Properties of Astrophysical Plasmas

- Basic Thermodynamics for Quantum Systems
- Distribution functions
- Thermodynamical Variables and Potentials
- Equations of State (General & limiting cases)

Why all the efforts ?

Results of nuclear processes depend on
- how the object (star, universe) reacts to nuclear reactions and
- on the competition between microscopic, nuclear reactions, and macroscopic, hydro and transport, system.

Basic Thermodynamical Quantities

Particle density $$n = \frac{N}{V} = \int_0^\infty \omega(p) f(p) dp$$

Energy density $$u = \frac{U}{V} = \int_0^\infty E\omega(p) f(p) dp$$

Pressure $$P = \frac{1}{3} \int_0^\infty pv\omega(p) f(p) dp$$

with
v: velocity, p: momentum, E: energy of state,
occupation probability $f(p)$,
state density $\omega(p)$ per unit volume

Quantum Dynamical Thermodynamical System: Stationary Schroedinger Equation

$$H\psi_{1,\dots n} = (\mathcal{T}+\mathcal{V})\psi_{1,\dots n} = E\psi_{1,\dots n}$$

kinetic E. $$\mathcal{T} = \sum_{i=1}^{n} t_i = \sum_{i=1}^{n} -\frac{\hbar^2}{2m}\Delta_i$$

potential $$\mathcal{V} = \sum_{i=1}^{n}\sum_{j>i}^{n} \mathbf{v}_{ij}.$$

$\left[-\frac{\hbar^2}{2m}\Delta+\mathbf{v}\right]\phi=\varepsilon\phi \quad \phi = X\cdot Y\cdot Z,$ spatial probability function of all particles

For non-interacting Particles:

t(i,j)=0 and v(i,j) = 0

Equations are separable !!!

State Density for non-interacting Gas

$$\left[-\frac{\hbar^2}{2m}\Delta+\mathbf{v}\right]\phi=\varepsilon\phi \qquad \phi = X\cdot Y\cdot Z,$$

thus (see deviation on blackboard)

Eigenvalues in energy in x,y,z $\varepsilon_{n_x,n_y,n_z} = \frac{\pi^2\hbar^2}{2md^2}(n_x^2+n_y^2+n_z^2) = \frac{\pi^2\hbar^2}{2md^2}R_\rho^2.$

Integrate over all points within radius R for an octant (factor 1/8):

Number of states $\Phi(E) = \frac{4\pi}{3}\frac{g}{h^3}(2m)^{3/2}E^{3/2}$

State density: $\omega(E) = 2\pi\frac{g}{h^3}(2m)^{3/2}E^{1/2}.$

Remark: $E = p^2/2m$

Occupation Probabilities

$$f(p) = \begin{cases} [e^{(E(p)-\mu)/kT} + 1]^{-1} & \text{Fermions} \\ [e^{(E(p)-\mu)/kT} - 1]^{-1} & \text{Bosons} \\ e^{-(E(p)-\mu)/kT} & \text{Maxwell-Boltzmann} \end{cases}$$

μ stands for the chemical potential without restmass. $\bar{\mu}$,

For equilibrium reactions between 2 states with C1/2/3/4 particles, we have

$$C_1\bar{\mu}_1 + C_2\bar{\mu}_2 = C_3\bar{\mu}_3 + C_4\bar{\mu}_4$$

$$\bar{\mu} = \mu + mc^2.$$

Thermodynamical Variables and Potentials

thermodynamic variable	differential
heat $Q(= q \cdot V)$	$dQ = TdS$
energy $U(= u \cdot V)$	$dU = TdS - PdV + \sum_i \bar{\mu}_i dN_i$
entropy $S(= s \cdot V)$	$dS = \frac{1}{T}dU + \frac{P}{T}dV - \sum_i \frac{\bar{\mu}_i}{T} dNi$
free energy $F = U - TS$	$dF = -SdT - PdV + \sum_i \bar{\mu}_i dN_i$
enthalpy $H = U + PV$	$dH = TdS + VdP + \sum_i \bar{\mu}_i dN_i$
free enthalpy $G = U - TS + PV$	$dG = -SdT + VdP + \sum \bar{\mu}_i dN_i$
thermodynamic potential $\Omega = U - TS - \sum_i \bar{\mu}_i N_i$	$d\Omega = -SdT - PdV - \sum_i N_i d\bar{\mu}_i$