The Final Exam

1. When, where and how

2. Material

3. Examples & your questions

When & Where & How & Materials?

- 1. Friday, 10am to noon(!) @ HCB309
- 2. 'Classical' Questions

(same style as midterm)

3. Tools: Paper, pencil & calculator, your notes, text-book, slides & wiki-pages (printed)

Materials to Help with the Prep.

- 1. Course book
- 2. Your transcripts
 Lecture notes
 Homework
- 3. If you need help or have questions, feel free to drop by my office

Overview of Nuclear Astrophysics

- 1/12 Introduction to Nuclear Physics
- 1/16 Methods of Abundance Determinations
- 1/19 Basic Thermodynamics of Plasmas
- 1/24 The Equation of State (EOS) and Implications for Stellar Structure
- 1/26 Basic Nuclear Physics and Decays
- 1/30 Nuclear Decays
- 2/2 Nuclear Reactions and Cross Sections
- 020/6 Prep. for 1st Midterm
- 2/13 Thermonuclear Reaction Networks
- 2/16 Nuclear Statistical Equilibrium
- 2/23 The Big Bang and Problems I
- 2/27 The Big Bang and Problems II
- 3/2 The Big Bang Nucleosynthesis
- 3/6 Stellar Structure and Evolution
- 3/16 The pp- and CNO Cycle
- 3/20 The Solar Neutrino Problem
- 3/23 Post-main-sequence: He burning
- 3/30 Late time Evolution
- 4/3 Core Collapse Supernovae I
- 4/6 Nucleosynthesis in Core Collapse SN II
- 4/10 Burning beyond Iron I
- 4/13 Burning Beyond Iron II
- 4/17 Thermonuclear Explosions
- 4/20 The rp-process (Nova and X-ray Bursters)

A Brief Guide to Study or How can I find things quickly?

- Definitions of basic quantities abundances, isotopes, tunneling, cross sections, rates, ..., magnitudes, luminosity big bang, stars, stellar exposions
- Basic concepts
 EOS, types of nuclear reactions, reaction networks, time scales & Co freeze out, equilibrium flows,
 QSE, NSE, time scales, waiting points
- Applications: Big bang, stellar evolution, explosive burning, H,He,C, Ne,O, Si, s-,p-,r-process

Homework-set 1

Recurrent Homeworks in Groups:

Every week, please provide a WIKI summary on the topic of the lectures of each week. It is due 7 days after the Friday lecture. For the weeks from 1/12-1/16 and 1/19-1/23, the Wiki pages are due on 1/23/2009 and 1/30/2009, respectively.

Please use the your notes, the slides, and the corresponding chapters of the text book as source of information. Feel free to add background information and be creative.

I hope this experiment works and will provide a useful tool to enhance learning, foster discussion and helps with the individual homeworks and exams.

For the topic, see class or the slides on Blackboard.

One time, individual homework due on Monday, 1/19/2009:

Go to the course Web-site, http://wiki.physics.fsu.edu, and write a note including one equation under 'Exercise PhysWiki':

If you have ideas or suggestions, please leave a comment under "Suggestions for Lecture Notes".

Individual Homeworks due on Monday, 2/2/2009:

 Show that the occupation probabilities of a photon gas (spin 1 particles, Boson's) in the momentum space p is given by

$$f(p) = \frac{1}{e^{E(p)/kT}-1}$$

(Remark: The momentum p of a photon p = E/c)

2) Derived a general expression for the pressure P of a non-relativistic Fermi Gas, and discuss the results for positive and negative chemical potentials in the limit of $|\mu|$ to be much larger than kT, and discuss the result. Estimate the radius of a neutron star.

Problem-set due on Monday, 2/16/2009:

Remark: Problems marked by + and * are for undergraduates and graduates, respectively.

- 1) Consider the chain of radioactive decays, $1 \to 2 \to \dots \to n$ with life times t_1, t_2 , to t_n , respectively with n being stable. Each decay of species i results in an energy release E_i . Assume that the abundance $N_i(t=0)$ is 0 for all i but $N_1(t=0) = N_o$.
- a) Setup and discuss the differential equations for n species.
- b1+) For n=3, write down the solution for $N_i(t)$ and E_i .
- b2*) For n species, write down the solution for $N_i(t)$ and E_i .
- c) The decay chain of ${}^{56}Ni \rightarrow {}^{56}Co \rightarrow {}^{56}Fe$ is the dominant energy source for light curves of thermonuclear supernovae. The life times are $t_6.1d$ and $t_2 = 77$ days.

Using the mass formula, estimate the radioactive energy $E_{1,2}$ released for the decay of nucleus 1 and 2, respectively.

Assume than a Supernovae produces 1/2 of a solar mass (2E33g) of ^{56}Ni during the explosion. Plot $N_i(t)$, the total energy generation E(t), and dE/dt(t) and discuss the results.

- 2) Calculate the transmission coefficient for a wave with energy E and a potentional with $V(x) = -V_o$ and V(x) = 0 for x less and greater than d, respectively. Consider the cases of $E \leq 0$ and $E \geq 0$, and discuss the result.
- 3) Show that, for direct and electromagnetic transitions $i \to f$, the S(E) factor is constant for s-waves (l=0) (Hint: see transmission coefficients), and discuss the results.
- *) What are the functional forms for S(E) for (l=1,2)? Discuss the results including the low energy limit.
- +) What are resonant reactions, and name two examples.
- 4+) How much energy is released in the following reactions: (i) ${}^3He(d,p){}^4He$; (ii) ${}^{17}O(p,\gamma){}^{18}F$, (iii) ${}^{12}C(\alpha,\gamma){}^{16}O$.

Problems are due on Monday, 3/2/2009:

- 1) Derive the rate for non-resonant, charged particle reaction rates at low energies (Hint: s-wave contributions dominate). Derive the expression for the energy E_o of the Gamov peak, and its width Δ . Determine E_o and its width (in MeV) for the reaction $^{12}C(\alpha, \gamma)^{16}O$. Above which temperature does photodisintegration becomes important? Why is this cross section uncertain despite its importance in Astrophysics?
- 2) For high temperatures, the abundances are independent from all rates electron capture. Why? Which isotopes are most abundant for the (T, ρ) pairs (in CGS) of (5E9,1E9), (5E9,1E4), and (1E10,1E4) for a $Y_e = \frac{p}{p+n}$ of a) 0.5, b) 0.48, and c) 0.45, respectively.
- 3) For high temperatures, the abundances are independent from all rates electron capture. Why? Which isotopes are most abundant for the (T, ρ) pairs (in CGS) of (5E9,1E9), (5E9,1E4), and (1E10,1E4) for a $Y_e = \frac{p}{p+n}$ of a) 0.5, b) 0.48, and c) 0.45, respectively.

1st Homeworkset 4 due on Monday 3/16/2009

Rem.: Solve all problems. For specific questions, feel free to provide a copy of your midterm.

- (1) 1 g of a dense electrially neutral plasma contains $610 \text{mg}^{-1}H$, 150 mg of ^{4}He , 140 mg of ^{15}N , and 100 mg of ^{14}N . The mass density is constant 1E2g in the inner core of a massive star.
- (a) Make a table with particle fractions $X_i = \frac{n_i}{\sum_j n_j}$, Y_i and relative abundances ϵ (log scale). What are their meanings.
- (b) Calculate the electron fraction Y_e .
- (c) At very high temperatures, N will be destroyed by the following reactions: ^{15}N will be destroyed by proton capture followed by an α emission. ^{14}N captures one proton.
- c1) Write down the chain of reactions in the form a(b,c)d, and mark the flow in the figure attached.
- c2) After all N is destroyed, write down the mass fraction, abundance (log scale) and particle fraction.
- c3) Does Y_e change? Explain why, and if it did, give the new value.
- 2) At low temperatures, we have a mixture of ¹⁵O, ²⁶Al, H and He with mass fractions of 0.02, 0.02, 0.70 and 0.26, respectively.
- a) Calculate Y_e for the initial mixture
- b) Does the composition change with time? If yes, why and write down and discuss the chain of reactions. Mark the reaction flow in the figure attached.
- c) What is the composition and mass fraction after about 1 day and 10Gyears, respectively. Does Y_e change with time, and give $Y_e(1d)$ and $Y_e(10Gyrs)$ (see figure).

5th Homeworkset due on Monday 3/30/2009

Rem.: This homework set covers topics relevant for the 2nd Midterm. Write down the steps.

- (1a) Use a table to compare 5 properties of the early with those in the current Universe.
- (b) Explain the term 'inflation' in context of the early Universe.
- (c) Why are predictions for the Big Bang nucleosynthesis more accurate than those for stars?
- (d) The early Universe is hot and dense. Why don't we see elements beyond ⁷Li?
- (e) In the lesson we have seen that the ratio between neutrino and photon background temperature $T_{\gamma}/T_{\nu} = 1.4$ with $T_{\gamma} = 2.7K$. Estimate the number density of the neutrino's per flavor. Why can we infer from the nucleosynthesis of the early universe that we have three neutrino flavors?
- (f) Write down the system of rate equations to be used for the early Universe.
- (2) Use a graph to show the density and temperature conditions needed for breaking out of the CNO cycle via a) the $^{15}O(\alpha, \gamma)$ reaction and b) via the $^{18}Ne(\alpha, p)$ reaction for proton and neon abundances of 0.66 and 8.4E-2, respectively.
- (3) In the first neutrino experiment, the measured neutrino solar fluxes at the earth have been smaller by a factore of about 3 compared to those expected from the luminosity of the sun. Possible solutions have considered have been astrophysical, nuclear, and in the properties of neutrino's.
- (a) However, possible solutions discussed have been astrophysical, nuclear, and in the properties of neutrino's. Explain how each of them could have solved it, and which experiments/measurements have helped to solve the puzzle.
- (b) Nowadays, neutrino oscillarions $\nu_e \to \nu_\mu$ have been identified as the solution with $\delta m^2 = 5E 5eV^2$ and a mixing angle $sin\theta_\nu = 30^\circ$. KAMLAND is a large liquid scintilator detector that measures $\bar{\nu}_e$ produced by reactors adn 100 to 1000 km. Assume a $\bar{\nu}$ energy of 5MeV and plot the measured $\bar{\nu}$ as a function of distance. (Hint: Ignore matter effects).
- (c) Calculate the density at which a 1MeV solar neutrino will become resonant in the sun (see lesson).
- (d) We may expect a 'day-night' effect if the solar neutrinos are measured during the day or night. Calculate the minimum neutrino energy that becomes resonant in the core of the Earth with $\rho \approx 13g/cm^3$. Would we expect that solar neutrino's from the ⁸B reactions are strongly effected?
- (4) In the sun, the CNO cycle provides only $\approx 1\%$ of the total luminosity. The nuclear flow of the CNO cycle is governed by $^{14}N(p,\gamma)^{15}O$, whereas, for the pp-cycle, it the $p(p,e^+\nu)D$ reaction is the bottleneck.

In terms of reactions, why do these reactions dominate the CNO-cycle and pp-cycle, respectively?

Assume an ^{14}N abundance of 0.01, and the S-factors are $S_{pp} = 4E - 22$ and $S_{pN} = 3$ in MeV and barns. Why can we assume for the sun that the S-factors do not depend on T? Calculate the temperature T at which both processes become comparable (show the calculation). Estimate the mass of a star where this happens (HINT: Assume that the central temperature of the sun is 15MK and a one zone model).

6st Homeworkset due on Monday 4/15/2009

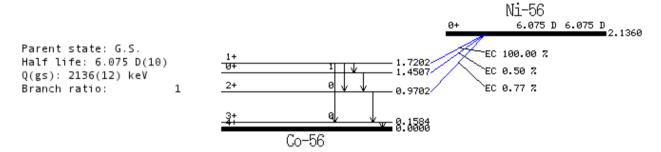
- Describe the role of radioactive decay of ⁵⁶Ni and subsequent decays with respect to supernova light curves.
- a) For ^{56}Ni , calculate the Q-values for β^+ decay, β^- decay, p-decay, n-decay, and α -decay.
- b) For ^{56}Co , calculate the Q-values for β^+ decay, β^- decay, p-decay, n-decay, and α -decay.
- (2) In a core collapse supernovae after the formation of a neutron star, a prompt shock wave forms with an energy of about 3E51erg which exceeds the potential binding energy of the envelope of star.

Why does this shockfront 'stalls' and does not result in the ejection of the envelope?

Estimate the distance (in mass coordinates) at which the shock front stalls.

- (3) A thermonuclear supernovae is the explosion of a white dwarf and is powered by a complete nuclear burning of a C/O white dwarf with a radius of 1600 km and a mass of about 1.35 M_{\odot} . Assume a C/O ratio of 1.
- a) Estimate the central density of the WD?
- b) The C/O ratio is about 1 for He-shell burning whereas it becomes smaller than 1 during the He-core burning of the progenitor. Why?
- c) Does the mean C/O ratio depend on the main sequence mass of the progenitor and, if, why? Estimate the radius change if the average C/O is reduced to 0.6.
- d) Estimate the nuclear energy release and total kinetic energy of the explosion for a C/O ratios 1 of 0.6, and discuss the result.
- (4) The s-process branches at ¹⁴⁷Nd because it has a β-decay half life of 5.3 days, which is much larger than neutron capture time scales during stellar burning.
- a) Why are ^{148}Sm and ^{150}Sm only produced by the s-process? Use the fraction of the reaction flow that branches into ^{147}Nd during the s-process to produce solar abundances (Hint: Use the steady flow approximation).
- b) Use the measured neutron capture rate for ^{147}Nd (see Iliadis) at temperatures kT=30 keV of 2E7 $cm^3/s/mole$ to determine the neutron density during the s-process.
- (5) Calculate the relative abundances of the isotopes along the Ba isotopic chain during the r-process at temperatures of 1.5 GK and neutron densities of $10^{24}cm^{-3}$ assuming $(n, \gamma) (\gamma, n)$ equilibrium. Set the partition functions equal to the one shown in the lesson.
- a) Give the relative particle abundances of the 5 most abundant isotopes
- b) Plot and discuss the result as a function of neutron numbers.

56NI EC DECAY



Beta+ ray: total intensity =1.32e-03

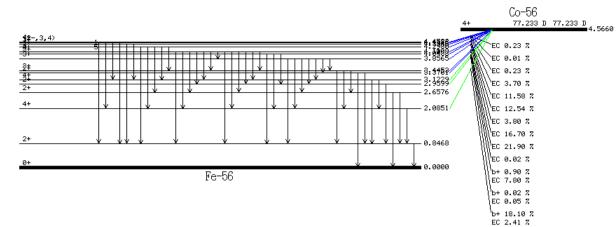
| Max.E(keV) | Avg.E(keV) | Intensity(rel) | Spin | 0+ |
|------------|------------|----------------|------|----|
| 1114.0(-) | 478(5) | 6.0E-5(LT) | | 4+ |
| 955.6(-) | 408(5) | 5.8E-5(LT) | | 3+ |
| 143.7(-) | 65(5) | 1.2F-3(LT) | Spin | 2+ |

EC: total intensity = 101.3

Gamma ray:

```
Energy (keV) Intensity(rel)
158.38(3) 98.8(10)
269.50(2) 36.5(8)
480.44(2) 36.5(8)
749.95(3) 49.5(12)
811.85(3) 86.0(9)
1561.80(5) 14.0(6)
```

HUO JUNDE, Nuclear Data Sheet 86, 315 (1999)



Parent state: G.S. Half life: 77.233 D(27) Q(gs): 4566.0(20) keV Branch ratio: 1.0

Beta+ ray: total intensity =19.0

| Max.E(keV) | Avg.E(keV) | Intensity(rel) | Spin | 4+ |
|------------|------------|----------------|------|----|
| 1458.9(-) | 631.2(9) | 18.1(9) | | 4+ |
| 584.1(-) | 247.1(9) | 0.017(6) | | 2+ |
| 421.1(-) | 178.8(9) | 0.90(5) | | 4+ |

Intensity(rel)

EC: total intensity = 81.0

Gamma rav:

Energy(keV)

```
263.41(10)
                0.022(4)
  411.38(8)
                0.025(5)
  486.54(11)
                0.061(10)
  655.0
                0.038(8)
  674.7
                0.038(7)
 733.511(5)
                0.190(7)
787.742(7)
                0.315(10)
846.771(4)
               99.935(25)
 852.78(5)
                0.050(3)
 896.531(12)
                0.086(20)
977.373(4)
                1.449(15)
 997.33(16)
                0.129(14)
1037.840(6)
                14.17(13)
1089.03(24)
                0.050(3)
1140.404(21)
                0.137(5)
1159.920(18)
                0.095(13)
1175.102(6)
                2.288(21)
1198.78(20)
                0.051(9)
1238.282(7)
                 66.9(6)
 1272.2(6)
                0.024(10)
                0.118(6)
1335.389(20)
1360.215(12)
                 4.29(4)
1442.75(8)
                0.185(7)
1462.34(12)
                0.065(8)
1640.404(21)
                0.072(12)
1771.351(16)
                15.47(14)
1810.772(17)
                0.638(8)
1963.714(12)
                0.724(10)
2015.181(16)
                 3.04(5)
2034.755(13)
                 7.89(13)
2113.123(10)
                0.376(10)
2212.933(18)
                0.395(14)
 2276.36(16)
                0.128(19)
 2373.7(4)
                0.082(22)
2523.86(20)
                0.068(11)
2598.459(13)
                 17.3(3)
 2657.4(8)
                0.021(6)
3009.596(7)
                 1.16(3)
3201.962(16)
                 3.32(7)
3253.416(15)
                 8.12(17)
3272.990(15)
                 1.93(4)
3369.69(30)
                0.011(2)
3451 152(17)
                0.972(20)
```