\frac{e}{L_y}\int_{-\infty}^{+\infty} dk \frac{L_y}{2\pi}\frac{1}{\hbar}\frac{\partial \epsilon_k}{\partial k} n_k$$

where we assume that in the bulk only a single Landau level is occupied and n_k is the occupied probability. At zero temperature, we have

$$I = -\frac{e}{h}\int_{\mu_R}^{\mu_L} d\epsilon = -\frac{e}{h}[\mu_L - \mu_R]$$

The definition of the Hall voltage drop is

$$(+e)V_H \equiv (+e)[V_R - V_L] = [\mu_R - \mu_L]$$

Hence

$$I = -\frac{e^2}{h}V_H$$

If there are ν Landau levels are occupied in the bulk, then

$$I = -\nu \frac{e^2}{h} V_H$$

Here, the applied voltage is in x axis and the net current is along y axis, therefore what we are calculating here is indeed the Hall resistance. So, we derive to what are observed in QHE

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = -\nu \frac{e^2}{h}$$

with the integer quantum number ν which is also known as Landau level filling factor.

Quantum Hall Ferromagnets

Even though FQHE is not necessary for the discussions in this report, it is worth to mention that in that class of QHE, the quantum number ν takes in some rational fraction ($\nu = 1/3, 2/5, 3/7, 2/3, 3/5, 1/5, 2/9, 3/13, 5/2, 12/5, \ldots$). The underlying physics in FQHE is the Coulomb interaction and the correlation between electrons. One may think that such interaction has nothing to do with integer quantum Hall system. However, the study of ferromagnetism in the system with $\nu = 1$ has shown that Coulomb interaction also plays an important role. In a fully ferromagnetic state, all the spins are lined up parallel to each other, hence the spin part of the wave function is symmetric under the particle exchanges. Therefore, the spatial part of the wave function must be fully antisymmetric and vanish when any pair of particles approaches each other. Such condition keeps the particles away from each other thus lowers the Coulomb interaction. It turns out that for filling factor $\nu = 1$, the Coulomb interaction is about 2 orders of magnitude greater than the Zeeman splitting and hence strongly stabilizes the ferromagnetic state. Indeed, at zero temperature, the ground state of the system with $\nu = 1$ is spontaneously fully polarized even in the absence of an external magnetic field. The spin wave excitations can be studied by using the method similar to that used in Heisenberg model despite the fact that spins in Heisenberg model are localized whereas quantum Hall ferromagnet is a system of itinerant spins. The spin wave dispersion shows a gap at $k = 0$ equal to the Zeeman splitting and starts out quadratically at small k. At large wave vectors, the energy saturates at the Coulomb interaction scale. Effective action theory can also be used to reproduce these results. In this theory, first we write down the Lagrangian of the system and then derive the equation of motion from that. The Lagrangian that realizes the correct precession of spin in magnetic field and also the global spin rotation (SU(2) symmetry)can be written as

$$\mathcal{L} = -\hbar S n \int d^2 r \{\dot{m}^\mu(\vec{r}) A^\mu[\vec{m}] - \Delta m^z(\vec{r})\} - \frac{1}{2}\rho_s \int d^2 r \partial_\mu m^\nu \partial_\mu m^\nu + \int d^2 r \lambda(\vec{r})(m^\mu m^\mu - 1)$$

where

$S = 1/2$ is the spin length

$n = \nu/2\pi l^2$ is the particle density

$$\dot{m}^\mu(\vec{r}) = \frac{dm^\mu(\vec{r})}{dt}$$

$\vec{m}$ is a vector field of fixed length $\vec{m} \cdot \vec{m} = 1$ which describes the local orientation of the order parameter (the magnetization)

$\vec{A}$ can be determined by requiring that it leads to the correct precession of the magnetization;

$\vec{\Delta} = \Delta\hat{z}$ is the external magnetic field

ρ_s is a phenomenological spin stiffness

λ is the Lagrange multiplier to impose the fixed length constraint.

…

Double-layer Quantum Hall Ferromagnet

Double-layer quantum Hall ferromagnet consists of two layers of 2D electron gas. Although the layer separation is comparable to the distance between electrons in the same layer, the interlayer electrons interaction is still smaller than the intralayer electrons interaction, therefore in this case the system does not have a full SU(2) symmetry. The Lagrangian is now written as

$$\mathcal{L} = -\hbar Sn \int d^2r \, \{\dot{m}^\mu(\vec{r}) A^\mu[\vec{m}] - \Delta m^z(\vec{r})\} - \frac{1}{2}\rho_s \int d^2 r \, \partial_\mu m^\nu \partial_\mu m^\nu + \int d^2 r \, \lambda(\vec{r})(m^\mu m^\mu - 1)$$
$$- \int d^2 r \, \beta m^z m^z + \int d^2 r \, ntm^z$$

Broken SU(2) symmetry; XY model

Linear instead of quadratic dispersion due to the β term

…

Interlayer Phase Coherent

Interaction

$$|\psi\rangle = \prod_k \{c_{k\uparrow}^+ + c_{k\downarrow}^+ e^{i\varphi}\} |0\rangle$$

$$H = \frac{1}{2}\rho_s \int d^2 r \, |\nabla\varphi|^2 + \cdots$$

Interlayer Tunneling and Tilted Field Effects

Broken U(1) symmetry

$$H_{eff} = \int d^2 r \left[\frac{1}{2}\rho_s |\nabla\varphi|^2 - nt\cos(\varphi)\right]$$

Tunneling

$$H_T = -t \int d^2 r \, \{\psi_\uparrow^+(r)\psi_\downarrow(r) + \psi_\downarrow^+(r)\psi_\uparrow(r)\}$$

$$H_T = -t \int d^2 r \, S^x(r)$$

$$H_{eff} = \int d^2\, r \left[\frac{1}{2}\rho_s |\nabla\varphi|^2 - nt\cos(\varphi - Qx)\right]$$

References