The Main Sequence: Hydrogen Burning

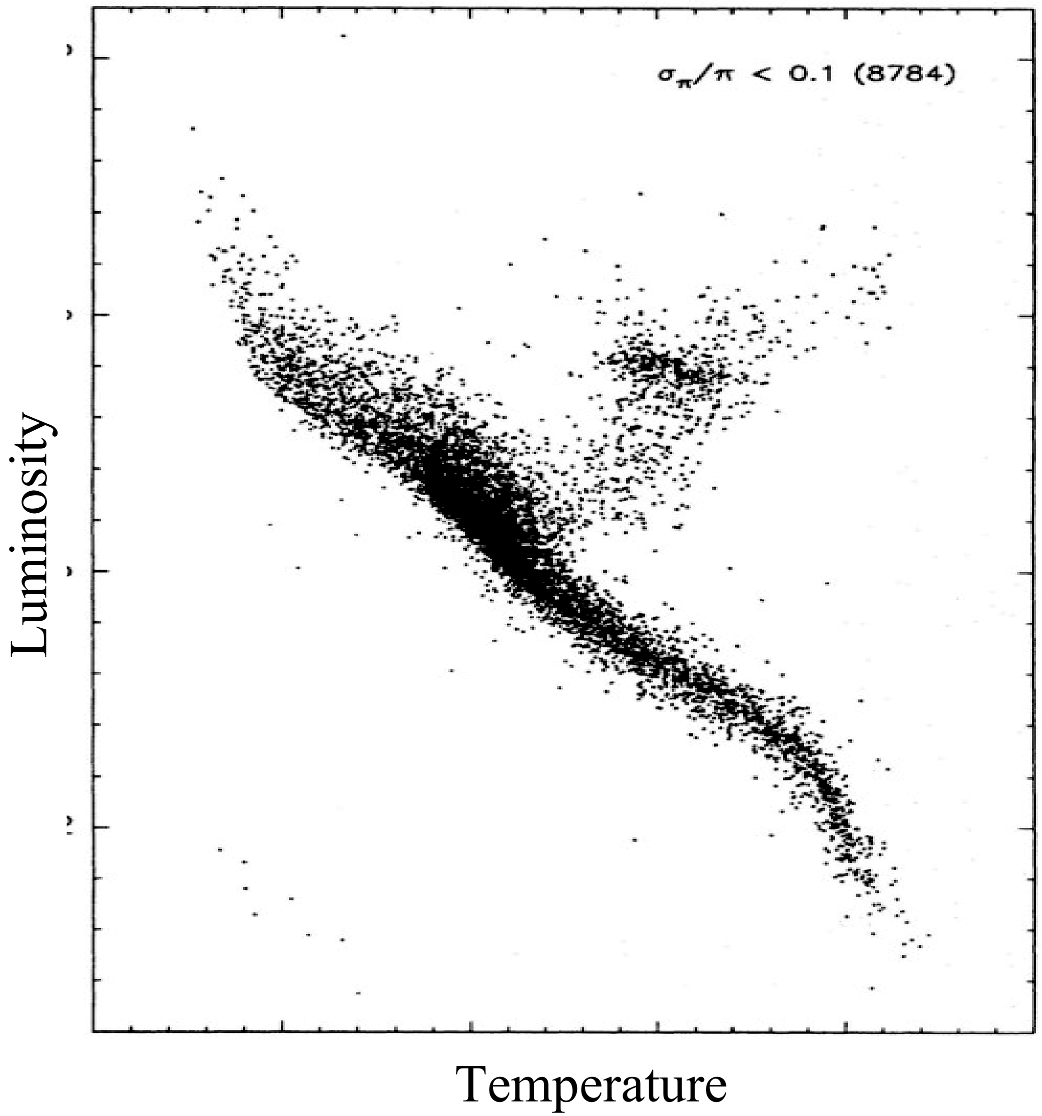
- Concept of steady flow in nuclear reactions

- the pp-chain(s)

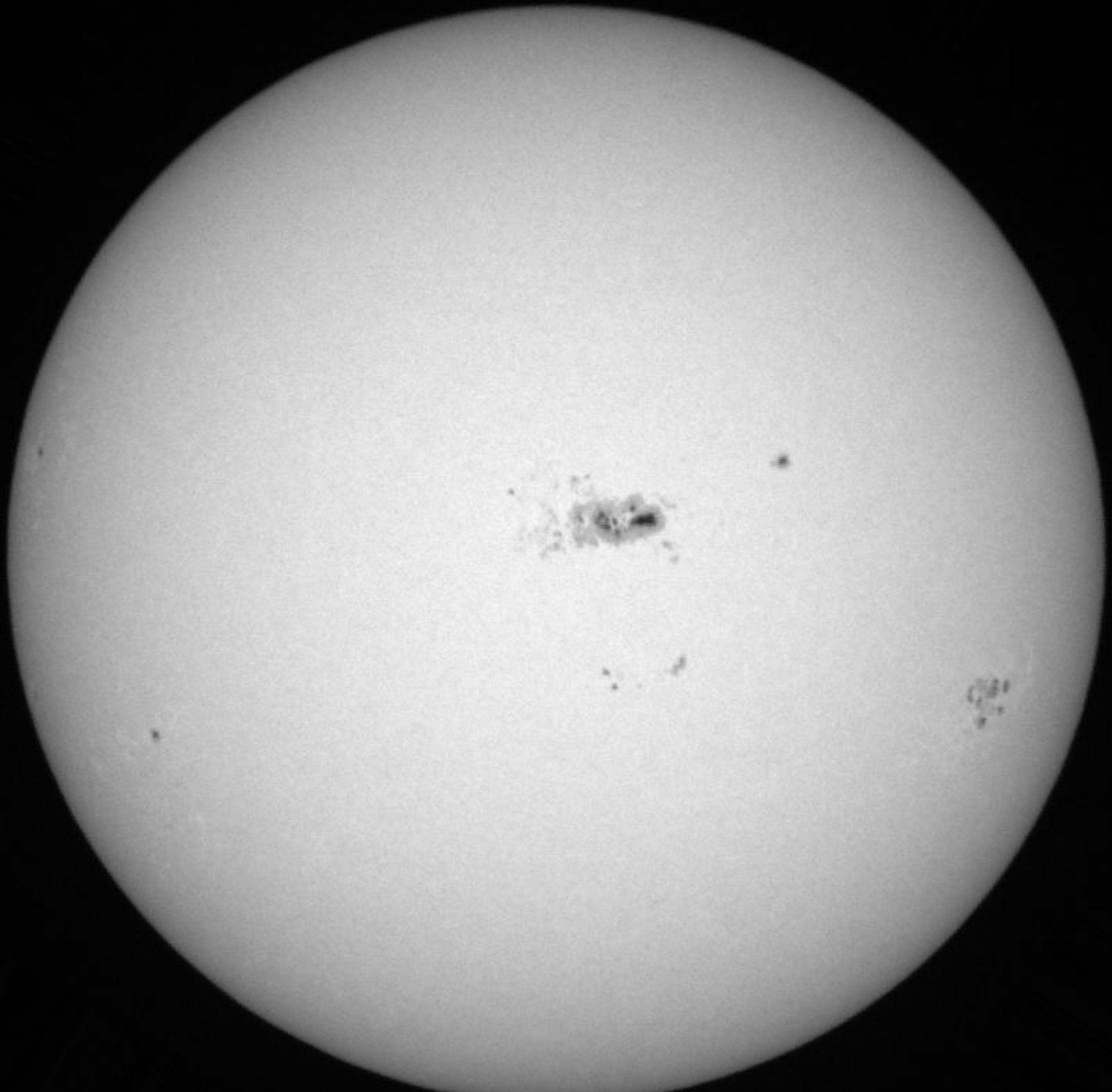
- CNO-cycle(s)

Literature: Iliadis: Chap. 5.1 - 5.2

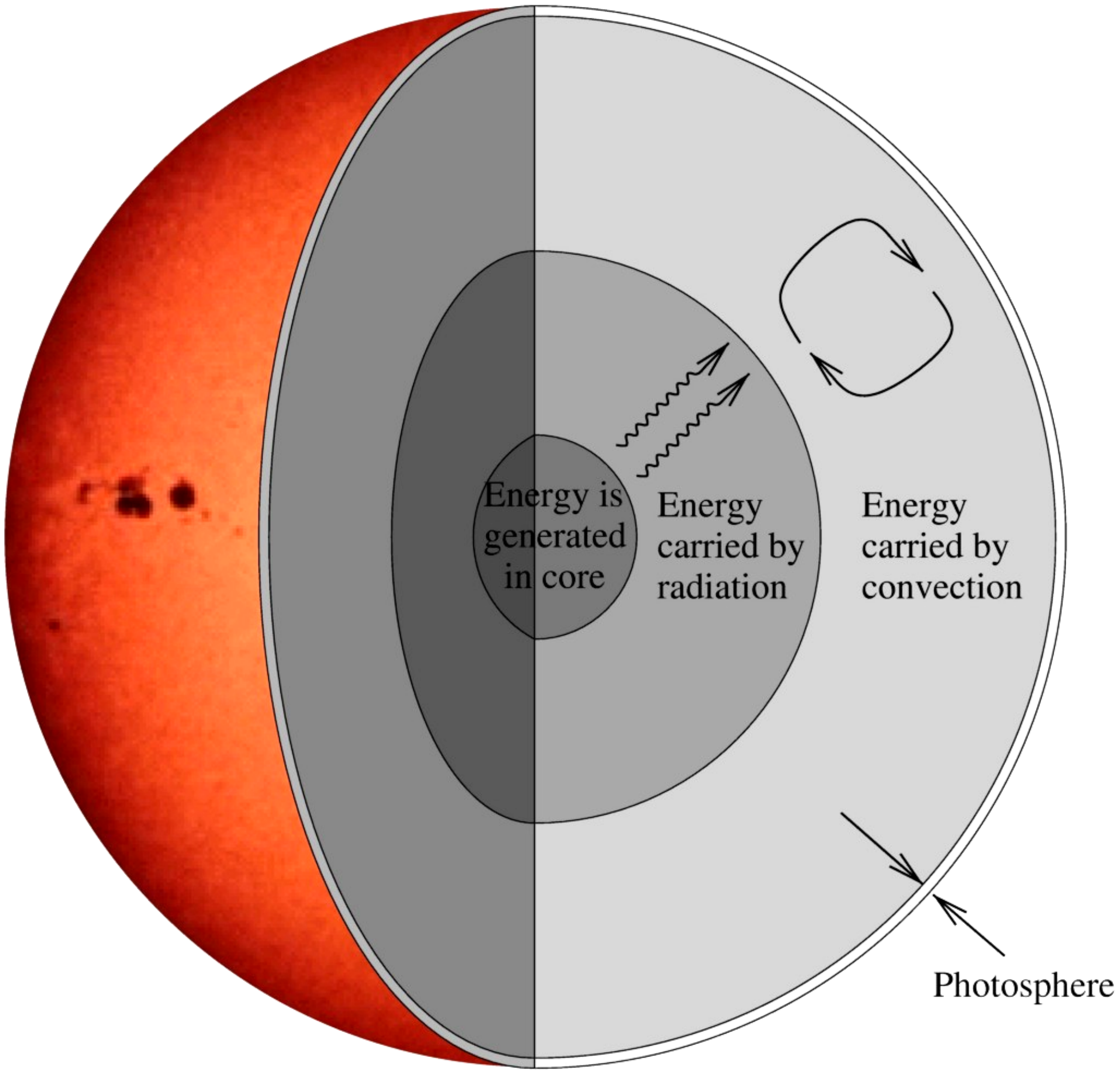
The Real H-R Diagram for Stars near the Sun



The Sun at Visible Wavelengths

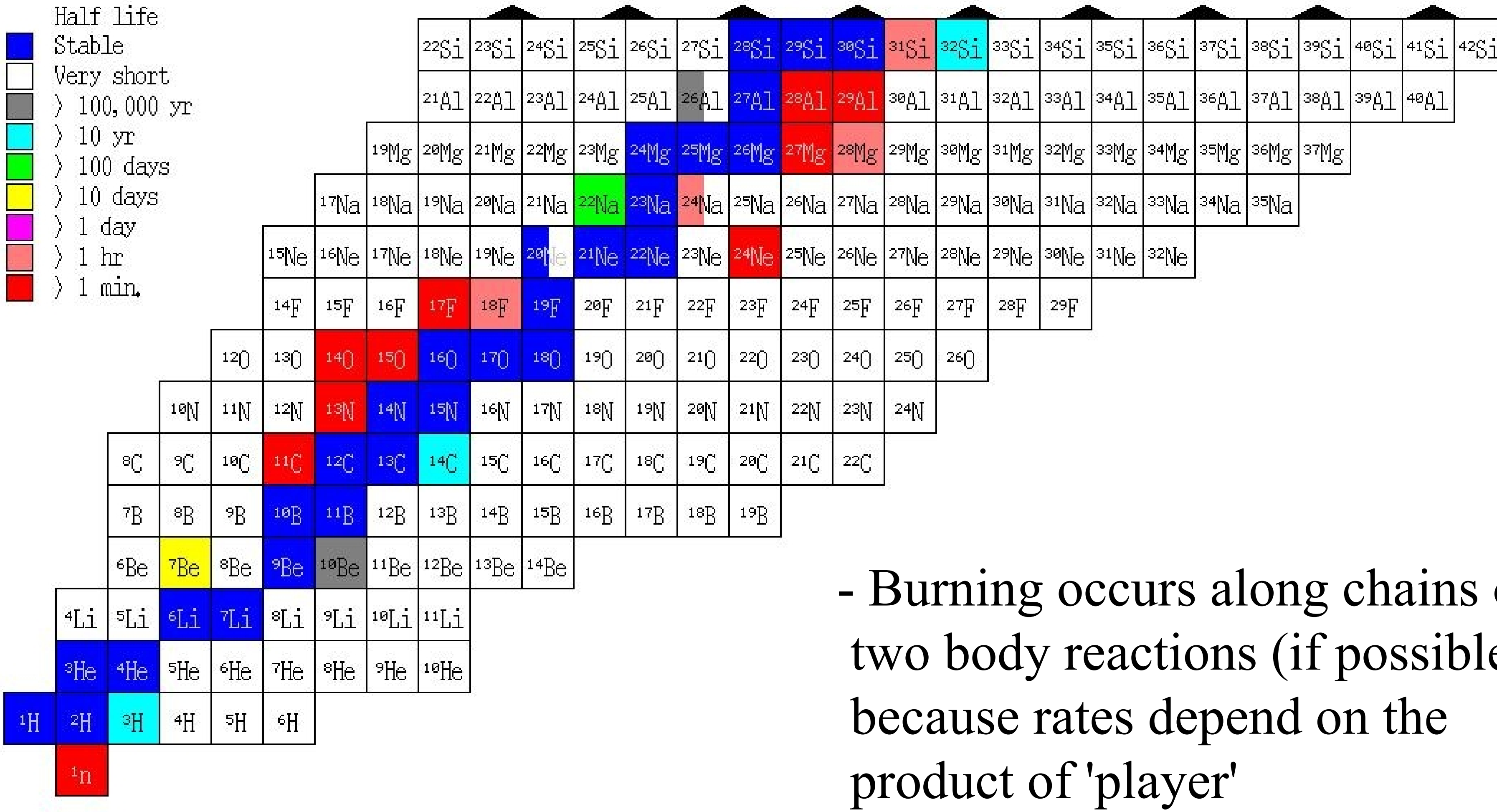


The Interior of the Sun



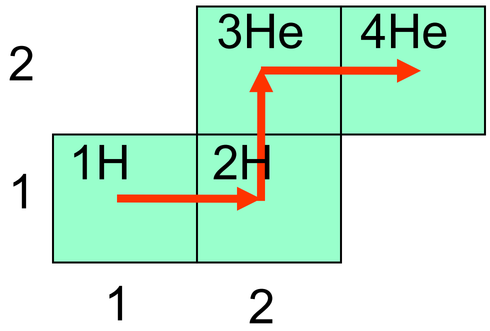
Remark: Nuclear processes proceed along two body reactions if possible

How to burn Hydrogen? PP-Cycle

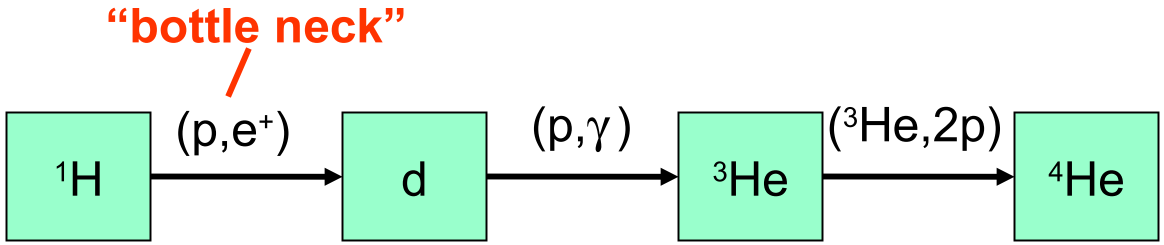


- Burning occurs along chains of two body reactions (if possible) because rates depend on the product of 'player'

On chart of nuclides:



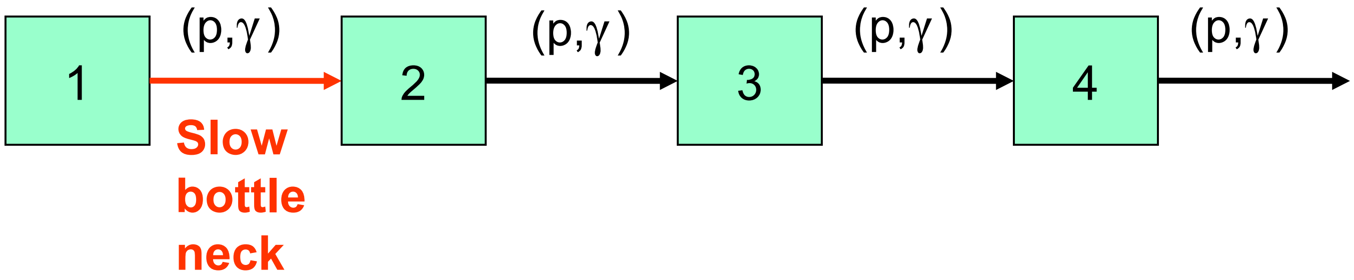
Or as a chain of reactions:



Steady Flow: A chain of reactions after a "bottle neck"

Example

For simplicity consider chain of proton captures:



Assumptions:

- Y1 const as depletion is very slow because of "bottle neck"
- Capture rates constant (Y_p ~ const because of large "reservoir", conditions constant as well)

Abundance of nucleus 2 evolves according to:

$$\frac{dY_2}{dt} = \underbrace{Y_1 \lambda_{12}}_{\text{production}} - \underbrace{Y_2 \lambda_{23}}_{\text{destruction}}$$

$$\lambda_{12} = \frac{1}{1+\delta_{p1}} Y_p \rho N_A \langle \sigma v \rangle_{1\to 2}$$

For our assumptions Y_1~const and Y_p~ const, Y_2 will then, after some time reach an equilibrium value regardless of its initial abundance:

In equilibrium:

$$\frac{dY_2}{dt} = Y_1 \lambda_{12} - Y_2 \lambda_{23} = 0$$

and

$$Y_2 \lambda_{23} = Y_1 \lambda_{12}$$

(this is equilibrium is called steady flow)

Same for Y_3 (after some longer time)

$$\frac{dY_3}{dt} = Y_2 \lambda_{23} - Y_3 \lambda_{34} = 0$$

and $Y_3 \lambda_{34} = Y_2 \lambda_{23}$ with result for Y_2: $Y_3 \lambda_{34} = Y_1 \lambda_{12}$

and so on …

So in steady flow: $Y_i \lambda_{i\,i+1} = \text{const} = Y_1 \lambda_{12}$ or $Y_i \propto \tau_i$

steady flow abundance (Y_i) destruction rate ($\lambda_{i\,i+1}$)

Timescale to achieve steady flow equilibrium

for λ~const

$$\frac{dY_2}{dt} = Y_1 \lambda_{12} - Y_2 \lambda_{23}$$

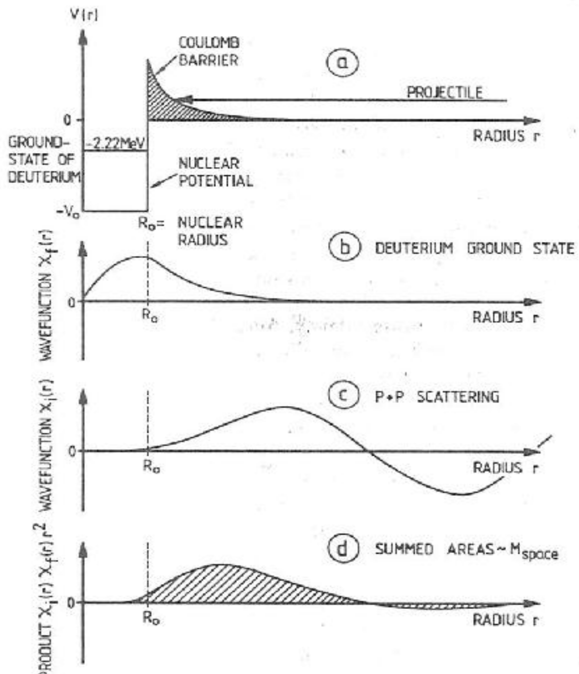
has the solution:

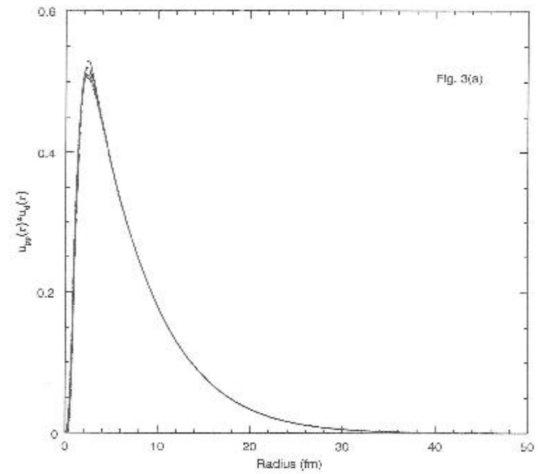
$$Y_2(t) = \bar{Y}_2 - (\bar{Y}_2 - Y_{2\,\mathrm{initial}})\, e^{-t/\tau_2}$$

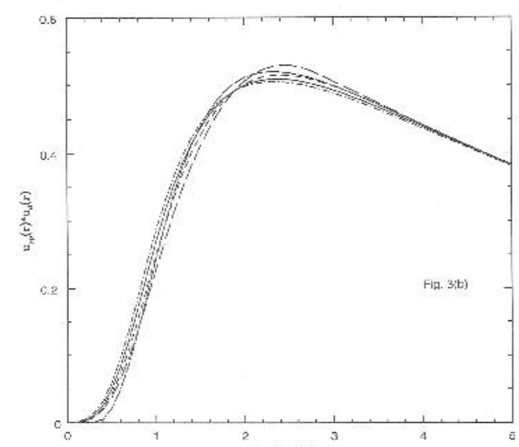
with $\bar{Y}_2$ equilibrium abundance

$Y_{2\,\mathrm{initial}}$ initial abundance

The Proton Proton Reaction p(p,nu)d







- s/d wave ratio
- only 'sensitive' to small distances
- different NN forces give same results

S factor for the p p reaction

The S factor at zero energy for the pp reaction can be written as, (Adelberger *et al., Rev. Mod. Phys.* **70**, *1265 (1998)).*

$$S_{11}(0) = 6\pi^2 m_p c\alpha(\ln 2)\frac{\Lambda^2}{(2\mu E_d)^{3/2}}\left(\frac{G_A}{G_V}\right)^2 \frac{f_{pp}}{(ft)_{0^+\rightarrow 0^+}}(1+\delta)^2$$

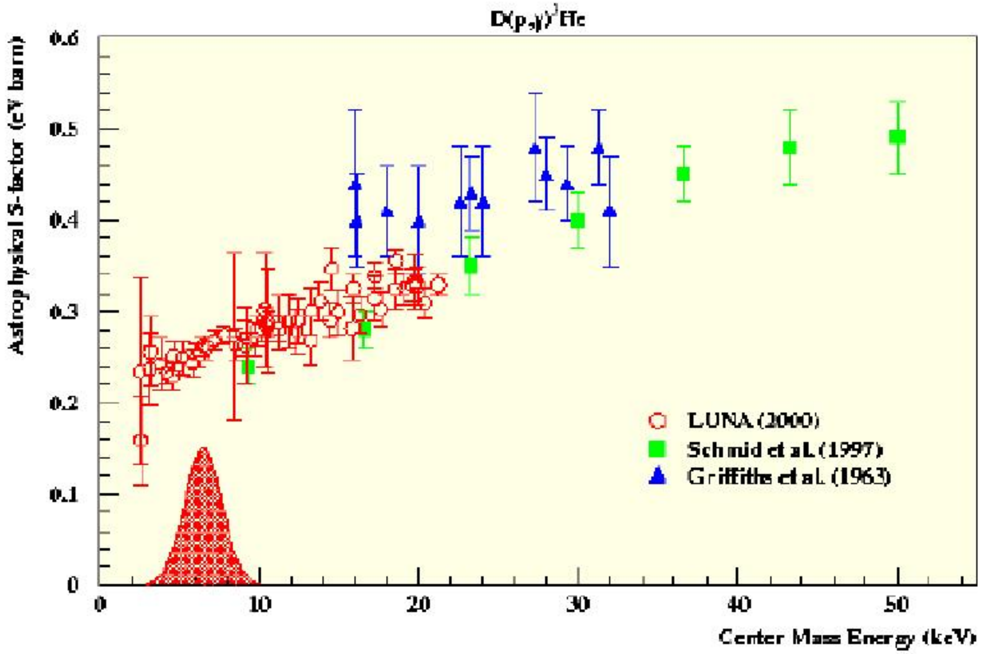
Here E_d is the deuteron binding energy and the weak coupling constants have been introduced via the ft-value for superallowed Fermi transitions ($0^+ \rightarrow 0^+$) which are experimentally wellknown ($ft = 3073.1 \pm 3.1$ s). $f_{pp} = 0.144$ is the proton-proton phase space factor.
One finds

$$S_{11}(0) = (4.00 \pm 0.05) \times 10^{-25}\ \mathrm{MeVb}$$

This corresponds to $\langle \sigma v \rangle_{11} = 1.2 \times 10^{-43}$ cm^3/s at $T_6 = 15$.

The d(p,gamma)3He Cross Section

The LUNA collaboration has measured the d(p,γ)^{3}He cross section at the solar Gamow energies.



$S_{12}(0) = 2.5 \times 10^{-4}$ keVb

LUNA: Laboratory for Nuclear Astrophysics (Grand Sasso)

The d(p,gamma)3He Reaction

Deuterons are burnt by the reaction $d(p,\gamma)^3$He:

$$\frac{dD}{dt} = r_{11} - r_{12}$$
$$= \frac{H^2}{2}\langle \sigma v \rangle_{11} - HD\langle \sigma v \rangle_{12}$$

In equilibrium ($\frac{dD}{dt} = 0$) one has

$$\left(\frac{D}{H}\right)_e = \frac{\langle \sigma v \rangle_{11}}{2\langle \sigma v \rangle_{12}}$$

$$(D/H)_e = 5.6 \times 10^{-18} \text{ for } T_6 = 5$$

The Lifetime of Deuterium in the Sun

Consider the reaction $1 + 2 \to 3 + 4$, then the lifetime of the nucleus a against destruction by b in some environment is given by

$$\tau_b(a) = \frac{1}{N_b \langle \sigma v \rangle_{ab}}$$

If we assume a density $\rho = 100$ g/cm^3 and an equal mixture by mass of hydrogen and helium ($X_H = X_{He} = 0.5$), one finds

$$\tau_p(p) = 0.9 \times 10^{10}\ y\ ;\ \tau_p(d) = 1.6\ s$$

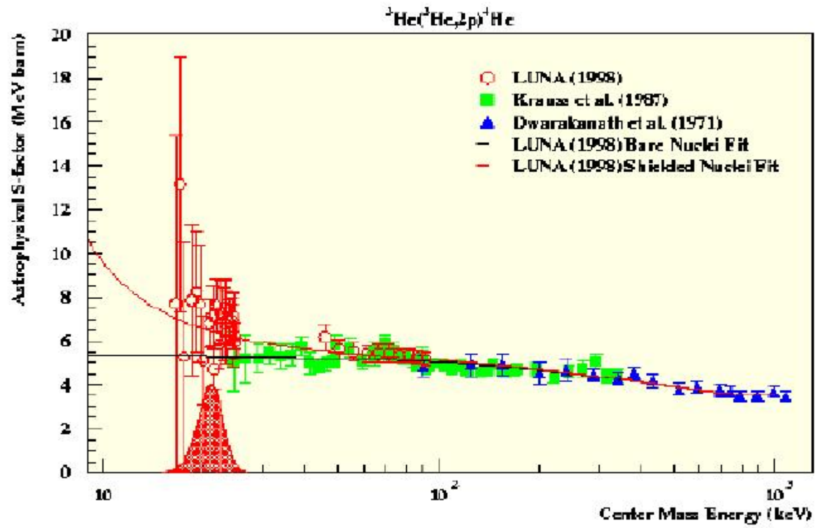
If one assumes a constant H abundance, one finds for the time evolution of D/H

$$D = \frac{H^2}{2} \langle \sigma v \rangle_{11} + e^{-t/\tau_p(d)} (Y_{\mathrm{D,initial}} - \frac{H^2}{2} \langle \sigma v \rangle_{11})$$

Equilibrium is reached in about $\tau_p(d) = 1.6$ s!

The 3He(3He,2p)4He cross section

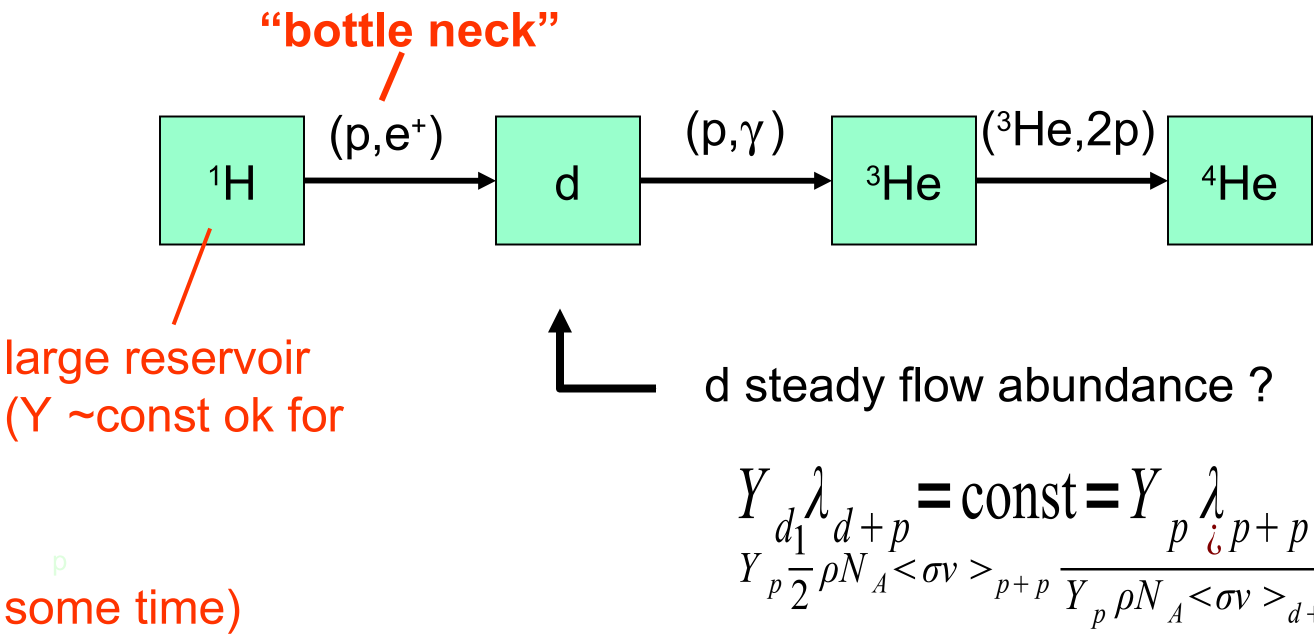
The LUNA collaboration has measured the $^3He(^3He,2p)^4He$ at solar Gamow energies. This was the first time that a nuclear reaction has been determined at the most effective stellar energies.



$S_{33}(0) = 5.4 \times \ \mathrm{MeVb}$

Much larger than $S_{12}(0)$ as resonant and mediated by strong

The ppI chain:



d steady flow abundance ?

$$Y_d \lambda_{d+p} = \text{const} = Y_p \lambda_{p+p}$$

$$Y_p \frac{1}{2} \rho N_A < \sigma v >_{p+p} \quad \overline{Y_p \rho N_A < \sigma v >_{d+p}}$$

$$\frac{Y_d}{Y_p} = \frac{\lambda_{p+p}}{\lambda_{d+p}} = \frac{Y_d}{Y_p} = \sigma v >_{p+p} \quad \frac{}{2 < \sigma v >_{d+p}}$$

S=3.8e-22 keV barn

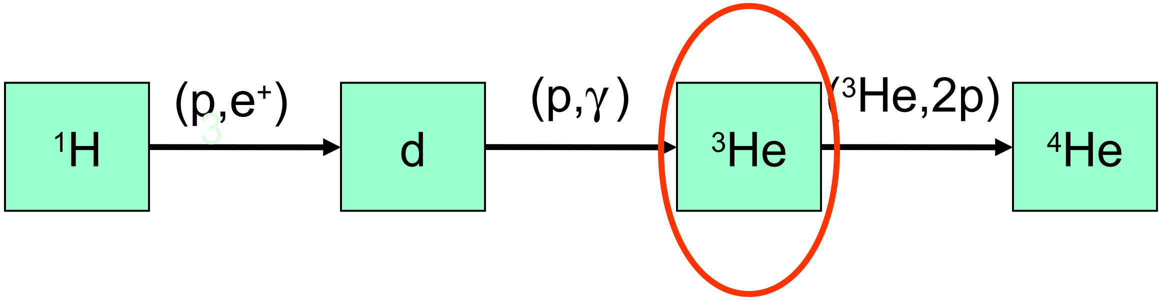
S=2.5e-4 keV barn

therefore, equilibrium d-abundance extremely small (of the order of 4e-18 in the sun)

equilibrium reached within lifetime of d in the sun:

$N_A<\sigma v>_{pd}$=1e-2 cm^3/s/mole $\quad \tau_d = 1/(Y_p \rho N_A < \sigma v >_{p+d}) = 2s$

He equilibrium abundance



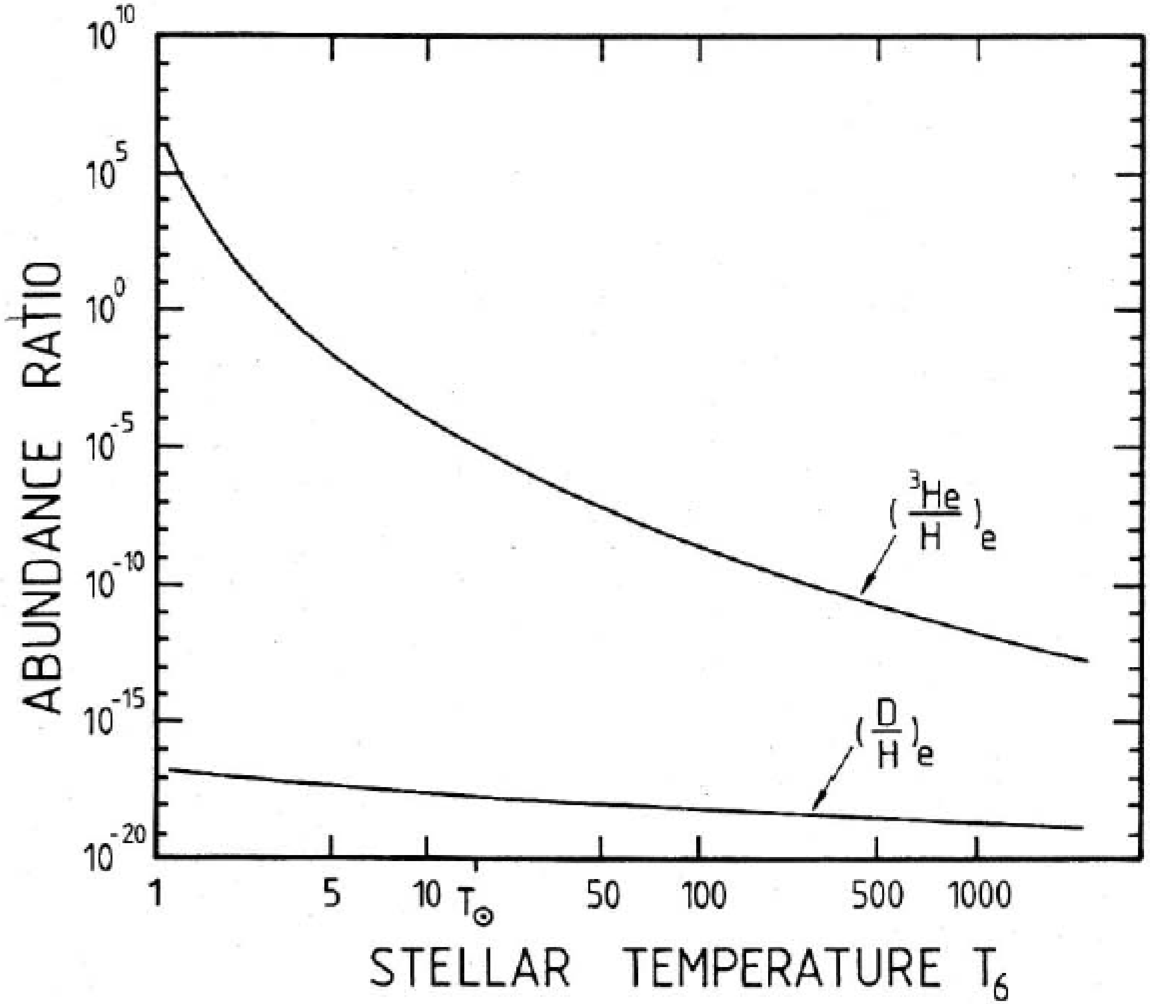
different because two identical particles fuse
therefore destruction rate $\lambda_{3He+3He}$ obviously NOT constant:

$$\lambda_{3He+3He} = \frac{1}{2} Y_{3He} \rho N_A < \sigma v >_{3He+3He}$$ ¿

¿

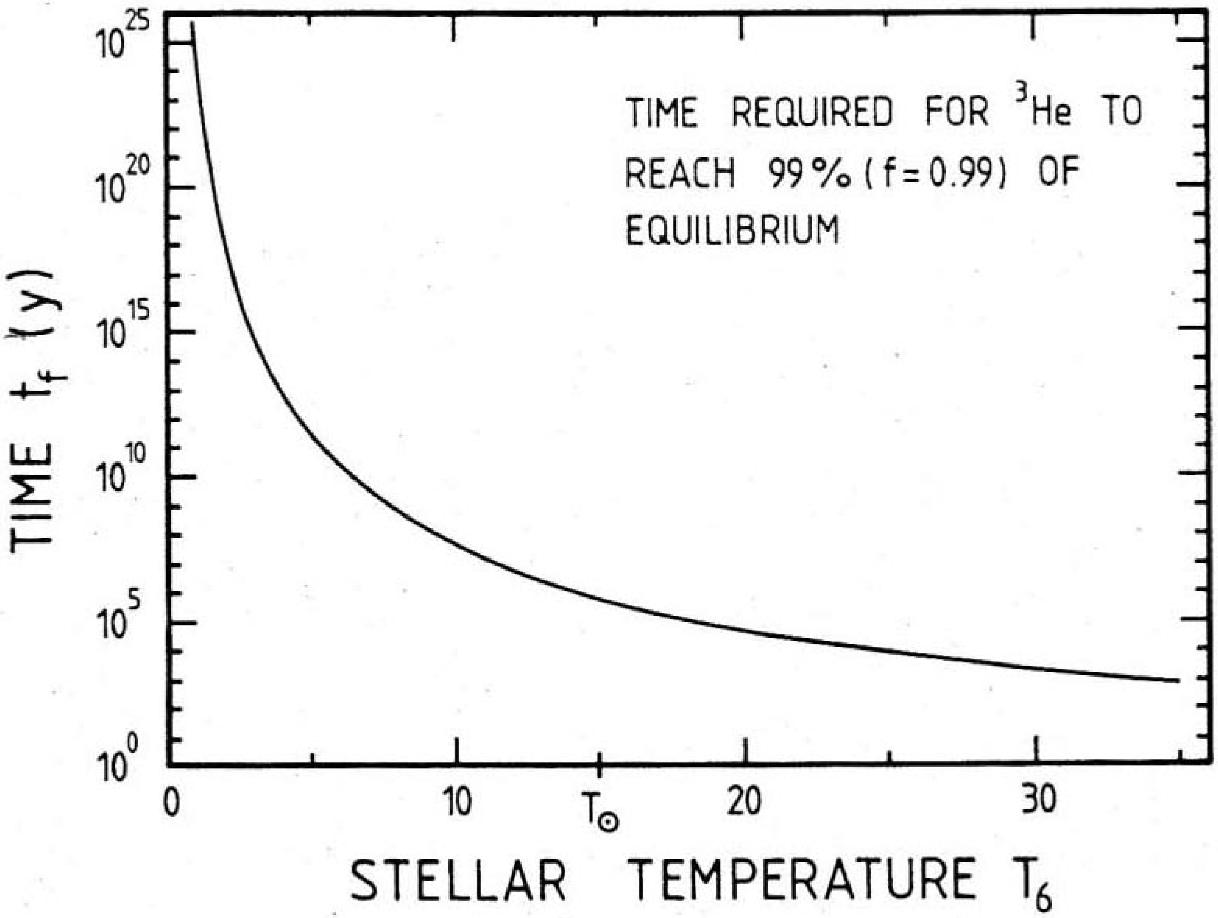
but depends strongly on Y (3He) itself

But equations can be solved again



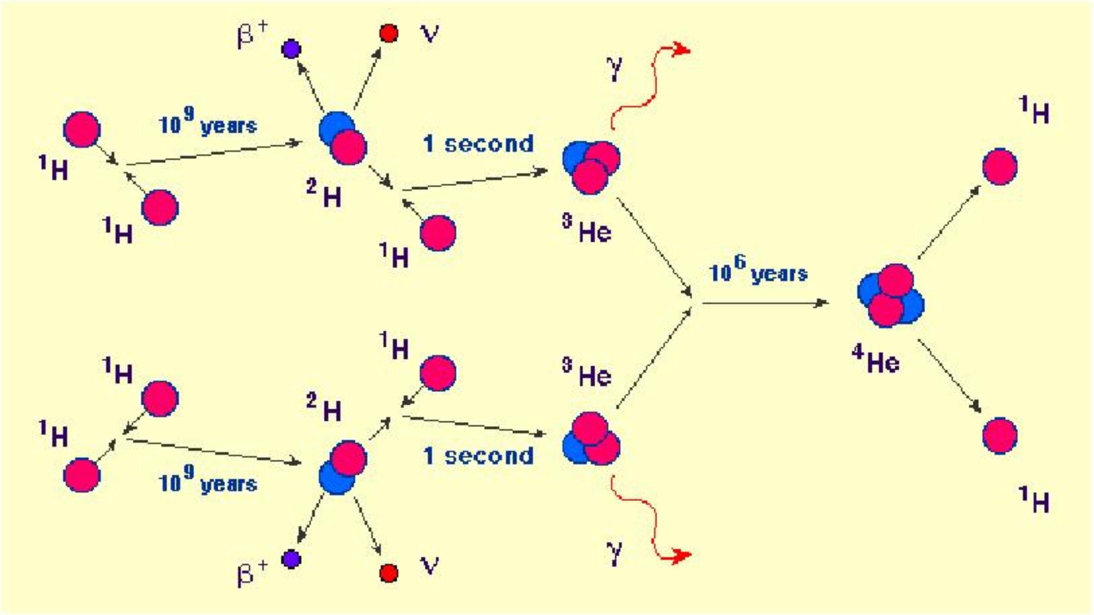
^{3}He has a much higher equilibrium abundance than d

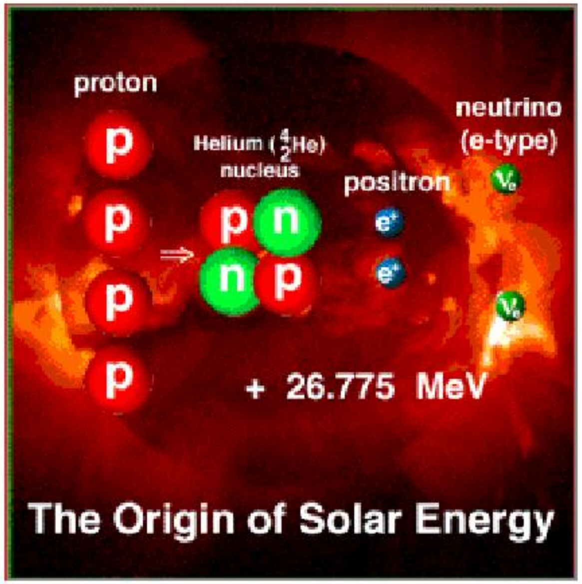
- therefore ^{3}He+^{3}He possible …



TIME REQUIRED FOR ³He TO REACH 99% (f = 0.99) OF EQUILIBRIUM
TIME t_f (y)
10^{25}
10^{20}
10^{15}
10^{10}
10^{5}
10^{0}
0
10
$T_\odot$
20
30
STELLAR TEMPERATURE T_6

The PP I Chain



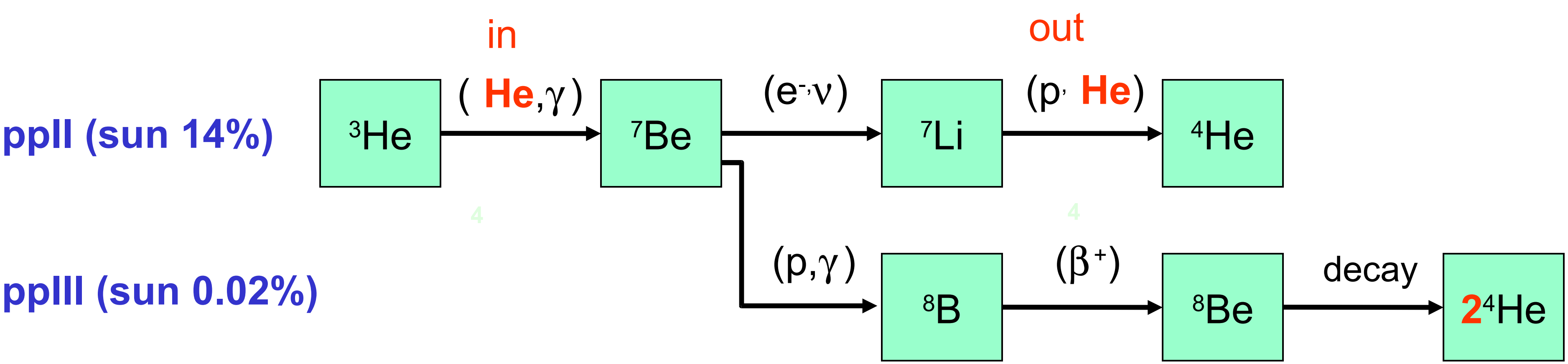


Hydrogen burning with catalysts

1. **ppII chain**
2. **ppIII chain**
3. **CNO cycle**

1. ppII and ppIII:

once ^{4}He has been produced it can serve as catalyst of the ppII and ppIII chains to synthesize more ^{4}He:



The PP Chain(s) in the Sun/He as Catalyst

$$^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_e$$

$$^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{3}_{2}\mathrm{He} + \gamma$$

85%

$$^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \longrightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

(PP I)

15%

$$^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \longrightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

15%

$$^{7}_{4}\mathrm{Be} + e^{-} \longrightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

$$^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP II)

0.02%

$$^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

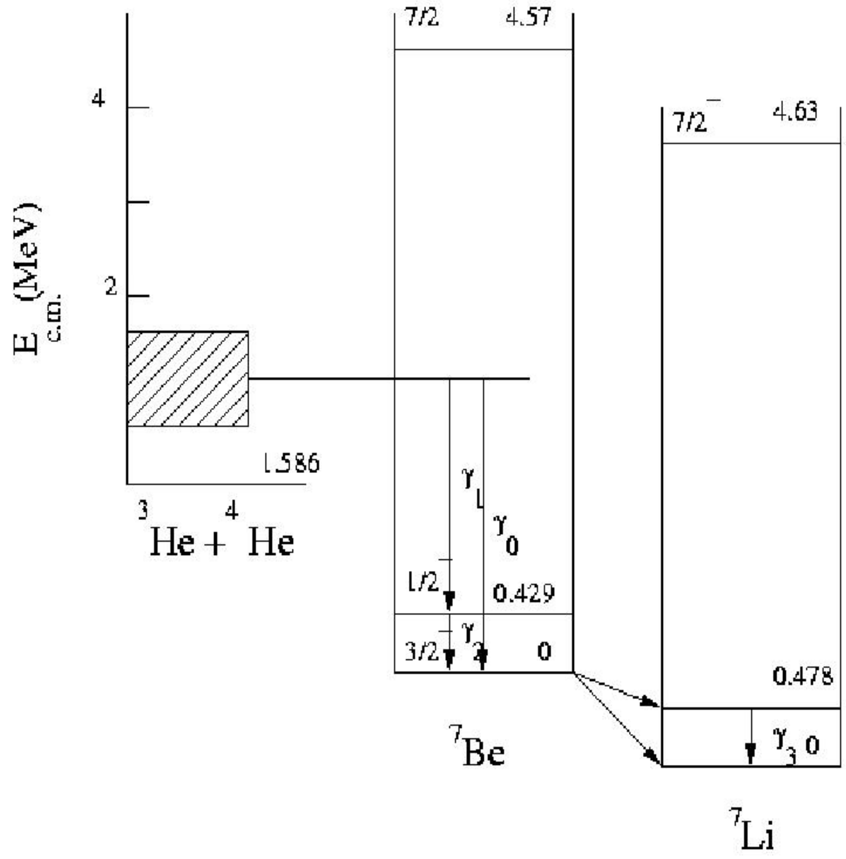
$$^{8}_{5}\mathrm{B} \longrightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

$$^{8}_{4}\mathrm{Be} \longrightarrow 2\,{}^{4}_{2}\mathrm{He}$$

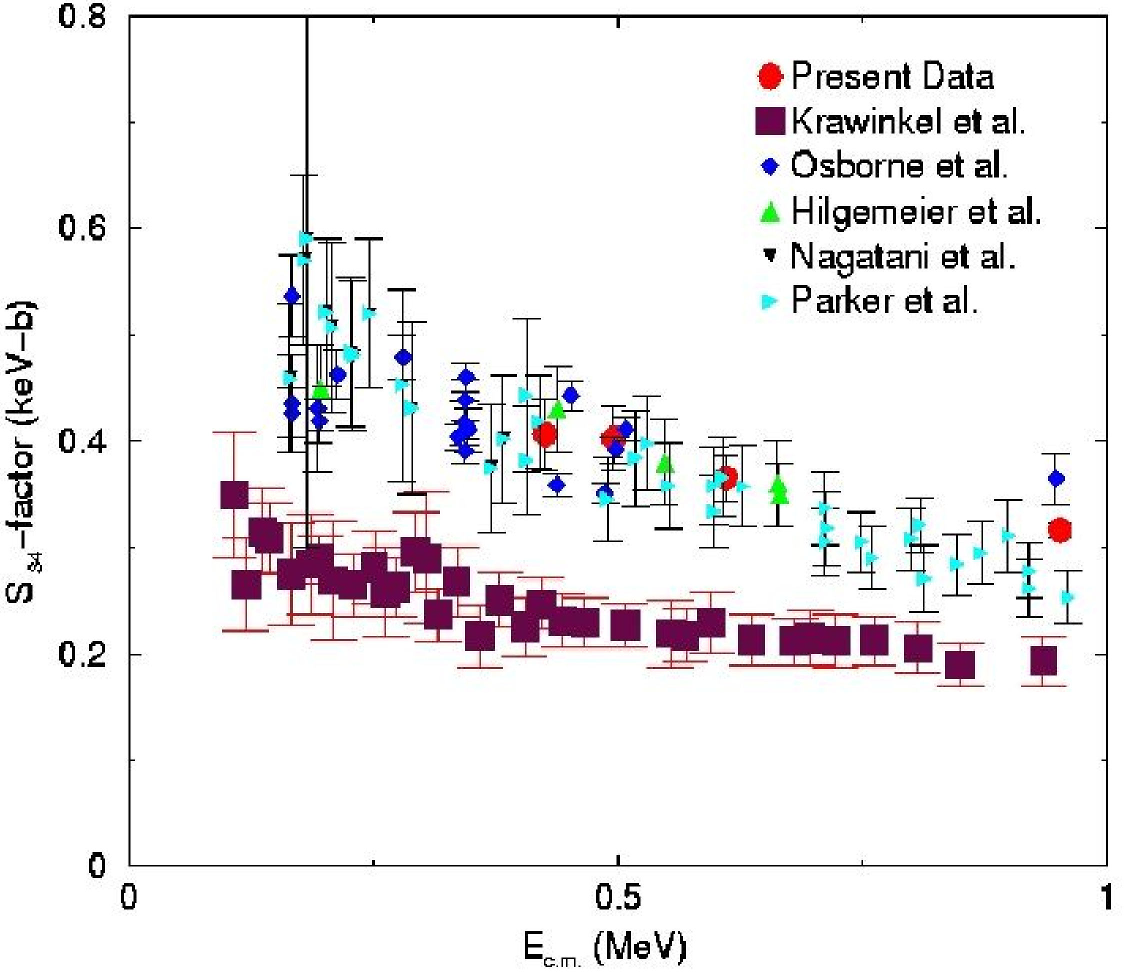
(PP III)

The 3He 4He Fusion Reaction:PP2/3

In 1958 Holmgren and Johnston measured the $^3He(^4He,\gamma)^7Be$ cross section and found it 1000 times larger than expected. Willy Fowler immediately realized the possibility of a break-out from the ppI chain.



The 3He-4He Fusion Data



Electron capture decay of ^{7}Be

Why electron capture:

Q_{EC}=862 keV

$Q_{\beta+}=Q_{EC}-1022 = -160$ keV

only possible decay mode

Earth:

- Capture of bound K-electron

$T_{1/2}$=77 days

Sun:

- Ionized fraction: Capture of continuum electrons

→ depends on density and temperature

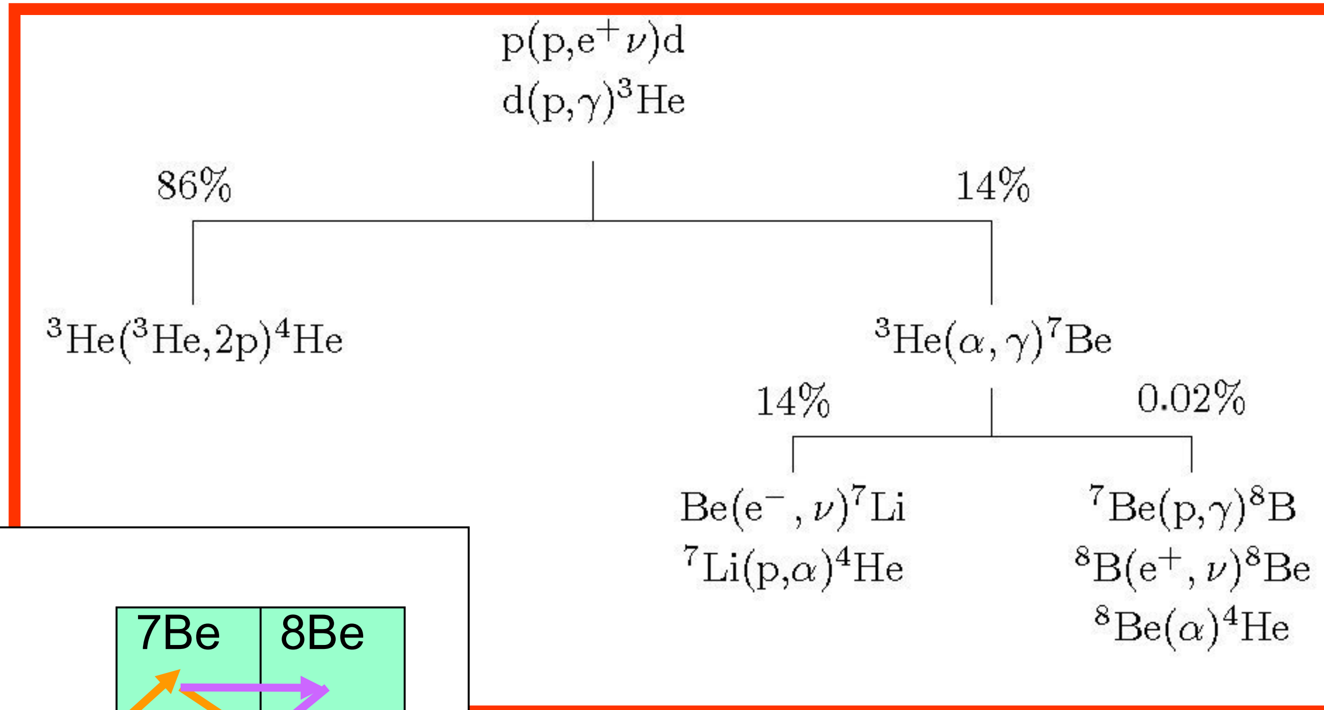
$$\tau_{^7Be} = 4.72e8 \, \frac{T_6^{1/2}}{\rho(1+X_H)} \, s$$

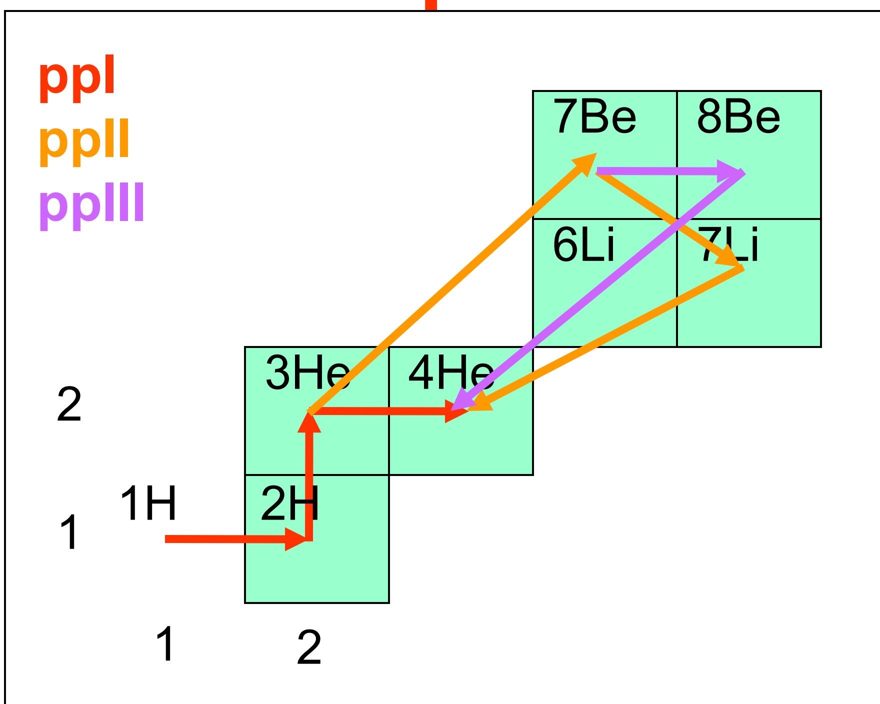
- Not completely ionized fraction: capture of bound K-electron

(21% correction in sun)

} $T_{1/2}$=120 days

Summary pp-chains:





<u>Why do additional pp chains matter ?</u>

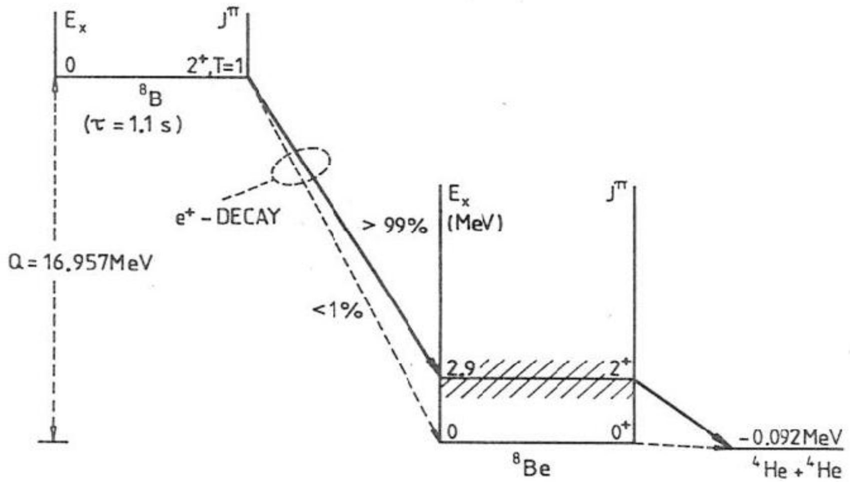
p+p dominates timescale BUT

ppI produces 1/2 ^{4}He per p+p reaction

ppI+II+III produces 1 ^{4}He per p+p reaction

→ **double burning rate**

The Proton Capture on 7Be



$$^{8}B \rightarrow {}^{8}Be^{*} + e^{+} + \nu$$
$$\quad \hookrightarrow {}^{4}He + {}^{4}He$$

- ^{8}B binding energy is only 137 keV

(Rolfs and Rodney)

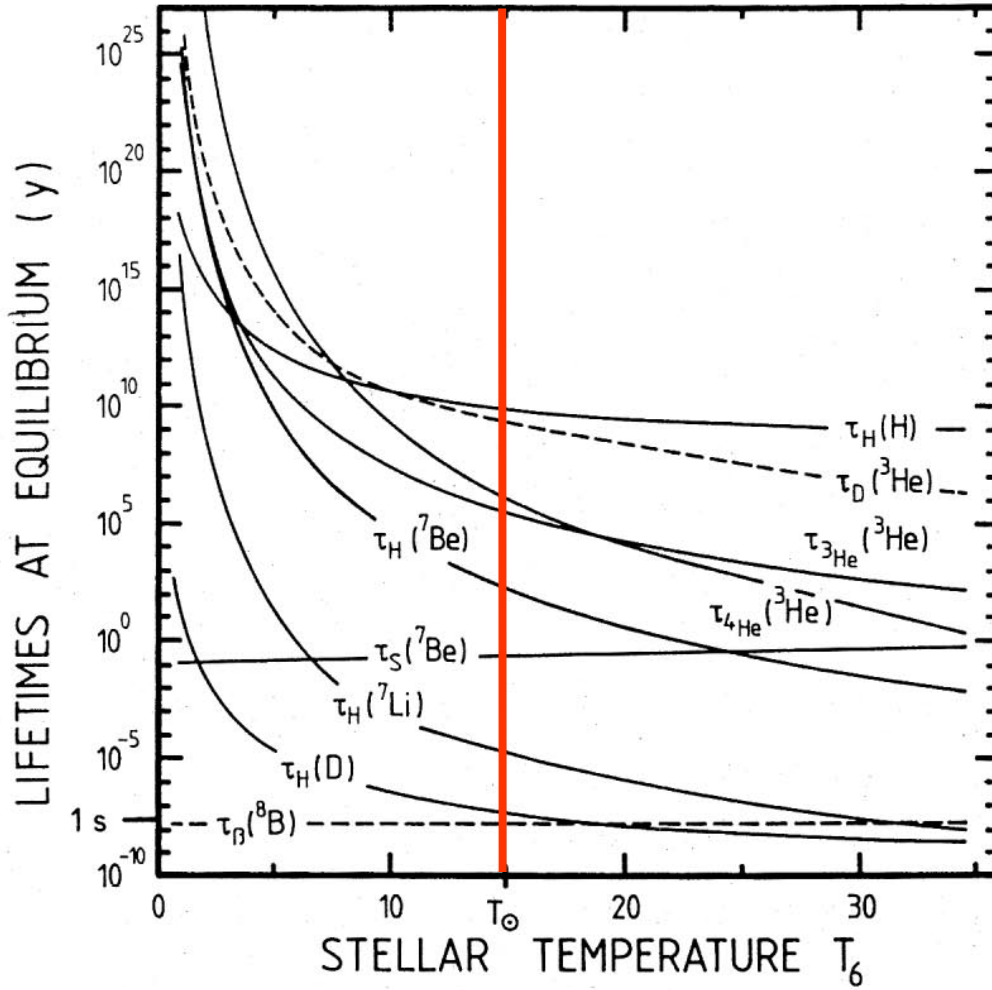
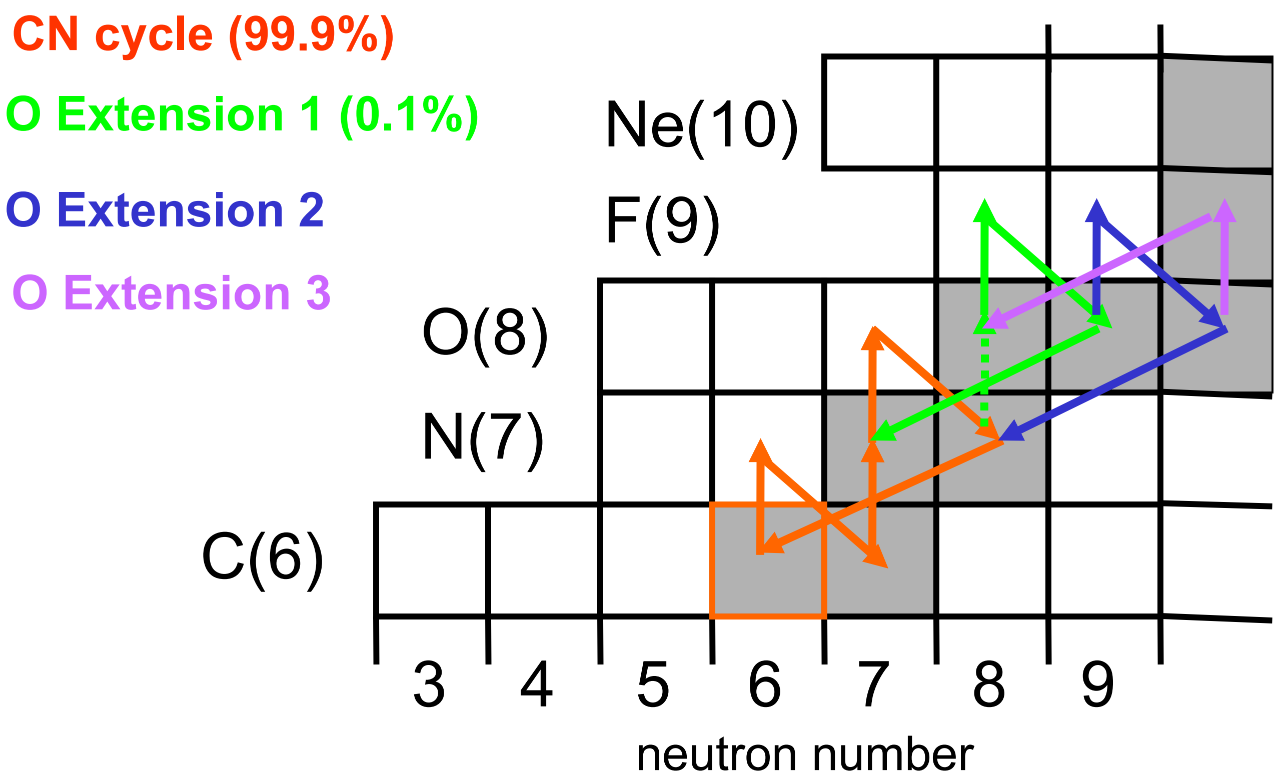


FIGURE 6.7. Plotted are the equilibrium lifetimes of ^{3}He resulting from different burning processes (Table 6.2). The $^3He(\alpha,\gamma)^7Be$ reaction leading to the $\tau_{^4He}(^3He)$–curve is important only in stars which have an appreciable amount of ^{4}He. Shown for comparison is the lifetime of hydrogen against destruction via the $p + p$ reaction and those of D, ^{7}Li, and ^{7}Be against destruction via hydrogen-burning interactions. The electron-capture lifetime of ^{7}Be in stars, $\tau_s(^7Be)$, and the laboratory lifetime of the positron decay for ^{8}B are also shown. All curves assume conditions of $\rho = 100$ g cm^{-3}, $X_H = X_{He} = 0.5$.

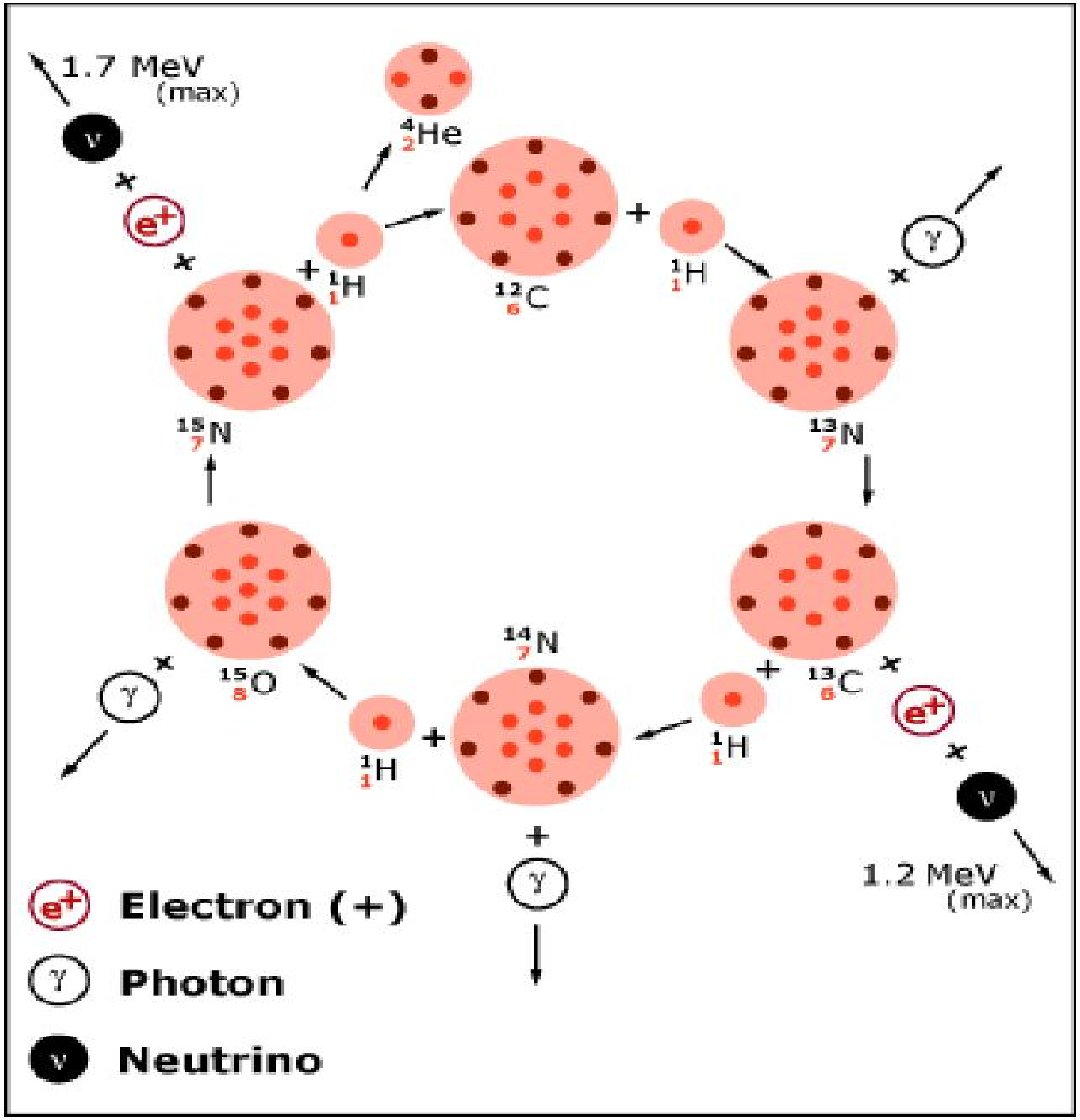
The Alternative H-Burning: CNO cycle. CNO as Catalysator



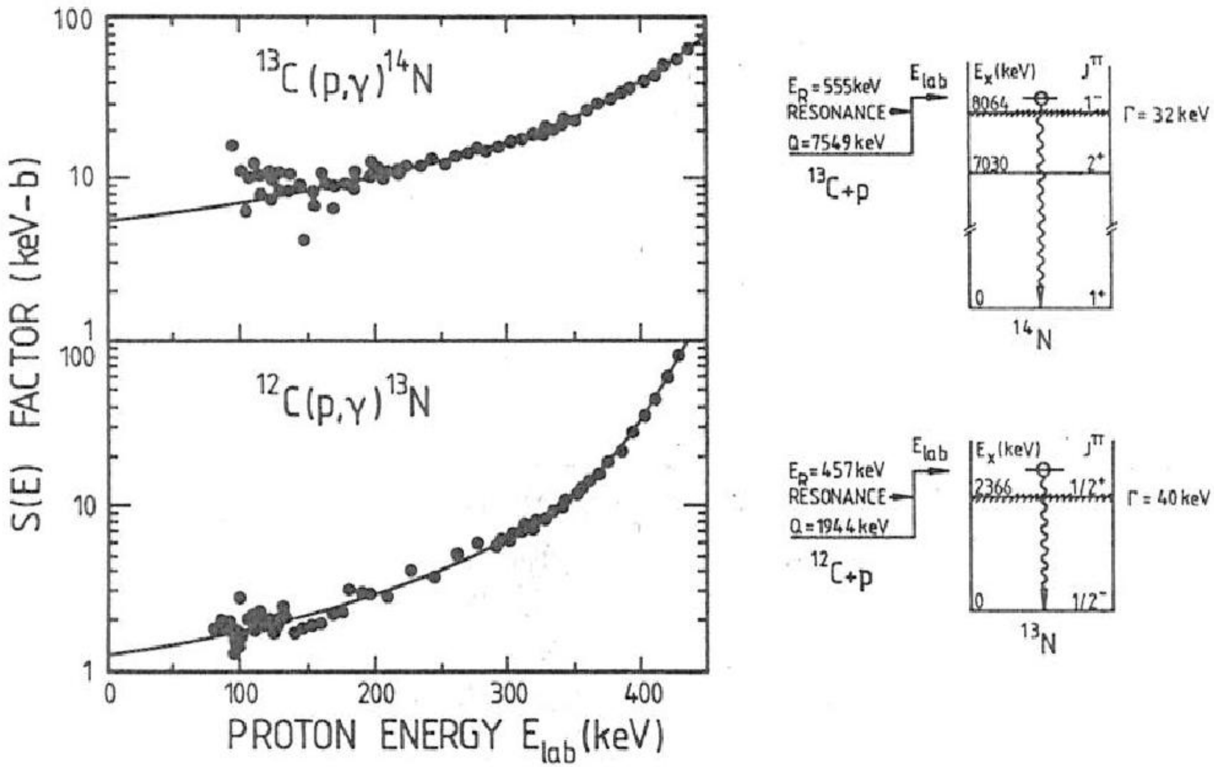
All initial abundances within a cycle serve as catalysts and accumulate at largest τ

Extended cycles introduce outside material into CN cycle (Oxygen, …)

The CNO Cycle:



Proton Capture on Carbon



Dipole transition, dominated by $\frac{1}{2}^+$ resonance.

^{12}C+p: S(0)=1.34±0.21 keV b; ^{13}C+p: S(0)=8.2 keV b.

Beta+ decay of 13N and 15N

β^+ decays:

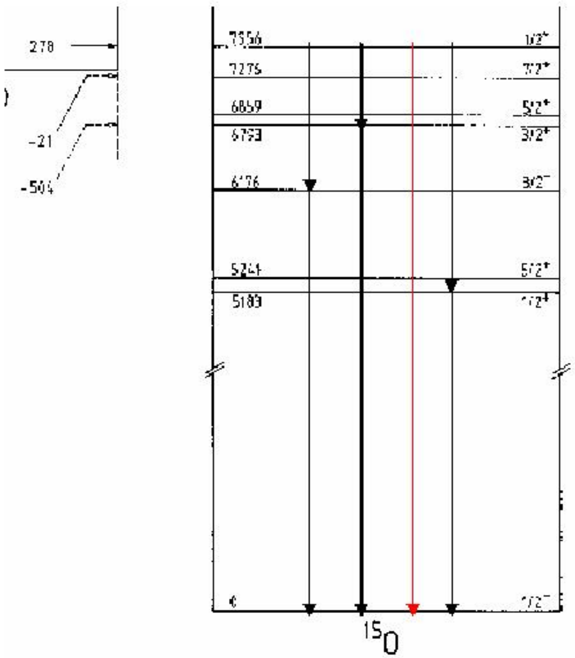
$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$; $^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$

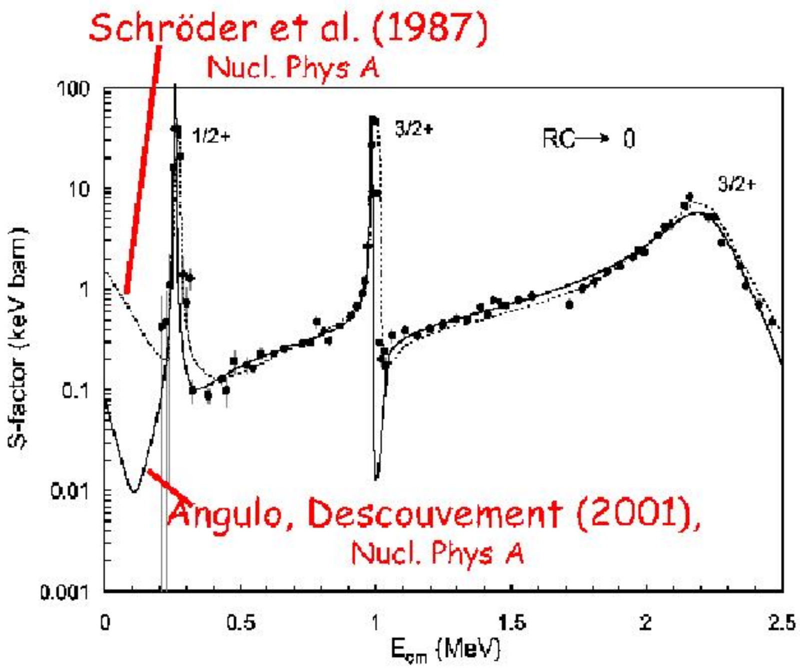
the lifetimes of the decays are experimentally wellknown

the Sun (or stars during hydrogen burning) is not dense enough to alter the laboratory lifetimes

τ (^{13}N) = 863 s; τ (^{15}O) = 176 s

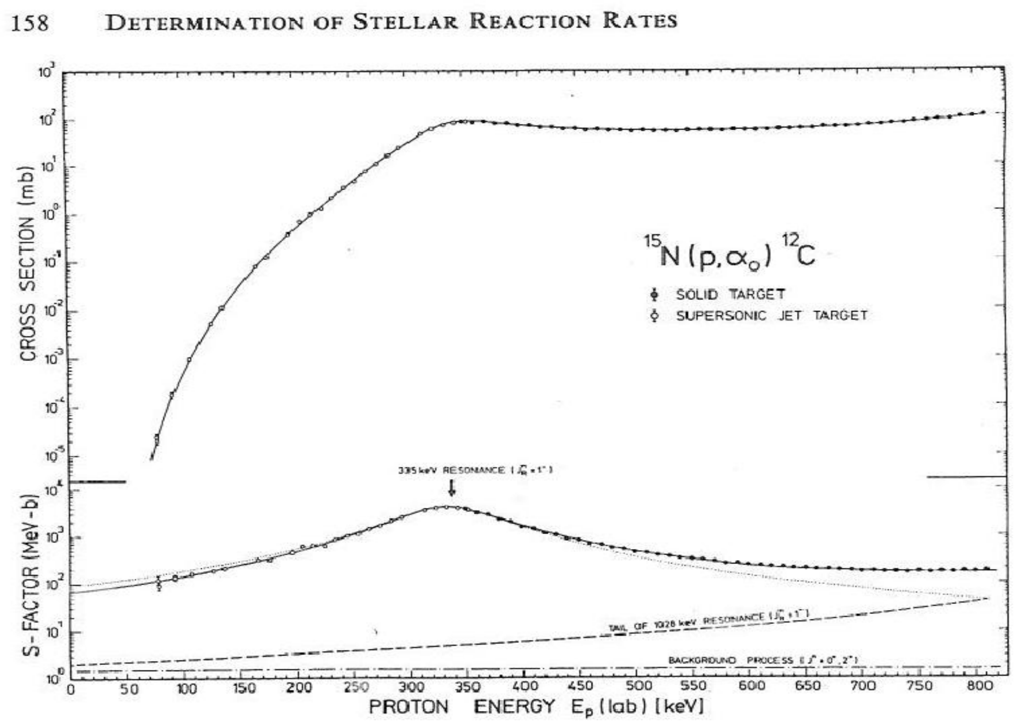
Proton Capture on 14N





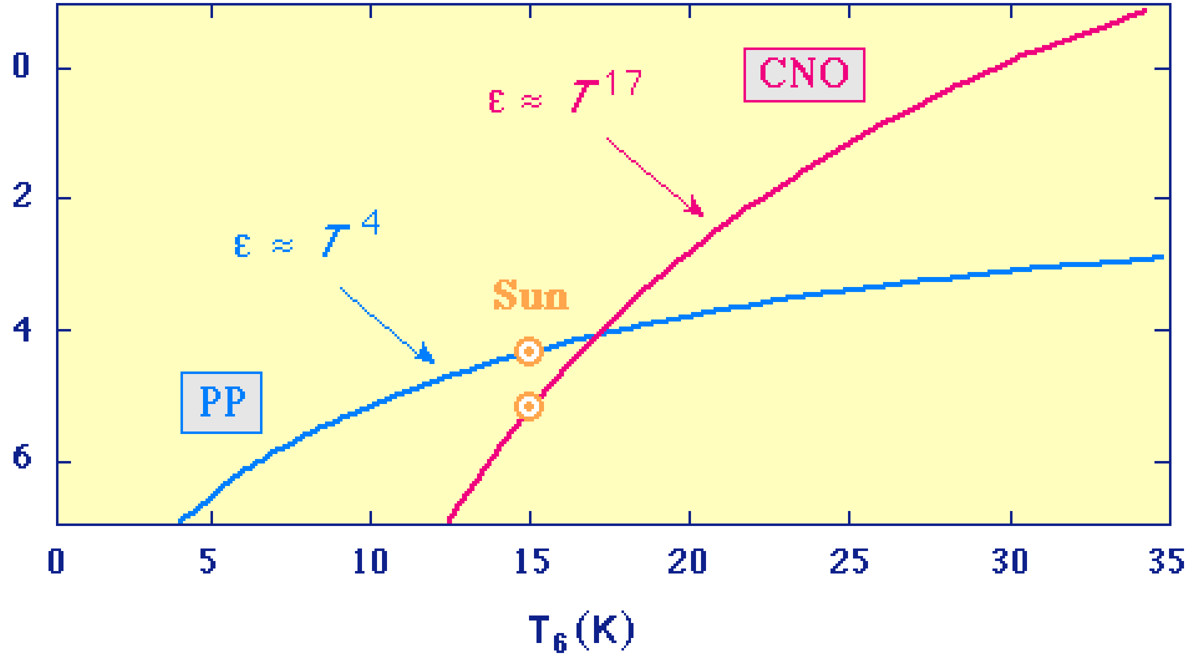
$R/DC \rightarrow 0$ {
- S(0) = 1.55 ± 0.34 keV-b (Schröder)
- S(0) = 0.08 ±0.06 keV-b (Angulo)

Proton Capture on 15N(p,alpha)



S(0)=65±4 MeV b

Competition between the p-p chain and the CNO Cycle



Consequences (see Kippenhahn, 1970)

mass [$M_\odot$]	timescale [y]
0.4	2×10^{11}
0.8	1.4×10^{10}
1.0	1×10^{10}
1.1	9×10^{9}
1.7	2.7×10^{9}
3.0	2.2×10^{8}
5.0	6×10^{7}
9.0	2×10^{7}
16.0	1×10^{7}
25.0	7×10^{6}
40.0	1×10^{6}

- stars with more than 3Mo go CNO

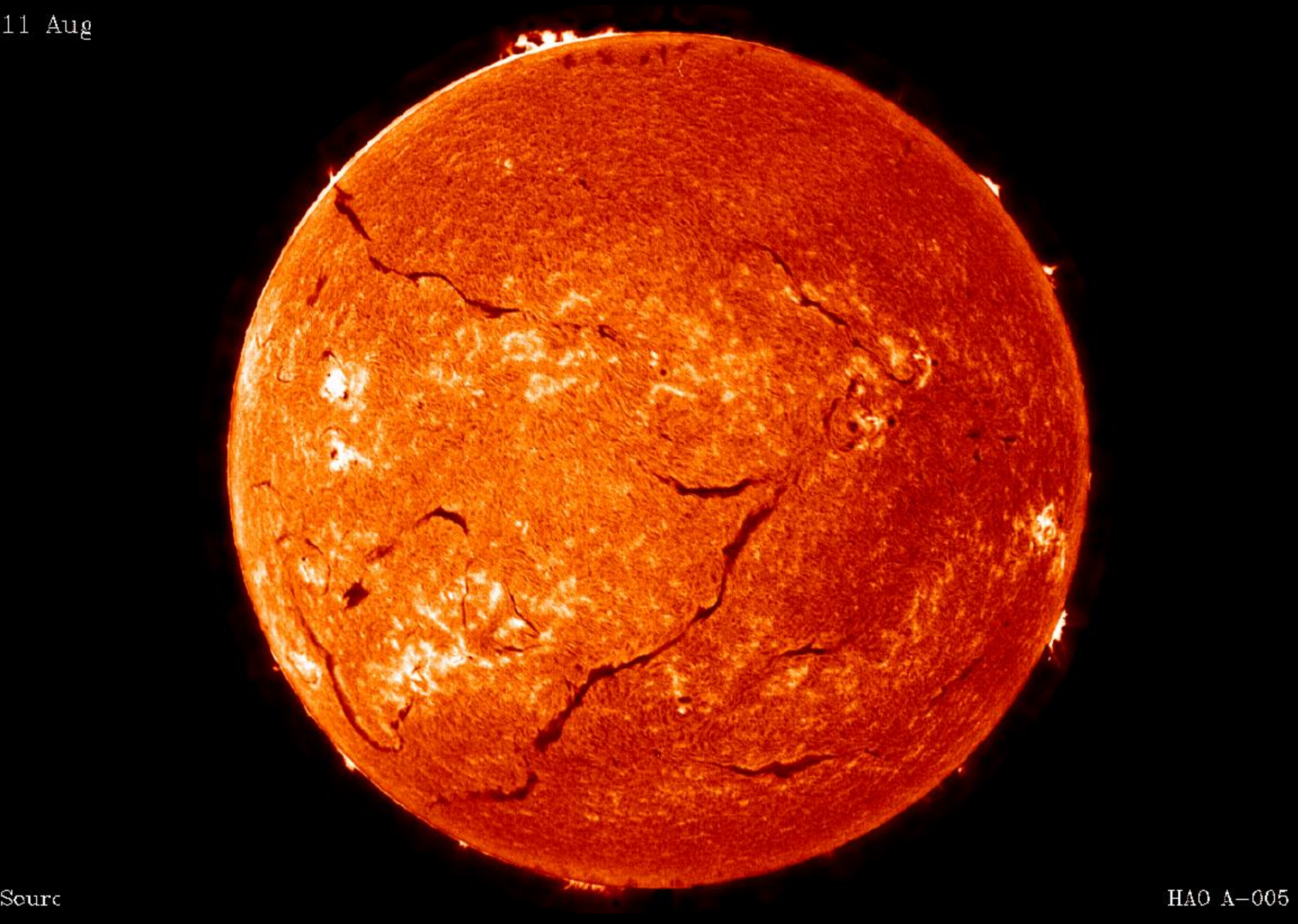
- stars without CNO do pp (early universe)

The Sun & Neutrino Astrophysics

- Properties of the solar Neutrinos
- The Solar Neutrino Problem
- Properties of Neutrinos

Literature: Iliadis, Chapter 5

The Surface the Sun



Basic Properties of the Sun

Parameter	Value
Photon luminosity ($L_\odot$)	3.86×10^{33} erg s^{-1}
Neutrino luminosity	$0.023 L_\odot$
Mass ($M_\odot$)	1.99×10^{33} g
Radius ($R_\odot$)	6.96×10^{10} cm
Oblateness $[(R_{\text{equatorial}}/R_{\text{polar}}) - 1)]$	$\leq 2 \times 10^{-5}$
Effective (surface) temperature	5.78×10^{3} K
Moment of inertia	7.00×10^{53} g cm^2
Age	$\approx 4.55 \times 10^{9}$ yr
Initial helium abundance by mass	0.27
Initial heavy element abundance by mass	0.020
Depth of convective zone	$0.26 R_\odot (0.015 M_\odot)$
Central density	148 g cm^{-3}
Central temperature	15.6×10^{6} K
Central hydrogen abundance by mass	0.34
Neutrino flux from pp reaction	6.0×10^{10} cm^{-2} s^{-1}
Neutrino flux from ^{8}B decay	6×10^{6} cm^{-2} s^{-1}
Fraction of energy from pp chain	0.984
Fraction of energy from CNO cycle	0.016

Solar Neutrinos from PP

$$^1_1\text{H} + {}^1_1\text{H} \longrightarrow {}^2_1\text{H} + e^+ + \nu_e$$ **0.27MeV**

$$^2_1\text{H} + {}^1_1\text{H} \longrightarrow {}^3_2\text{He} + \gamma$$

85% | 15%

$$^3_2\text{He} + {}^3_2\text{He} \longrightarrow {}^4_2\text{He} + 2{}^1_1\text{H}$$

(PP I)

$$^3_2\text{He} + {}^4_2\text{He} \longrightarrow {}^7_4\text{Be} + \gamma$$

15% | 0.02%

$$^7_4\text{Be} + e^- \longrightarrow {}^7_3\text{Li} + \nu_e$$ **0.39,0.86MeV**

$$^7_3\text{Li} + {}^1_1\text{H} \longrightarrow 2{}^4_2\text{He}$$

(PP II)

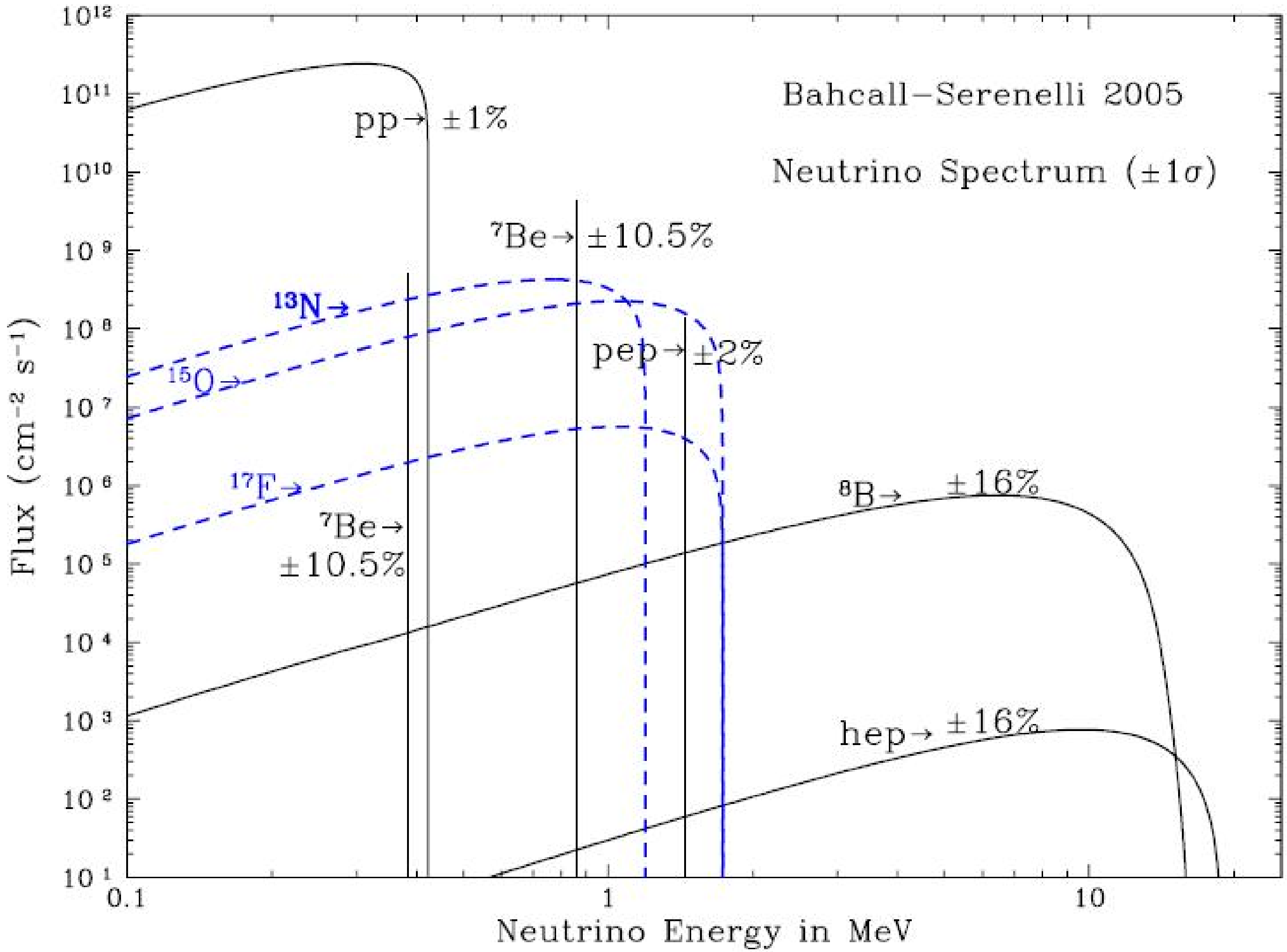
$$^7_4\text{Be} + {}^1_1\text{H} \longrightarrow {}^8_5\text{B} + \gamma$$

$$^8_5\text{B} \longrightarrow {}^8_4\text{Be} + e^+ + \nu_e$$ **6.84Mev**

$$^8_4\text{Be} \longrightarrow 2{}^4_2\text{He}$$

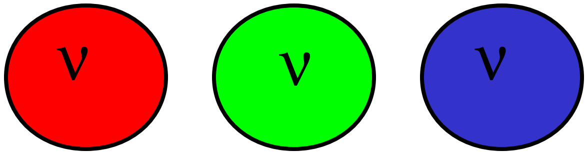
(PP III)

Solar Neutrino Production in the Standard Model

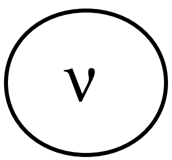


Neutrino Properties: Flavors and Masses

Flavor states:

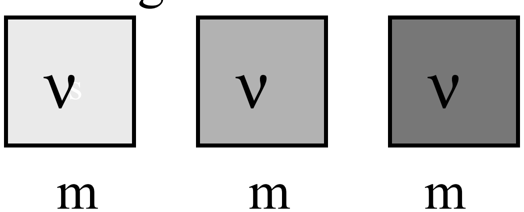


Eigenstates of the CC weak interactions

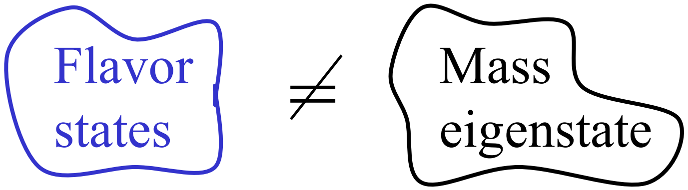


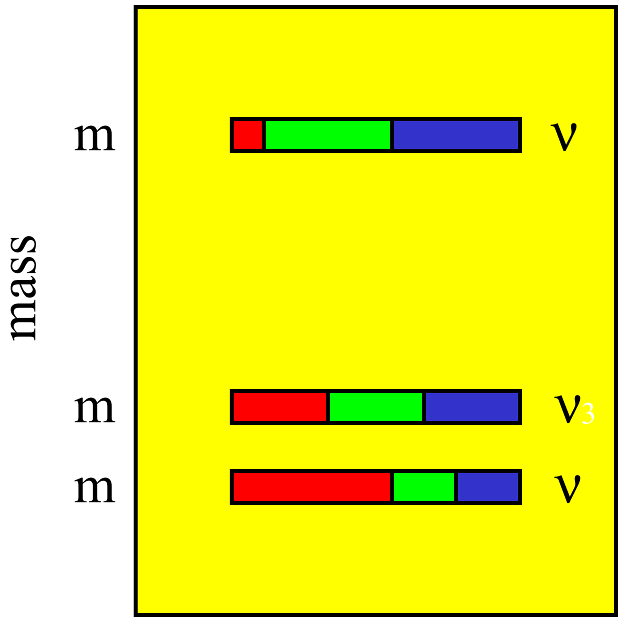
Sterile neutrinos
no weak interactions ?

Mass eigenstates:



Mixing:





Neutrino mass and flavor spectrum

Mixing and oscillations

$\nu = \sin\theta \; \nu \; + \; \cos\theta \; \nu$

$\nu = \cos\theta \; \nu \; - \; \sin\theta \; \nu$

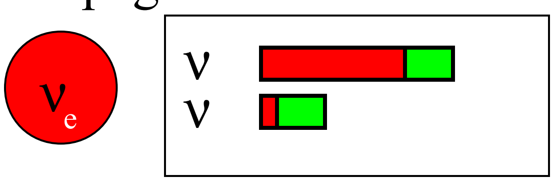
θ is the vacuum mixing angle

$\nu = \cos\theta \; \nu \; + \; \sin\theta \; \nu$

coherent mixture of mass eigenstates

:

Propagation:



wave packets

$$\Delta v = \frac{\Delta m^2}{2E}$$

$\Delta m = m \;-\; m$

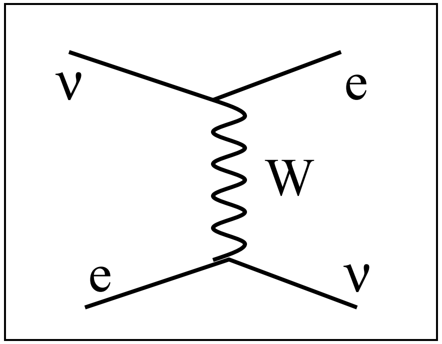
$\Delta\phi = \Delta v \;\; t$

Interference of the parts of wave packets with the same flavor depends on the phase difference Δϕ between ν and ν

Oscillations: effects of the phase difference increase which changes the interference pattern

Matter effect

for ν ν :



Elastic forward scattering ⇒ Potentials V , V

Difference of potentials is important ➡

$$V - V = \sqrt{2}\, G\, n$$

Matter changes

- mixing angle — $\rightarrow \theta \ (n , E)$ (mixing angle in matter)
- eigenstates — $\nu \ , \nu \rightarrow \nu \ , \nu \ (n , E)$
- effective masses — $m , \ m_{m \rightarrow e} \ m \ , \ m \ (n , E)$

- modifies oscillations in the case of uniform medium
- leads to qualitatively new effects in media with varying densities

MSW conversion

Resonance condition:

$$V(n) = \cos 2\theta \frac{\Delta m^2}{2E}$$

Matter frequency

Eigenfrequency of neutrino system

Density, n_R, (energy, E_R) which satisfies the resonance condition

is called the resonance density (energy)

If density changes slowly enough (typical scale of density change is larger than the oscillation length) the adiabaticity condition is fulfilled

Adiabatic propagation

Flavor of neutrino state follows density change:

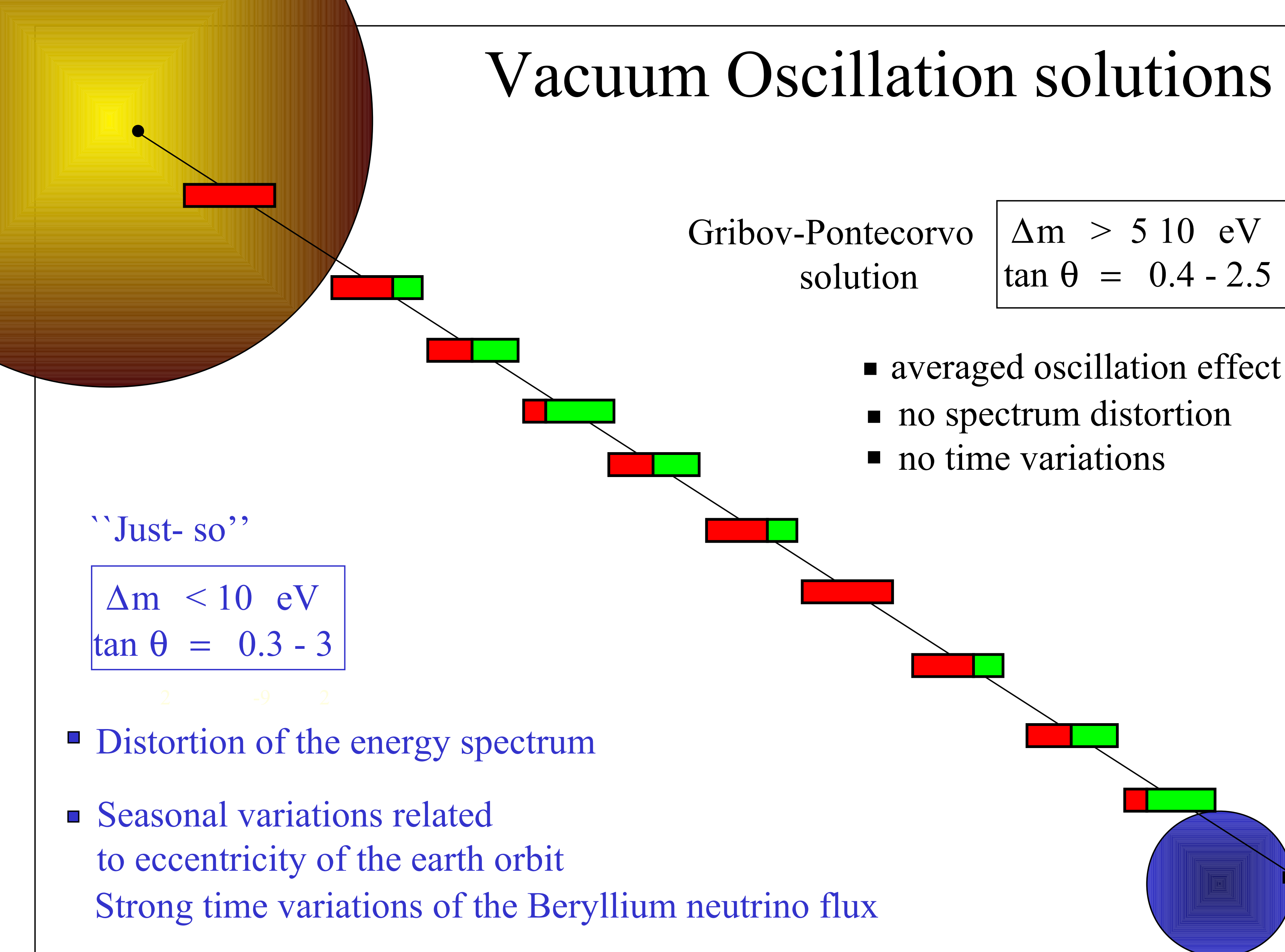
Flavor = F (density)

If density, e.g., decreases from $n >> n$ down to $n << n$

Strong transformation of flavor:

ν ⇒ ν

Vacuum Oscillation solutions



Gribov-Pontecorvo solution

Δm > 5 10 eV
$\tan\theta$ = 0.4 - 2.5

- averaged oscillation effect
- no spectrum distortion
- no time variations

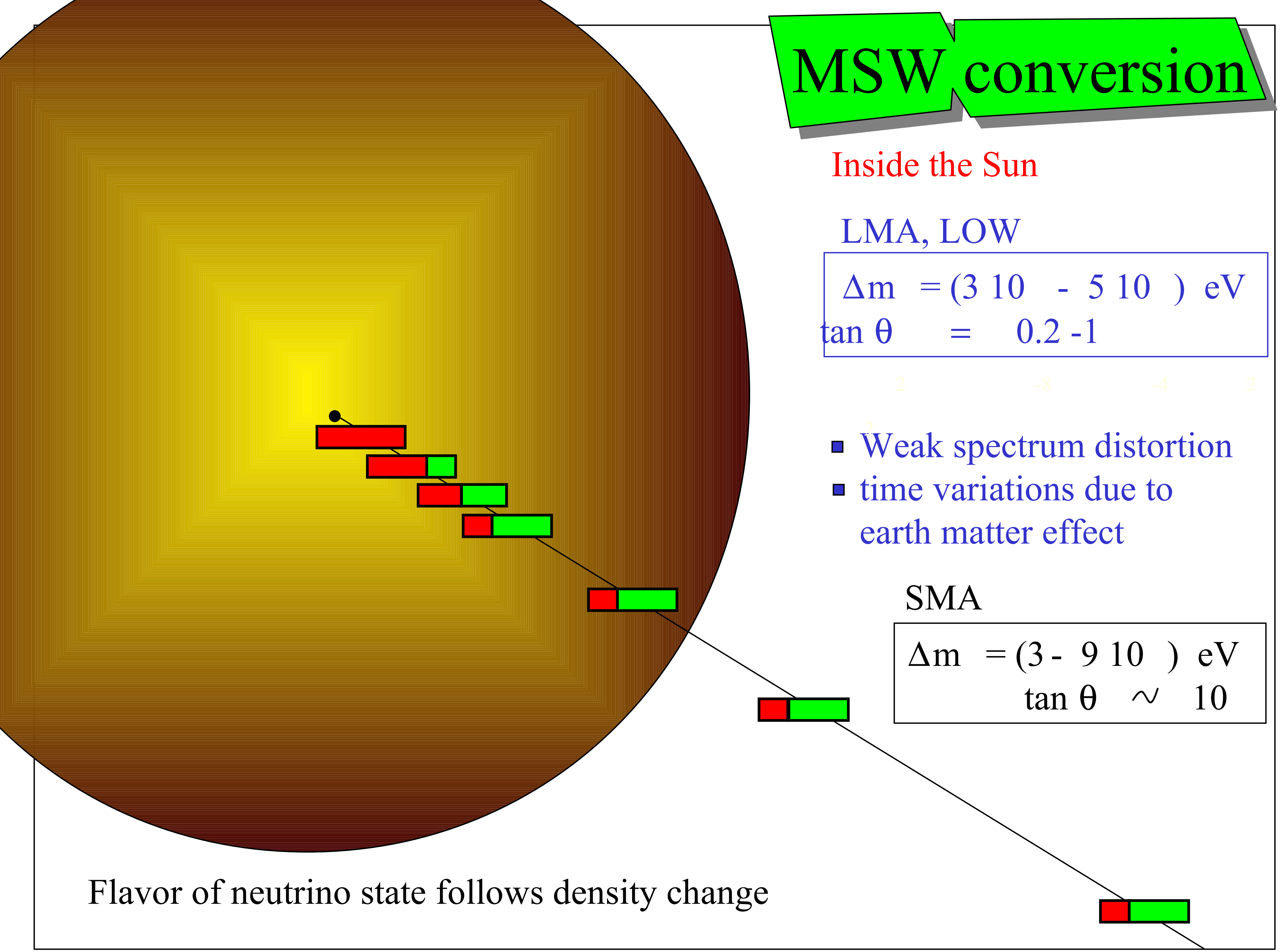
``Just- so''

Δm < 10 eV
$\tan\theta$ = 0.3 - 3

- Distortion of the energy spectrum
- Seasonal variations related to eccentricity of the earth orbit

Strong time variations of the Beryllium neutrino flux

MSW conversion



Inside the Sun

LMA, LOW

$\Delta m^2 = (3\,10^{-8} - 5\,10^{-4})\ eV^2$

$\tan^2\theta = 0.2 - 1$

- Weak spectrum distortion
- time variations due to earth matter effect

SMA

$\Delta m = (3 - 9\,10)\ eV$

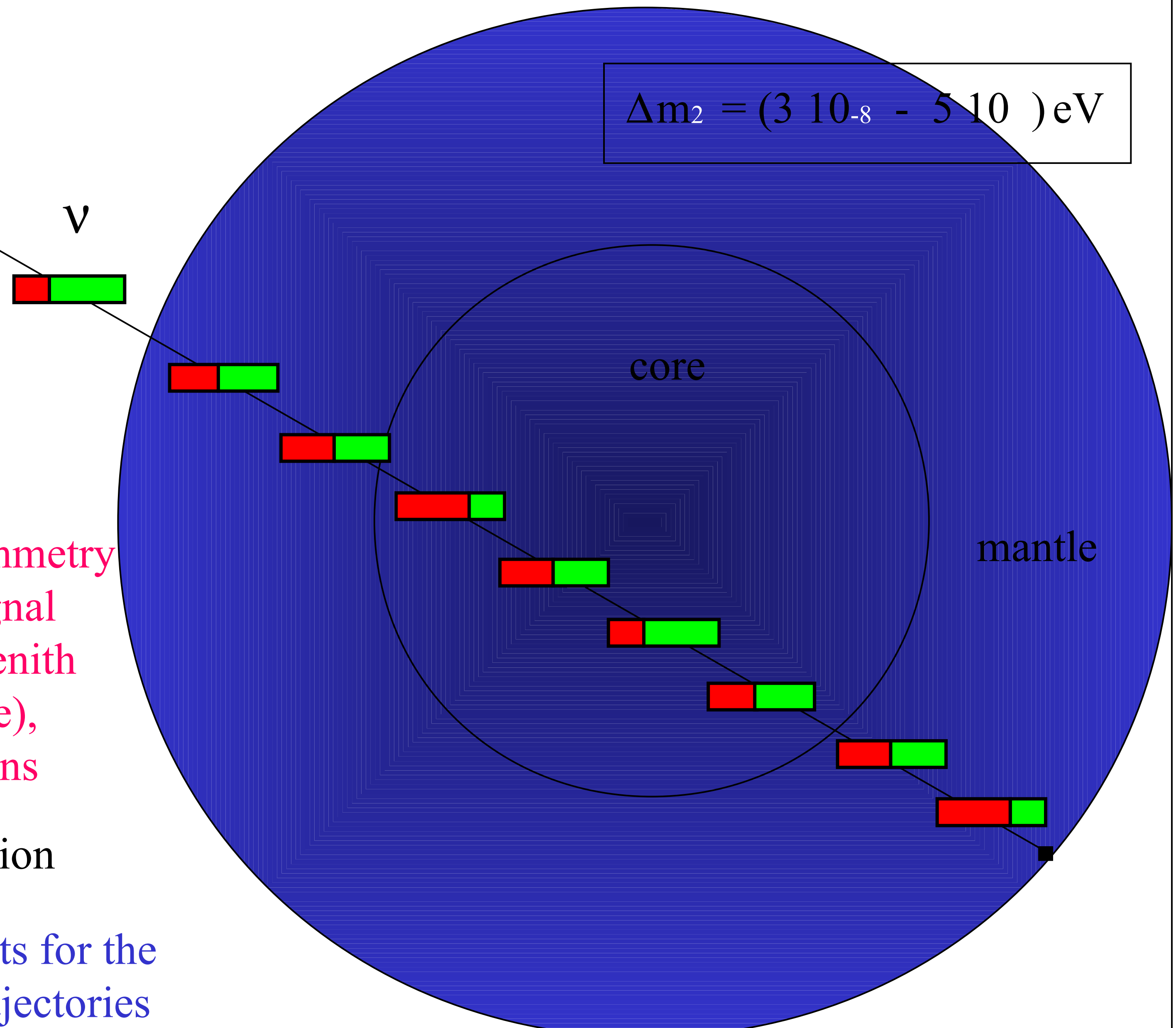
$\tan\theta \sim 10$

Flavor of neutrino state follows density change

Inside the Earth. Regeneration

Oscillations in the matter of the Earth

Regeneration of the ν flux



- Day - Night asymmetry
- Variations of signal during nights (zenith angle dependence),
- Seasonal variations
- Spectrum distortion
- Parametric effects for the core crossing trajectories

Homestake experiment

R. Davis Jr. et al
Establishing the problem

$\nu + {}_{37}Cl \rightarrow e + {}_{37}Ar$

E = 0.814 MeV
Sensitive to ν only

Deficit of the Argon production rate:

$R = 0.298 \pm 0.049$

Solutions of the problem

- Astrophysics
- Nuclear physics
- Neutrino physics

For about 20 years the only experimental result
Practically all solutions have been suggested during this period

Time variations of the signal: 11 years/ 2 years/1 month?
Favored solution: Spin flip in the magnetic field

Kamiokande

Water Cerenkov detector

1987 - 1995

$\nu + e \;{-}{-}{-}{>}\; \nu + e$

$a = e, \mu, \tau$

$E = 7.5$ MeV

sensitive to all active neutrinos
detects the Boron and hep- neutrino fluxes

Deficit of the boron neutrino flux

$$R = \frac{F}{\quad} = 0.47 \pm 0.06$$

No time variations

Homestake vs. Kamiokande

Barabanov
Bahcall
Bethe

F

F ⇨ Q ⇨

$Q_B > Q_{Homestake}$

Distortion of the energy spectrum Beryllium neutrino flux should be strongly suppressed

or/and

The fluxes of ν , ν from the Sun exist which contribute to Kamiokande but not to Homestake

- Hint: astrophysics and nuclear physics solutions do not work
- SMA MSW -- favorite solution

SAGE, GALLEX, GNO

1990, 1991, 1998

$\nu_e + {}^{71}Ga \rightarrow \nu + {}^{71}Ge$ $E = 0.233$ MeV

sensitive to all components of the solar neutrino spectrum

Deficit of signal

time variations ?

$R = \frac{Q}{Q_e} = 0.581 \pm 0.055$

``Just at the edge''

Contribution from the pp neutrino flux (reliably predicted)

$Q = 70$ SNU

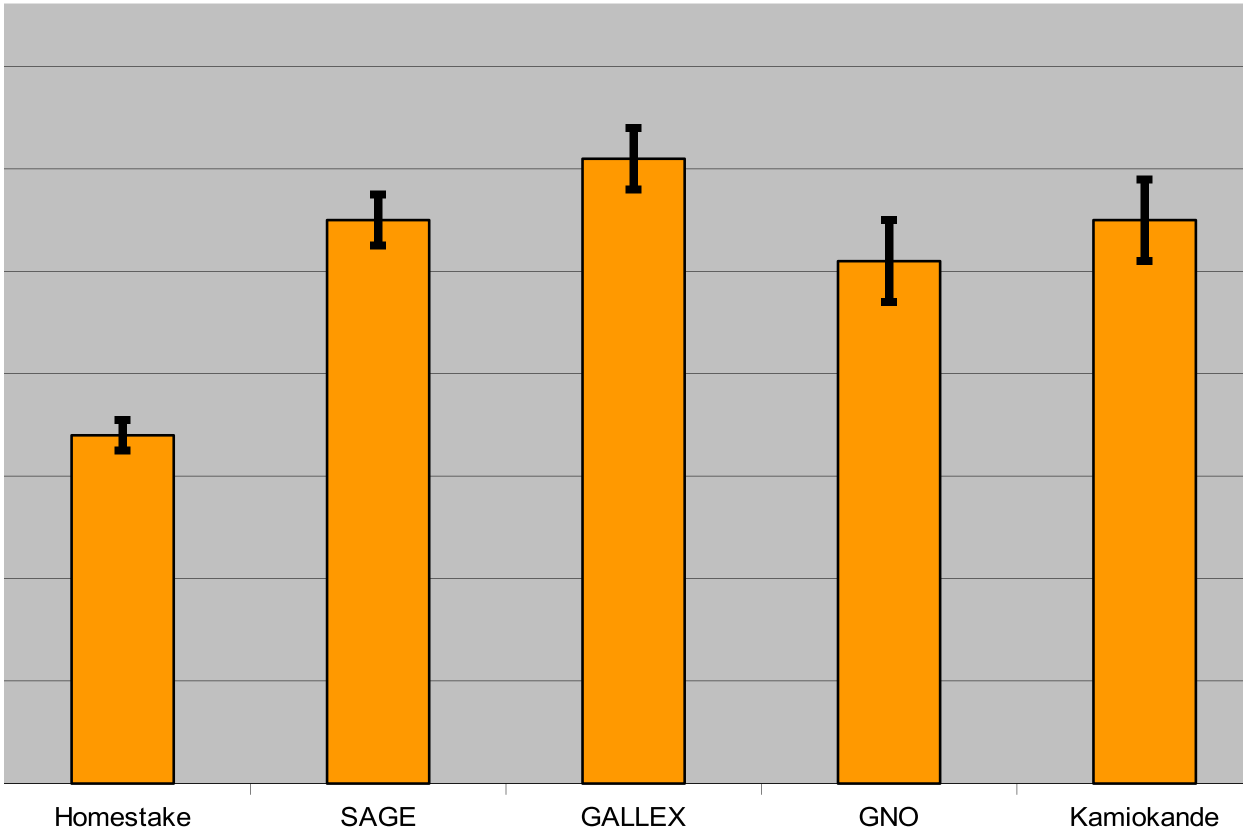
$Q < 70$ SNU would exclude astrophysical solutions

Observations:

$Q = (75 + 5)$ SNU

- Confirm deficit and inferences from Homastake-Kamiokande comparison
- Strong suppression of the Beryllium neutrino flux or/and the pp-neutrino flux
- SMA MSW -- favorite solution

many more experiments over the years with very different energy thresholds:

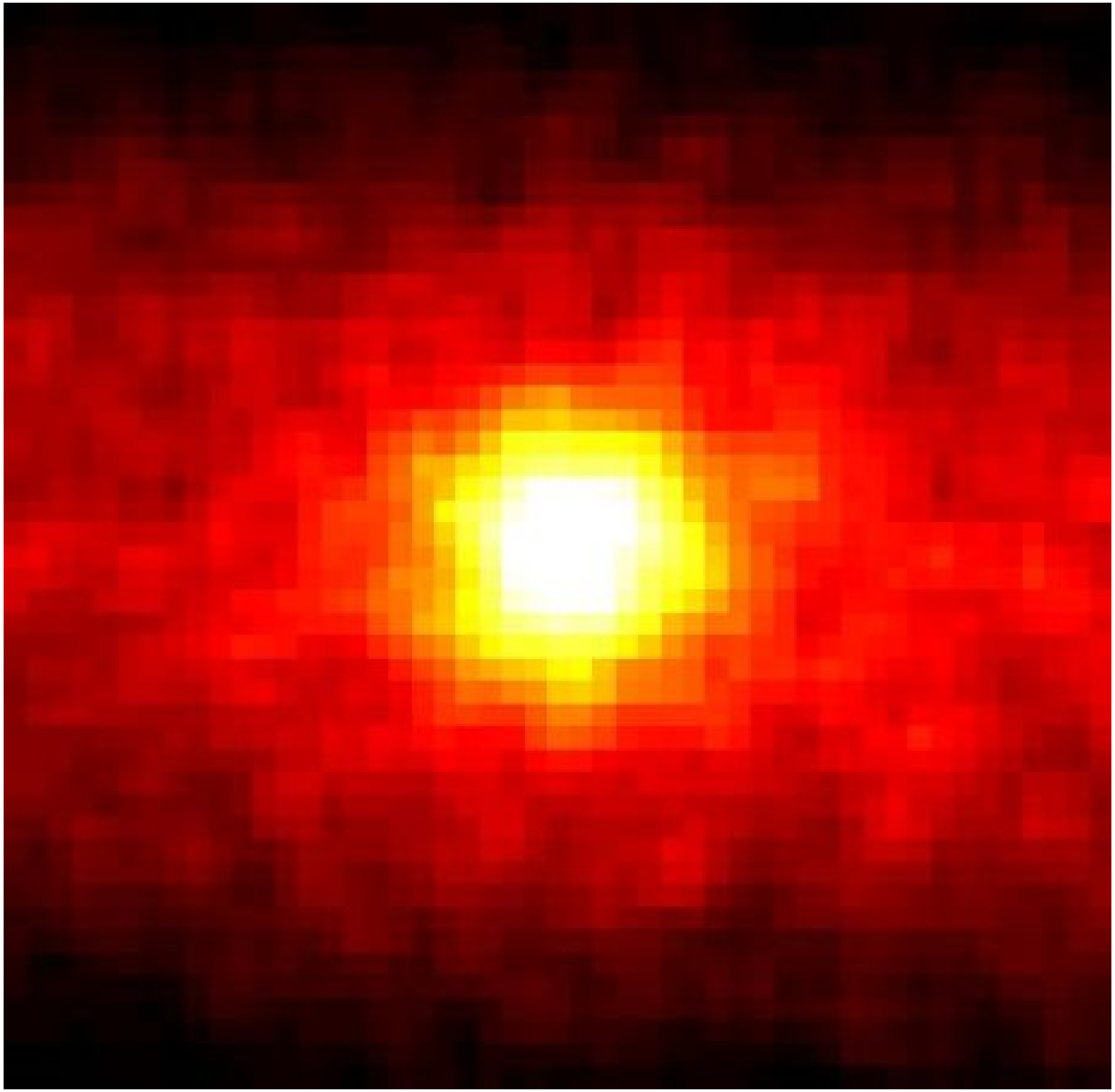


all show deficit to standard solar model

ν_e only

all flavors, but ν_τ,ν_μ only 16% of ν_e cross section because no CC, only NC

Astronomy Picture of the Day June 5, 1998



Neutrino image of the sun by Super-Kamiokande – next step in neutrino astronomy

SuperKamiokande

Water Cerenkov detector

1996

$\nu + e \text{ ---> } \nu + e$

$a = e, \mu, \tau$
$E = 5$ MeV

- Deficit

$$R_a = \frac{F}{\quad} = 0.391 \pm 0.060$$

- No spectrum distortion.
  Excess of events in the high energy bins?

F

- No time variations of the flux apart from seasonal variations related to the eccentricity of the Earth orbit

Signatures disappear

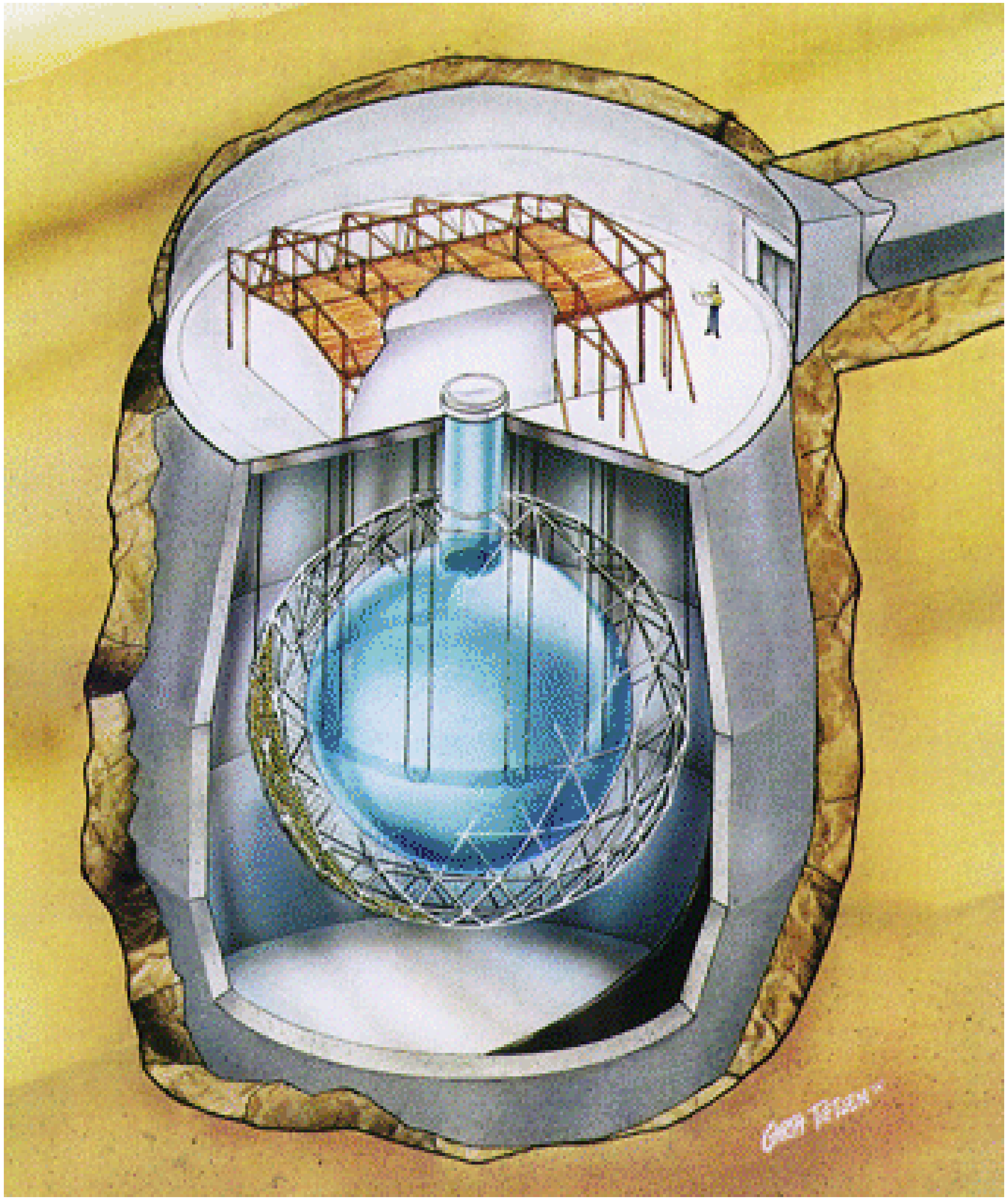
Day- Night asymmetry: $2.5\,\sigma \longrightarrow 1\,\sigma$

Zenith angle distribution of events: no enhancement in the deepest night bin expected for SMA solution, flat distribution...

- Change of favorites: SMA MSW is disfavored by SK data

SMA MSW → Vacuum oscillations → LMA MSW

Sudbury Neutrino Observatory



SNO

1000 tons of heavy water

1999

Sudbury Neutrino Observatory

CC $\nu + d \rightarrow e^- + p + p$

E = 6.75 MeV

ES $\nu + e \rightarrow \nu + e$

NC $\nu + d \rightarrow + e + n + \nu$

- Deficit: $R = \frac{F}{} = 0.295 \pm 0.051$

First strong evidence of solar neutrino flavor conversion

- SNO vs. SuperKamiokande

$F = 1.75 \pm 0.15$

ν contribute only

CC,SNO

F

$F = 2.32 \pm 0.085$

all active neutrinos contribute

BES,SK

units

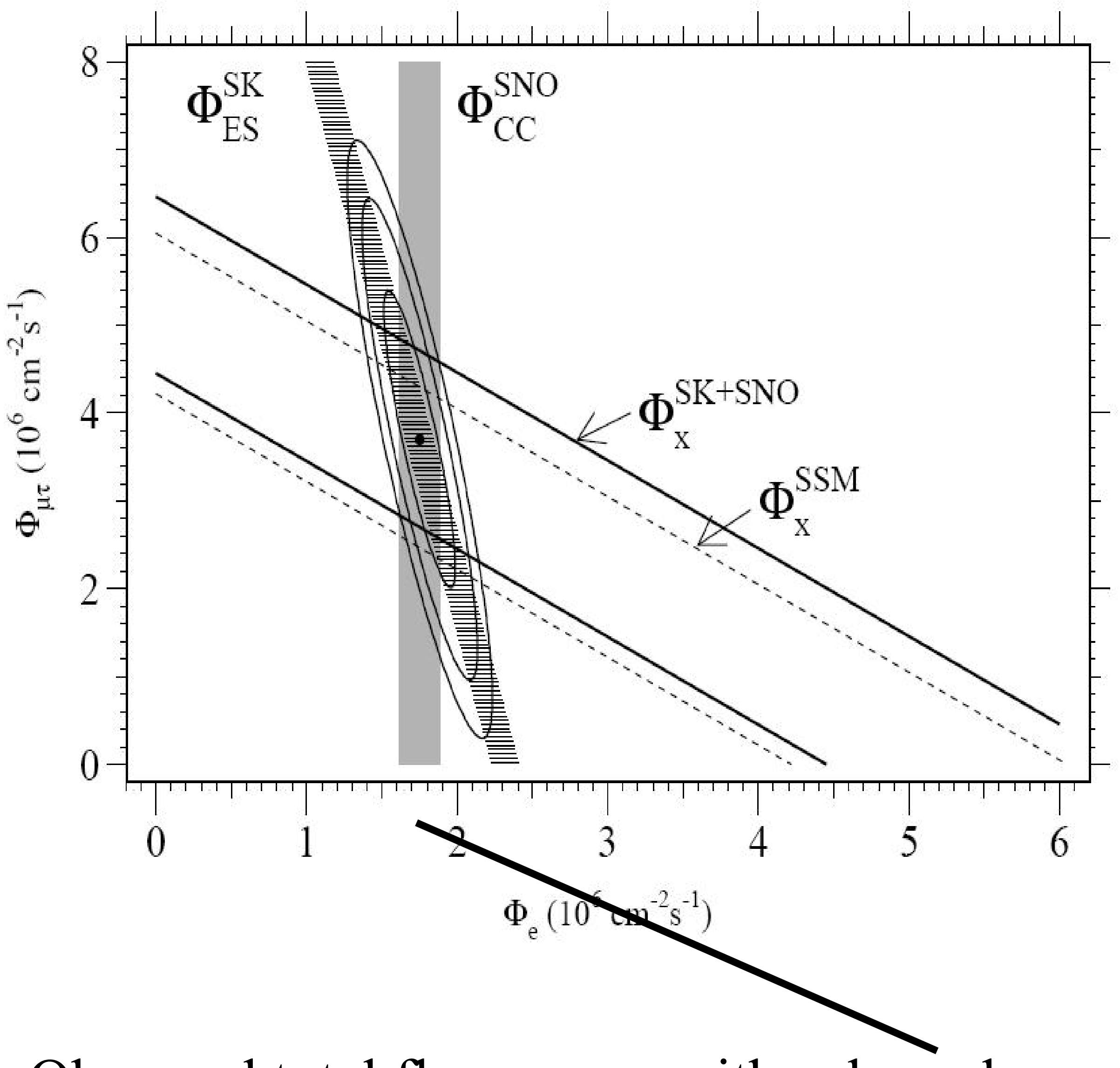
10 cm s

$F_{ES,SK} - F_{CC,SNO} = 3\sigma$

- Imply appearance of ν /ν flux from the Sun (which contributes to SK)
- LMA is further favored, SMA -- disfavored
- ``sterile'' solutions are also disfavored

With SNO results:

SNO proof of Neutrino Oscillations

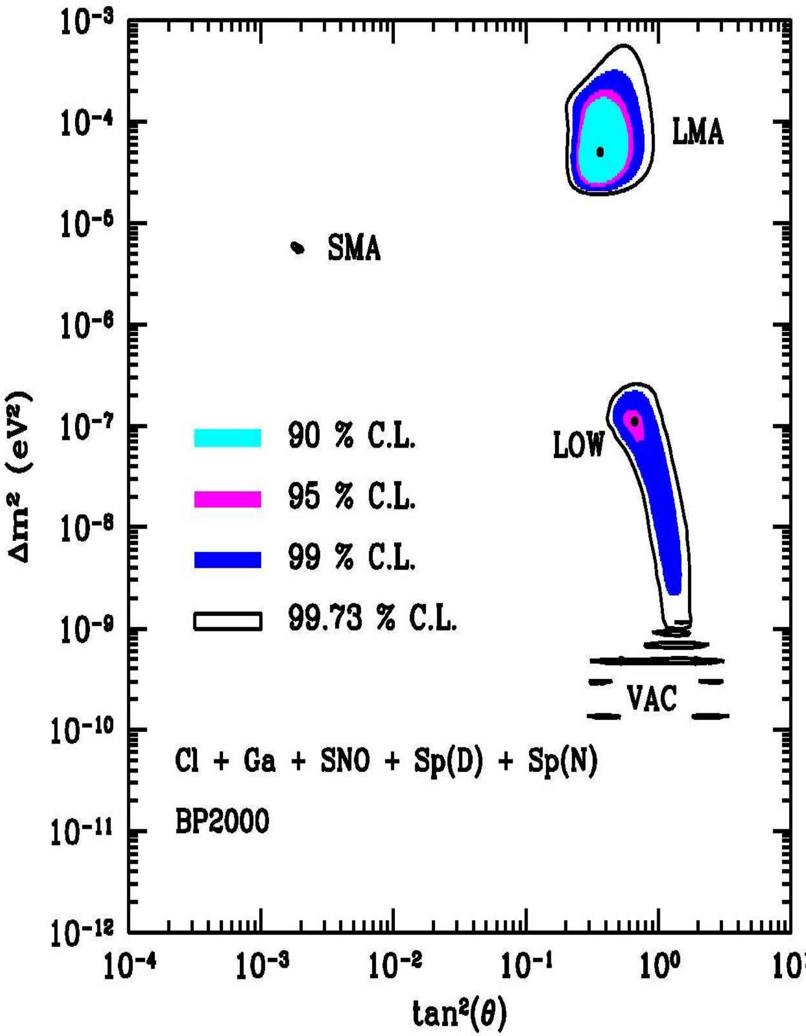


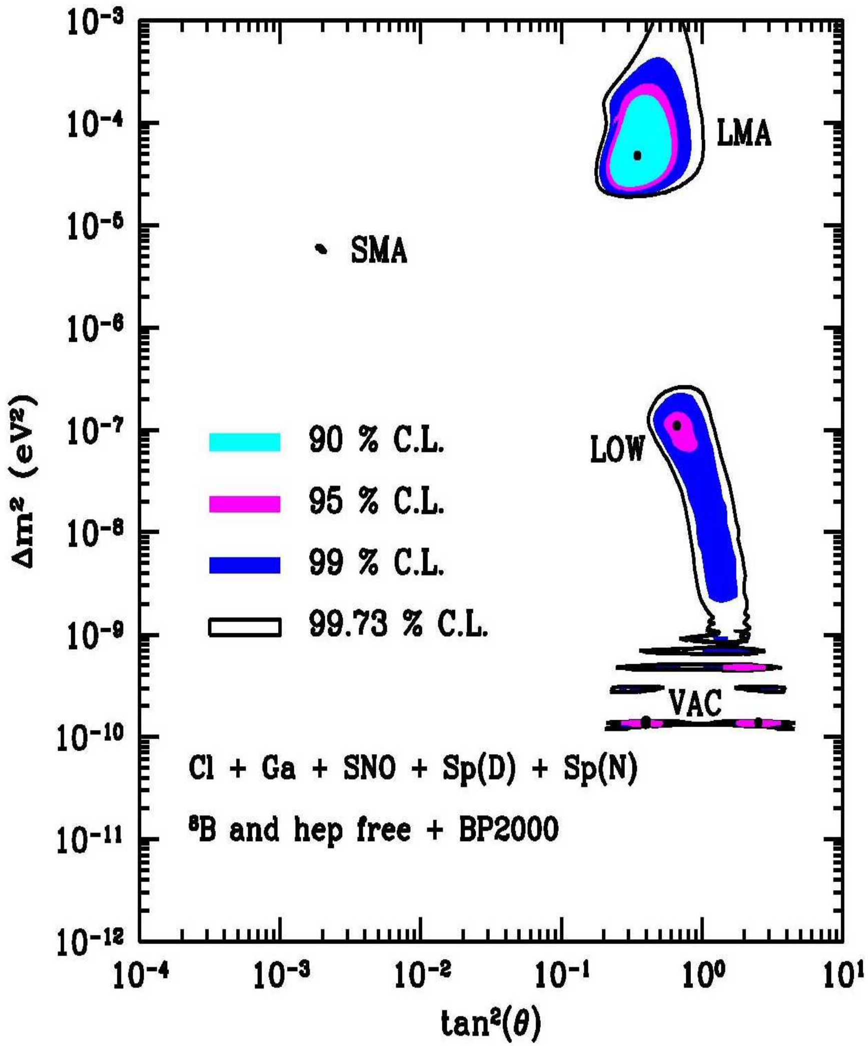
Observed total flux agrees with solar value

Global Fit after SNOW

Allowed regions of the oscillation parameters

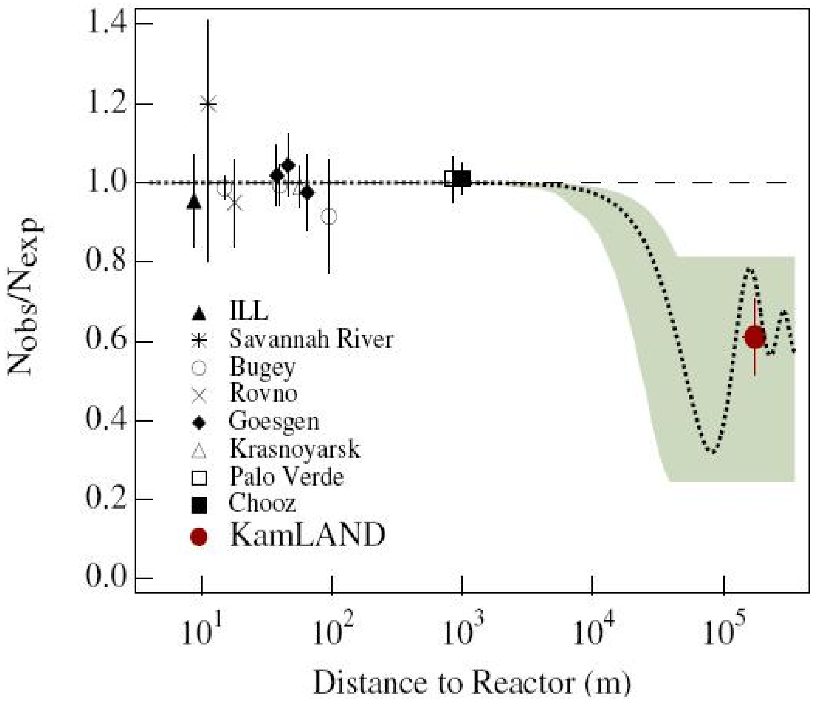
P Krastev, A.S

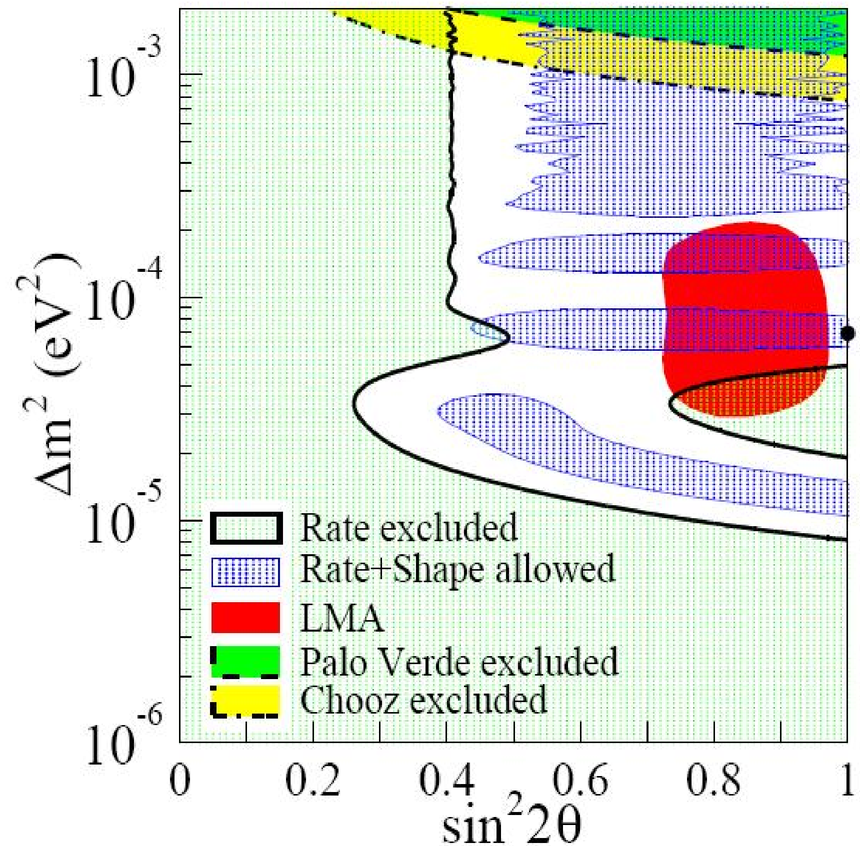




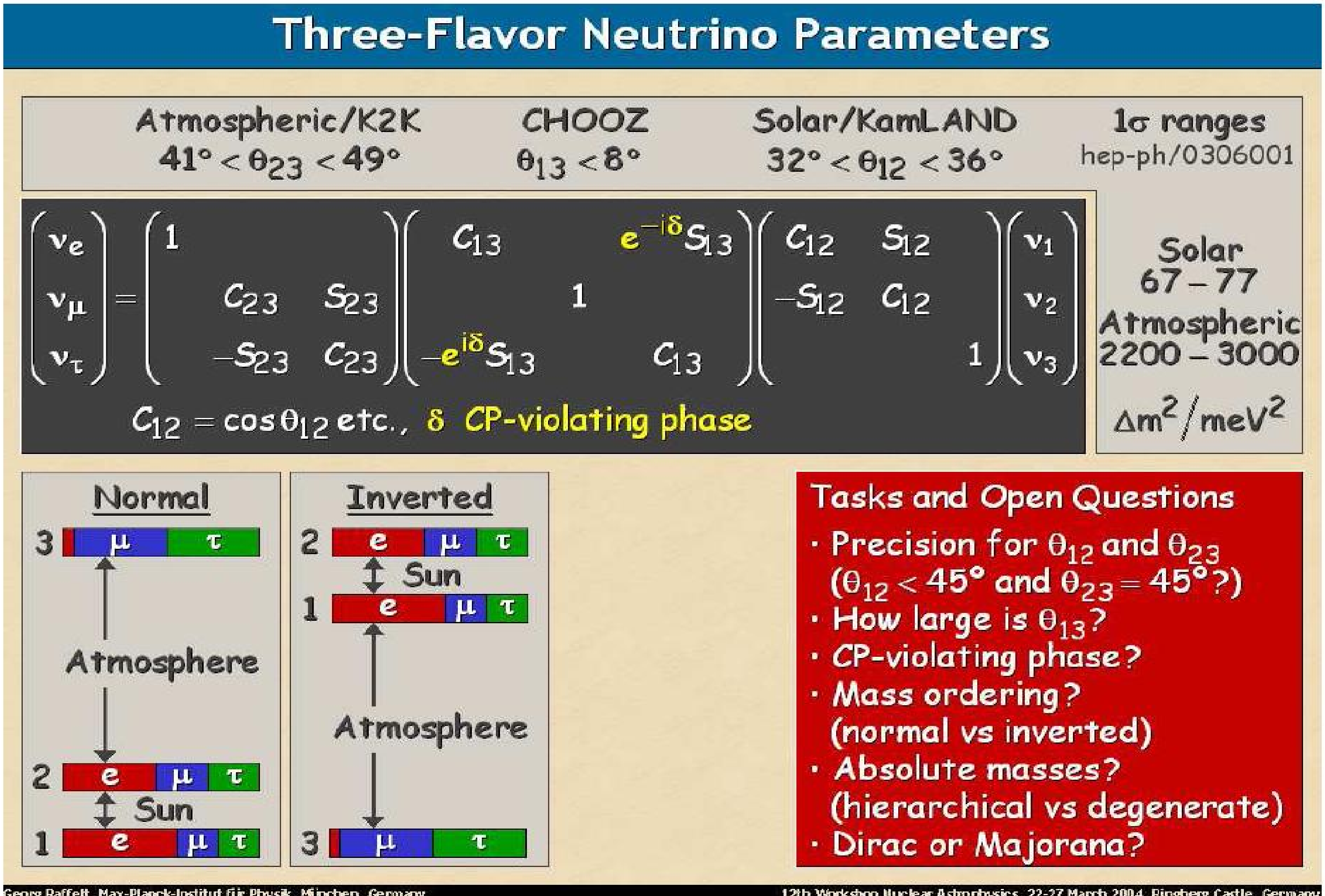
KamLAND confirmed Oscillations (2003)

(KamLAND: Detectors at various distances from Nuclear reactor)





Current Status (from Raffelt)



Best current Fits (KAMland vs. solar Neutrions)